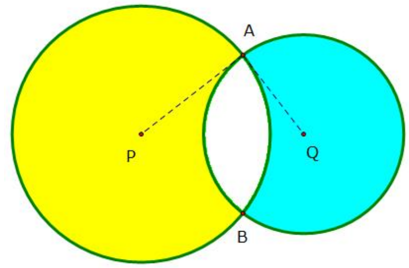
kg to 7 kg in exactly one way in either case. For example, using 1st case this is the only possible combination of weights to measure 1 kg to 7 kg: 1 = 1, 2 = 2, 3 = 1 + 2, 4 = 4, 5 = 1 + 4, 6 = 2 + 4, and 7 = 1 + 2 + 4. So find the number of ways a block of 14 kg can be broken under similar conditions e.g. {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1} is a valid case but {1, 2, 3, 4, 4} is not.

There are exactly 3 ways including the one mentioned in the problem statement. The other two are: {1, 1, 1, 1, 5, 5}, {1, 1, 3, 3, 3, 3}.



109. I am twice as old as you were when I was as old as you are. What is the ratio of ages of mine and yours?

4 : 3.

110. Circles with centers P and Q have radii 20 and 15 cm respectively and intersect at two points A, B such that ÐPAQ = 90°. What is the difference in the area of two shaded regions?

The required difference is = $\pi(20^2 - 15^2) = 175\pi$.

111. What is the largest integer that is a divisor of

$$(n + 1)(n + 3)(n + 5)(n + 7)(n + 9)$$

for all positive even integers n?

Given expression is product of five consecutive odd numbers, so it's always divisible by $1 \times 3 \times 5 = \boxed{15}$.

112. For how many ordered pairs of positive integers (x, y), xy/(x+y) = 9?

Simplifying we get, $(x - 9)(y - 9) = 81$. As $81 = 3^4$ has 5 positive integral divisors, number of ordered pairs of positive integers (x, y) is $\boxed{5}$.

113. Amu, Bebe, Chanda and Dori played with a deck of 52 cards. In one game, Dori was dealing out the cards one by one to the players, starting with Amu, followed by Bebe, Chanda and Dori in this order, when suddenly some of the cards she had not dealt out yet slipped out of her hands and fell on the floor. The girls noticed that the number of cards on the floor was 2/3 of the number of cards Amu had already got, and the number of cards that Chanda had got was 2/3 of those in the remaining part of the deck in Dori's hand that she had not dealt out yet. How many cards had Dori dealt out altogether?

Amu and Bebe had 9 cards each while Chanda and Dori had 8 cards each. 6 cards were fallen off and 12 are yet to be dealt. So altogether $9 + 9 + 8 + 8 = \boxed{34}$ cards had been dealt.

114. In a city, 2/3 of the men and 3/5 of the women are married. (Everyone has one spouse and the spouses live in the same city.) What fraction of the inhabitants of the city is married?

$\boxed{12/19}$.

115. The sum of all interior angles of eight polygons is 3240°. What is the total number of sides of polygons?

Let the number of sides in the polygons are: $n_1, n_2, \ldots, n_8$. So sum of all interior angles will be $(n_1 - 2)180° + (n_2 - 2)180° + \ldots + (n_8 - 2)180° = (n_1 + n_2 + \ldots + n_8)180° - 16*180° = 3240° = 18*180°$. So total number of sides of all polygons is simply $18 + 16 = \boxed{34}$. Remember that sum of all interior angles of an n-gon is = $(n - 2)180°$.

116. Consider a triangle ABC with BC = 3. Choose a point D on BC such that BD = 2. Find the value of

$$AB^2 + 2AC^2 - 3AD^2.$$

Using Stewart's theorem, we have $(1 \times AB^2) + (2 \times AC^2) = (1 + 2)(AD^2 + 1 \times 2)$. So $AB^2 + 2AC^2 - 3AD^2 = \boxed{6}$. Alternately we can use Pythagoras Theorem multiple times to get the result.

117. Determine the number of divisors of 2012^8 that are less than 2012^4.

$2012 = 2^2 \times 503$. So number of divisors of $2012^8 = 17 \times 9 = 153$ and less than $2012^4 = (153 - 1)/2 = \boxed{76}$ as 2012^4 is square root of 2012^8.

118. How many numbers in the following sequence are prime numbers?

{1, 101, 10101, 1010101, 101010101, ……}

When numbers of 1's in the number is even, then number is divisible by 101 for sure but for odd number of 1's, the number can be factorised as $\underbrace{10101...1}_{2n+1 \text{ times } 1's} = \underbrace{1111..111}_{2n+1 \text{ times } 1's} \times \underbrace{909090..9090}_{n-1 \text{ times } 90's}91$. So there is only $\boxed{1}$ prime number in the sequence i.e. 101.

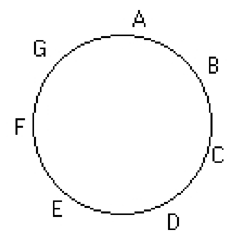
119. How many triples of natural numbers (a, b, c) such that a, b and c are in geometric progression, and a + b + c = 111.

There are total $\boxed{5}$ triplets and the solution triples are (37; 37; 37); (1; 10; 100); (100; 10; 1); (27; 36; 48); (48; 36; 27).

120. What is the smallest integer n for which $\sqrt{n} - \sqrt{n-1} < 0.01$?

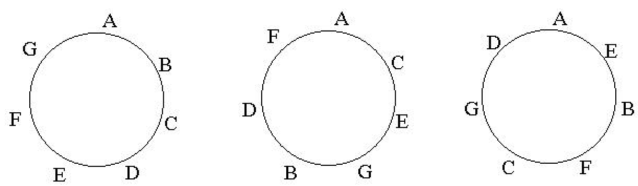
Given expression can be written as $\sqrt{n} + \sqrt{n-1} > 100$ i.e. $\sqrt{n} > 50 > \sqrt{n-1}$. So the smallest such n is = $\boxed{2501}$.

121. Seven people, A, B, C, D, E, F and G can sit down for a meal at a round table as shown. Each person has two neighbours at the table: for example, A's neighbours are B and G. There are other ways in which the people can be seated round the table. Last month they dined together on a number of occasions, and no two of the people were neighbours more than once. How many meals could they have had together during the month?



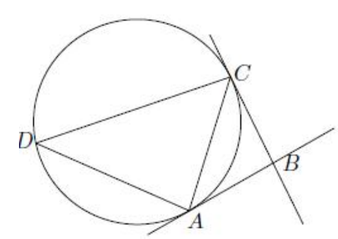
There are total 6!/2 = 3(5!) number of ways of seating arrangement as A-B is same as that of B-A. Also when A, B are together, they have been counted (5!) times. But as this pair has to be counted once only, we need to divide the total arrangements by (5!) to get the desired arrangements as = 3(5!)/(5!) = 3.

To show that it is possible to have had 3 meals together, a possible seating plan is:



Hence the greatest number of meals they could have had together during the month is $\boxed{3}$.

122. Points A, D and C lie on the circumference of a circle. The tangents to the circle at points A and C meet at the point B. If ∠DAC = 83° and ∠DCA = 54°. Find ∠ABC.



∠ADC = 180° - ∠DAC - ∠DCA = 180° - 83° - 54° = 43° = ∠ACB = ∠CAB (Alternate segment theorem).

So ∠ABC = 180° - 2(43°) = $\boxed{94°}$.

123. How many 4-digit numbers uses exactly three different digits?

Required 4-digit numbers will be of just all arrangements of a number of the form 'aabc' which has three distinct digits; a, b and c. So number of such numbers is given by $(9/10)(^{10}C_3)(^3C_1)(4!/2!) = \boxed{3888}$.

124. The ratio of two six digit numbers abcabc and ababab is 55 : 54. Find the value of a + b + c.

$$\frac{abcabc}{ababab} = \frac{1001 \times abc}{10101 \times ab} = \frac{7 \times 11 \times 13 \times abc}{3 \times 7 \times 13 \times 37 \times ab} = \frac{11 \times abc}{3 \times 37 \times ab} = \frac{55}{54}.$$

=> $\frac{abc}{ab} = \frac{5 \times 37}{18} = \frac{185}{18}$. So a + b + c = 1 + 8 + 5 = $\boxed{14}$.

125. Find the infinite sum:

$$\mathbf{\frac{1}{4} + \frac{1}{8} + \frac{2}{16} + \frac{3}{32} + \frac{5}{64} + \frac{8}{128} + \frac{13}{256} + \frac{21}{512} + \ldots}$$

Let S = $\frac{1}{4} + \frac{1}{8} + \frac{2}{16} + \frac{3}{32} + \frac{5}{64} + \frac{8}{128} + \frac{13}{256} + \frac{21}{512} + \ldots$

So ½ S = $\frac{1}{8} + \frac{1}{16} + \frac{2}{32} + \frac{3}{64} + \frac{5}{128} + \frac{8}{256} + \frac{13}{512} + \frac{21}{1024} + \ldots$

Subtracting the two we get, ½ S = $\frac{1}{4} + \frac{1}{16} + \frac{1}{32} + \frac{2}{64} + \frac{3}{128} + \frac{5}{256} + \frac{8}{512} + \ldots = \frac{1}{4} + \frac{1}{4}S$.

Hence S = $\boxed{1}$.

126. What is the probability of tossing a coin 6 times such that no two consecutive throws result in a head?

If I write the favorable number of outcomes in a sequence for 1 toss, 2 tosses, 3 tosses, 4 tosses,….., there is a pattern. Just try to observe it. Let F(n) gives the favorable number of outcomes for n tosses and T(n) denotes total possible outcomes of n tosses. It is clear that T(n) = 2^n. Now sequence of F(n) for n = 1, 2, 3, … is as follows: 2, 3, 5, …. It can be clearly inferred that F(n) is a Fibonacci sequence and F(6) = 21. So the required probability = F(6)/T(6) = $21/2^6$ = $\boxed{21/64}$.

127. In how many ways 3 letters can be selected from 3 identical A's, 3 identical B's and 3 identical C's?

It is simply the whole number solutions of A + B + C = 3 i.e. $^5C_2$ = $\boxed{10}$.

128. For how many pairs of positive integers (x, y) both x^2 + 4y and y^2 + 4x are perfect squares?

Let x ≤ y, then $y^2 < y^2 + 4x < y^2 + 4y < (y + 2)^2$.

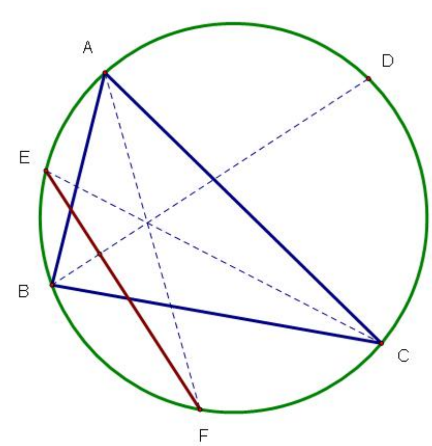
So $y^2 + 4x = (y + 1)^2$ i.e. x = (2y + 1)/4 which is never possible. Hence $\boxed{0}$ pairs are there.

129. Jar X contains six liters of a 46% milk solution; Jar Y contains three liters of a 43% milk solution and Jar Z contains one liter of p% milk solution. q/r liters of solution from Jar Z is transferred to Jar X and remaining solution from Jar Z is transferred to Jar Y such that resulting two solutions both contain 50% milk solution. Also q and r are positive integers co-prime to each other. Find the value of p + q + r.

Total quantity of solution is 6 + 3 + 1 = 10 liters out of which 50% i.e. 5 liters is milk. So 0.46*6 + 0.43*3 + p/100 = 5. Simplifying we get *p = 95*.

Now let 'n' liters has been transferred from Jar Z to Jar X, so we get 0.46*6 + 0.95*n = 0.5(6 + n). Simplifying we get n = *q/r = 8/15*.

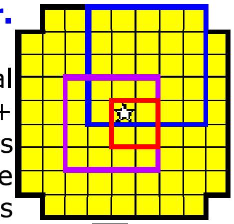
So p + q + r = 95 + 8 + 15 = $\boxed{118}$.



130. ABC is an isosceles right triangle inscribed in a circle such that ∠B = 90°. BD, CE and AF are angle bisectors of triangle ABC as shown. What is the measure of smaller angle of intersection of BD and EF?

Let I be the incenter and O be the intersection of BD and EF. In ▲IEO, ∠IEO = ∠CEF = ∠CAF = 22.5°. And ∠EIO = ∠ICB + ∠IBC = 22.5° + 45° = 67.5°. Hence ∠EOI = 180° - 22.5° - 67.5° = 90°. So EF ⊥ BD and required angle is $\boxed{90°}$.

131. In the diagram, three squares are shown, all containing the star. Altogether, how many squares containing the star can be found in the diagram?



If the given grid would have been a complete 9 × 9 grid, then because of symmetrical placement of star, required number of squares would have been $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2$ counting squares of unit length, two units, … up to nine units respectively. But as the grid given is not a complete square, we need to subtract the number of squares formed using the corner ones i.e. 4 + 4 + 4 + 4 + 1 counting squares of side-length five units, six units,.. nine units respectively. So the final answer comes out to be 68.

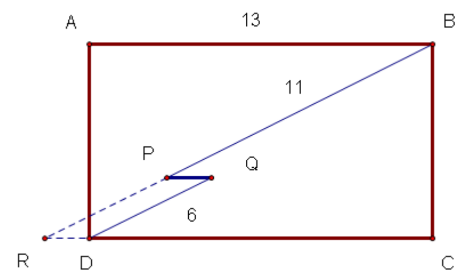
132. Find all integers x, y, z (such that x < y < z) greater than 1 for which xy - 1 is divisible by z, yz - 1 is divisible by x, and zx - 1 is divisible by y.

First point to note is that x, y, z are all co-prime. Also $(xy - 1)(yz - 1)(zx - 1) \equiv 0 \bmod(xyz)$

- ⇨ $(xyz)^2 - xyz(x + y + z) + xy + yz + zx - 1 \equiv 0 \bmod(xyz)$
- ⇨ $xy + yz + zx \equiv 1 \bmod(xyz)$
- ⇨ $xy + yz + zx > xyz$
- ⇨ $1/x + 1/y + 1/z > 1$

Only one triplet $(x, y, z) \in (2, 3, 5)$ is possible.

133. In a rectangle ABCD, AB = 13 and BC = 8. PQ lies inside the rectangle such that BP = 11, DQ = 6, AB||PQ, and BP||DQ. Find the length of PQ.



First draw the diagram as shown (no other placement of PQ is possible). Extend BP to meet CD at R such that PQDR becomes a parallelogram with PQ = RD and DQ = PR = 6.

In right angle triangle BCR, CR = $\sqrt{BR^2 - BC^2} = \sqrt{17^2 - 8^2}$ = 15, so RD = PQ = CR − CD = 15 − 13 = 2.

134. What are the last two digits of the sum obtained by adding all the possible remainders of numbers of the form 2^n, n being a non-negative integer, when divided by 100?

We are to find remainder of a GP of 2 with 100. Important point to note is that there will be a power of 2 after which the last two digits will start repeating. Let's say $2^a \equiv 2^b$ mod25, that means $2^{(a-b)} \equiv 1$ mod25 or there exists a *k* such that $2^k - 1$ is multiple of 25.

Now the number we are finding = N = $2^0 + 2^1 + 2^2(1 + 2 + \ldots + 2^{k-1})$ mod100 ≡ (1 + 2) mod100 ≡ 03.

135. On planet LOGIKA, there live two kinds of inhabitants; black and white ones and they answer every question posed to them in a Yes or No. Black inhabitants of northern hemisphere always lie while white inhabitants of northern hemisphere always tell the truth. Also white inhabitants of southern hemisphere always lie while black inhabitants of southern hemisphere always tell the truth. On a dark night, there is an electricity failure and you meet an inhabitant without knowing your location on the planet. What single yes/no question can you ask the inhabitant to determine color of the inhabitant?

Are you a northerner? If he answers in Yes, he is White otherwise he is Black.

136. N is product of first 50 prime numbers. A is a factor of N and B is a factor of A. How many ordered pairs (A, B) exist?

Every prime number has three possibilities (1: contained in A & B, 2: contained in A and not in B, not contained in either of A, B). So number of ordered pairs of (A, B) = 3^{50}.

137. For how many positive integers, N > 2, (N - 2)! + (N + 2)! Is a perfect square?

(N - 2)! + (N + 2)! = [(N + 2)(N + 1)(N)(N - 1) + 1](N - 2)!

We know that product of any four consecutive natural numbers is always 1 less than a perfect square. So the term in [] is always a perfect square and for the complete expression to be a perfect square – (N

- 2)! has to be a perfect square which is only possible for N = 3 as N has to be greater than 1. Hence there is only $\boxed{1}$ value of N which makes the expression a perfect square.

138. Let P = $\frac{1}{1\times2}+\frac{1}{3\times4}+\frac{1}{5\times6}+\ldots+\frac{1}{2013\times2014}$
and Q = $\frac{1}{1008\times2014}+\frac{1}{1009\times2013}+\frac{1}{1010\times2012}+\ldots.+\frac{1}{2014\times1008}$. Find P/Q.

P = $\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots+\left(\frac{1}{2013}-\frac{1}{2014}\right)$

P = $\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{2014}\right)-2\left(\frac{1}{2}+\frac{1}{4}+\ldots+\frac{1}{2014}\right) = \frac{1}{1008}+\frac{1}{1009}+\frac{1}{1010}+\ldots+\frac{1}{2014}$.

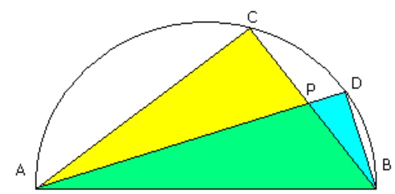
So 2P = $\left(\frac{1}{1008}+\frac{1}{2014}\right)+\left(\frac{1}{1009}+\frac{1}{2013}\right)+\left(\frac{1}{1010}+\frac{1}{2012}\right)+\ldots+\left(\frac{1}{2014}+\frac{1}{1008}\right)$

Or 2P = $3022\left(\frac{1}{1008\times2014}+\frac{1}{1009\times2013}+\frac{1}{1010\times2012}+\ldots+\frac{1}{2014\times1008}\right)$ = 3022Q.

Hence P/Q = $\boxed{1511}$.

139. C and D are two points on a semicircle with AB as diameter such that AC – BC = 7 and AD – BD = 13. AD and BC intersect at P. Find the difference in area of triangles ACP and BDP.

Difference in area of triangles ACP and BDP is same as difference in area of triangles ACB and BDA which is ½ *(AC*BC – AD*BD).



In ▲ABC, $(AC - BC)^2 = AC^2 + BC^2 - 2AC*BC = AB^2 - 2AC*BC = 7^2 = 49$.
And in ▲ABD, $(AD - BD)^2 = AD^2 + BD^2 - 2AD*BD = AB^2 - 2AD*BD = 13^2 = 169$.
Subtracting the above two equations we get, 2(AC*BC – AD*BD) = 169 – 49 = 120.
So ½ (AC*BC – AD*BD) = ¼ (120) = $\boxed{30}$.

140. TG *fashions* hold its annual sale on the eve of Pi-day (14th March) and offered a discount of 90% on all its apparels. But this month it is offering the usual 80% discount. How much percent more I need to pay now than that on the annual sale's eve for purchase of similar clothing?

Earlier I need to pay 10% of the price and now I'll have to pay 20% i.e. $\boxed{100\%}$ more price is to be paid.

141. In how many ways 1,000,000 can be expressed as sum of a square number and a prime number?

Let 1,000,000 = $1000^2 = a^2 + p$ where p is a prime number.
So p = $1000^2 - a^2 = (1000 - a)(1000 + a)$.
As LHS is a prime number, (1000 – a) should be equal to 1 and s = 999. Now p comes out to be 1999 which is a prime number. So there is only $\boxed{1}$ desired way; 1,000,000 = $999^2 + 1999$.

142. How many ordered triples of three positive integers (a, b, c) exist such that $a^3 + b^3 + c^3 = 2011$?

For a positive integer N, $N^3 \equiv 0$ or ±1 mod9. But $2011 \equiv 4$ mod9. So LHS can never be equal to RHS. Hence $\boxed{0}$ triples are there.

143. In a quadrilateral ABCD, sides AD and BC are parallel but not equal and sides AB = DC = x. The area of the quadrilateral is 676 cm². A circle with centre O and radius 13 cm is inscribed in the quadrilateral such that it is tangent to each of the four sides of the quadrilateral. Determine the length of x.

Given quadrilateral ABCD is an isosceles trapezium. Equating the lengths of common tangents in the trapezium, we get x = ½ (sum of parallel sides).
Also a key point to observe is that diameter of circle is acting as height of the trapezium which is 26cm. We know that area of a trapezium = ½ (sum of parallel sides) (height of trapezium).
So putting all knowns and looking for unknowns, we get that 26x = 676 and x = 26.

144. Kiran, Shashi and Rajni are Kiran's spouse, Shashi's sibling and Rajni's sister-in-law in no particular order. Also Kiran's spouse and Shashi's sibling are of same sex. Who among the three is a married male?

Rajni's sister-in-law is certainly female. And other two are of same sex. Certainly they cannot be females (because there is a married couple also). So Kiran's spouse and Shashi's sibling are male. That means Kiran is a female while Rajni and Shashi are males. And Shashi is the married male i.e. husband of Kiran.

145. A and B start running from two opposite ends of a 1000m racing track. A and B travel with a speed of 8m/s and 5m/s respectively. How many times they meet, while running, in first 1000s after start?

In first 1000s after start, A takes 8 end-to-end journey while B takes 5. In every one of his end-to-end journey, A meets B exactly once. So, in all, they meet 8 times including the final meeting after 1000s.

146. According to death-will of Mr. Ranjan, all of this money was to be divided among his children in the following manner: `N to the first born plus 1/17 of what remains, `2N to the second born plus 1/17 of what then remains, `3N to the third born plus 1/17 of what then remains, and so on. When the distribution of the money was complete, each child received the same amount and no money was left over. Determine the number of children.

Let there be total x children, then equating the amount received by $(x-1)^{th}$ and x^{th} child we get, (x - 1)N + xN/16 = xN. Simplifying we get x = 16.

147. One number is removed from the set of integers from 1 to n. The average of the remaining numbers is 40.75. Which integer was removed?

Total numbers should a number of the form 4k + 1 near 80. So most probable such number is 81. Sum of first 81 natural numbers is 3321. Sum of given 80 numbers is 40.75*80 = 3260, so number 61 was removed.

148. What is the 2037^{th} positive integer that can be expressed as the sum of two or more consecutive positive integers? (The first three are 3 = 1+2, 5 = 2+3, and 6 = 1+2+3.)

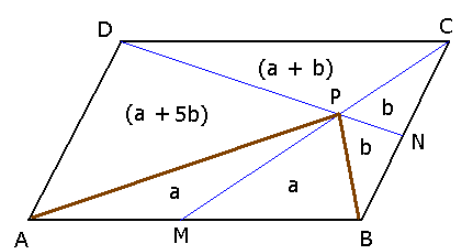
All positive integers except powers of 2 can be written as sum of two or more consecutive positive integers. So the 2037^{th} such number is 2049.

149. Determine the number of ordered triplets (A, B, C) of sets which have the property that
(i) $A \cup B \cup C$ = {1, 2, 3… 1000}, and
(ii) $A \cap B \cap C = \varnothing$.

Every element of the set {1, 2, 3 … 1000} has exactly 6 choices viz. to be included in A, B, C, A&B, B&C, or A&C. Hence the required number of triplets = 6^{1000}.

150. In a parallelogram ABCD, let M be the midpoint of the side AB and N the midpoint of BC. Let P be the intersection point of the lines MC and ND. Find the ratio of area of ▲s APB : BPC : CPD : DPA.

As M and N are mid points of AB and BC respectively, areas of ▲s APM, BPM and BPN, CPN are equal and equal to a, a and b, b respectively (say) as shown in the diagram. Also area(BMC) = area(CND) = ¼ area(ABCD) = a + 2b. So area(CPD) = a + b and area(DPA) = 4(a +



2b) − (a + a + b + b + a + b) = a + 5b. Now we have that area(APB) + area(CPD) = area(BPC) + area(DPA) = ½ area(ABCD) so we get that a = 3b. Hence ratio of area of ▲s APB : BPC : CPD : DPA = 3 : 1 : 2 : 4.

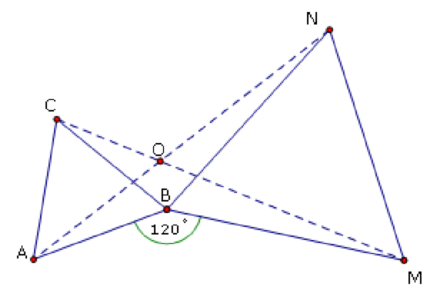
151. Kali-Jot is a game played by two players each of them having some number of marbles with her. One of the two players has to determine whether the number of marbles with other player is even or odd. A particular game of Kali-Jot has seven players and starts with players P_1 and P_2 on field and the other players P_3, P_4, P_5, P_6, P_7 waiting in a queue for their turn in order. After each game is played, the loser goes to the end of the queue; the winner adds 1 point to her score and stays on the field; and the player at the head of the queue comes on to contest the next point. Game continues until someone has scored 11 points. At that moment, it was found out that a total of 43 points have been scored by all seven players together. Who is the winner?

Whenever a player loses a game, she has to wait for five games before her next turn. If a is the number of games before her first turn, then the player will win if a + 6b + 11 = 43, where b ≥ 0 is an integer and 0 ≤ a ≤ 5. Here b is the number of times she lost. See whenever she lost, from that point onward till her next turn – 6 games has been played or 6 points have been distributed. From this, we obtain a = 2 and b = 5. Thus the second player in the queue wins. That is P_4 wins. 120°

152. For positive real numbers; A, B, C, D such that A + B + C + D = 8, find the minimum possible value of $\frac{1}{A}+\frac{1}{B}+\frac{1}{C}+\frac{1}{D}$.

For n positive real numbers (a_1, a_2, …a_n) we have $(a_1+a_2+\ldots+a_n)\left(\frac{1}{a_1}+\frac{1}{a_2}+\ldots+\frac{1}{a_n}\right) \geq n^2$. So here we have that $\frac{1}{A}+\frac{1}{B}+\frac{1}{C}+\frac{1}{D} \geq 4^2/8 = 2$. Hence required minimum value is 2.

153. In two equilateral triangles ABC and BMN, ∠ABM = 120°. AN & CM intersects at O. Find ∠MON.



▲ABN ~ ▲CBM (S-A-S similarity), that means ∠ANB = ∠CMB. So O, B, M, N are con-cyclic and ∠MON = ∠MBN = 60°.

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