58. A closed bag contains 3 green hats and 2 red hats. Amar, Akbar, Anthony all close their eyes, take a hat, put it on, and close the bag. When they open their eyes, Amar looks at Akbar and Anthony, but can't deduce the color of his own hat. Akbar now tries to deduce his own hat's color but can't be certain. What color is Anthony's hat?

Since there were only 2 red hats in the bag, if Amar saw red hats on Akbar and Anthony, he would have known is own hat was green. Therefore Amar saw at least 1 green hat on either Akbar or Anthony. If Akbar saw a red hat on Anthony, he would have known that the green hat Amar saw was his own. Amar's and Akbar's silence allowed Anthony to deduce that his own hat's color must be **green**.

59. Find the sum of all remainders when $n^5 - 5n^3 + 4n$ is divided by 120 for all positive integers $n \geq 2010$.

$n^5 - 5n^3 + 4n = (n - 2)(n - 1)(n)(n + 1)(n + 2)$ i.e. product of 5 consecutive positive integers, hence is always divisible by 5! i.e. 120. So the required sum is **zero**.

60. The equation $x^2 + ax + (b + 2) = 0$ has real roots. What is the minimum value of $a^2 + b^2$?

$a^2 - 4(b + 2) = a^2 + b^2 - (b^2 + 4b + 8) \geq 0$.

So $a^2 + b^2 \geq b^2 + 4b + 8$

$\Rightarrow a^2 + b^2 \geq (b + 2)^2 + 4$

$\Rightarrow a^2 + b^2 \geq 4$.

Hence the required minimum value is **4**.

61. There are 12 balls of equal size and shape, but one is either lighter or heavier than the other eleven. For how many minimum number of times weighing required with ordinary beam balance to determine the faulty ball?

Begin by balancing 4 and 4. If they balance, one of the 4 remaining balls is different. Now choose 3 of the remaining 4 to balance against any 3 of the known good balls. If they don't balance, you've at least determined whether you're looking for a lighter or heavier ball, and it takes one balance to determine which of the 3 is faulty.

If the first 2 groups of 4 don't balance, it's a bit trickier. Let's suppose the left side is heavier, but remember that there could be a lighter ball on the right side. For the 2nd balance, replace 3 balls on the left (heavy) side with 3 balls from the remaining 4, and in addition, swap the 4th ball on the left side with any ball from the right side. If the scale now balances, you know that one of the 3 balls removed from the left side is heavier. If the left side is now lighter, one the 2 balls swapped is different. If the right side is still lighter, you know that one of the 3 balls on the right side that wasn't swapped is lighter. In any case, minimum **three** balances are required.

Sanjeev: **I am thinking of a two digit number. Bet you can't guess it.**
Kamal: **Bet I can.**
Sanjeev: **Well, I'll only tell you the remainders of my number with anything from 1 to 10. How many questions do you think that you will have to ask?**
Kamal: **Hmmm! That depends on how lucky I am. But I'm not going to take chances. I am sure that I can guess your number with exactly _______ questions.**

62. How many questions does Kamal tell Sanjeev he will ask?

Just **two** questions are sufficient. First Kamal will ask the remainder of number with 10 and in second question he'll ask for remainder of number with 9.

63. What is the smallest possible difference between a square number and a prime number, if prime is greater than 3 and the square number is greater than prime?

Least value is **2** because $3^2 - 7 = 2$. Say if least value is less than 2 i.e. 1, then $x^2 - 1$ should be prime which is not.

64. 101 digits are chosen randomly and two numbers a, b are formed using all the digits exactly once. What is the probability that $a^4 = b$?

If a has 'n' digits then a^2 can have 2n or 2n – 1 digits in turn a^4 or b will have 4n or 4n – 1 or 4n – 2 or 4n – 3 digits. So total digits used for a and b will be 5n or 5n – 1 or 5n – 2 or 5n – 3. But 101 is of the form 5n – 4. Hence the required probability is **zero**.

65. Let ABCD be a quadrilateral. The circumcircle of the triangle ABC intersects the sides CD and DA in the points P and Q respectively, while the circumcircle of CDA intersects the sides AB and BC in the points R and S. The straight lines BP and BQ intersect the straight line RS in the same points M and N respectively. If ∠BQP = 90°, find ∠PMR.

∠NQP = ∠BQP = ∠BAP = ∠BAC + ∠PAC = ∠RDC + ∠PBC = ∠MSB + ∠MBS = 180° - ∠BMS = 180° - ∠NMP.

=> ∠NQP + ∠NMP = 180°. Hence P, Q, M, N are concyclic and **∠PMR = 90°**.

66. Kamal and Rajeev are playing the following game. They take turns writing down the digits of a six-digit number from left to right; Kamal writes the first digit, which must be nonzero, and repetition of digits is not permitted. Kamal wins the game if resulting six-digit number is divisible by 2, 3 or 5, and Rajeev wins otherwise. Who has a winning strategy?

Kamal will consume 3 and 9 in his first two moves so that Rajeev is left with at most 1 and 7 as his last move i.e. 6^{th} digit of the number. Now to win Kamal need to put his third move such that by putting this 5^{th} digit number becomes of the form 3k + 2 so that if Rajeev puts anything from 1 or 7, the number will be divisible by 3 or he'll need to put an even digit or 5. In either case **Kamal** has a winning strategy.

67. What is the least number of links you can cut in a chain of 21 links to be able to give someone all possible number of links up to 21?

Two. OOO C OOOOO C OOOOOOOOOOO (where Os are chained unbroken links, and the Cs are the unchained broken links). And equivalently: OOO C OOOOOO C OOOOOOOOOO

68. Every blip is a blop. Half of all blops are blips, and half of all bleeps are blops. There are 30 bleeps and 20 blips. No bleep is a blip. How many blops are neither blips nor bleeps?

They are **5** in number.

69. Several weights are given, each of which is not heavier than 1 kg. It is known that they cannot be divided into two groups such that the weight of each group is greater than 1 kg. Find the maximum possible total weight of these weights.

Three weights of 1 kg each will satisfy the condition. So the maximum possible total weight is **3 kg**.

70. Find the largest prime number *p* such that p^3 divided 2009! + 2010! + 2011!

2009! + 2010! + 2011! = 2009!(1 + 2010 + 2010 × 2011) = $2011^2 \times 2009!$ is divisible by 2011^2 which is square of a prime number and all other prime numbers contained in 2009! are lesser than 2011. So we need to find the largest prime number which is contained thrice in 2009! i.e. largest prime number less than [2009/3] which is 661. Hence *p* is **661**.

71. How many integers less than 500 can be written as the sum of 2 positive integer cubes?

First observe that there are seven perfect cubes which are less than 500. Also, except $6^3 + 7^3$ and $7^3 + 7^3$, all sums of 2 perfect cubes are different numbers less than 500. So required answer is: ${}^8C_2 - 2 = 28 - 2 =$ **26**.

72. Three boys Ali, Bashar and Chirag are sitting around a round table in that order. Ali has a ball in his hand. Starting from Ali the boy having the ball passes it to either of the two boys. After 6 passes the ball goes back to Ali. How many different ways can the ball be passed?

Table below shows the number of ways in which the ball can be passed to any three of the boys after every pass. For example; initially ball is with Ali so after 1^{st} pass it can go to Bashar in one way or Chirag in one way but it can't remain with Ali. Continuing on same thoughts after 2^{nd} pass, ball can be with Ali in two ways (passed either from Bashar or Chirag in one way each).

Pass	Chirag	Ali	Bashar
0^{th}	-	1	-
1^{st}	1	0	1
2^{nd}	1	2	1
3^{rd}	3	2	3
4^{th}	5	6	5
5^{th}	11	10	11
6^{th}	21	**22**	21

Clearly the required answer is **22**.

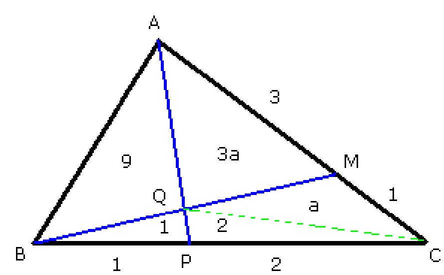
73. There are 21 girls standing in a line. You have only nine chairs. In how many ways you can offer these chairs to nine select girls (one for each girl) such that number of standing girls between any two selected girls is odd?

Observe that if we number the girls from 1 to 21, then the nine selected girls must be of same parity i.e. either all odd numbered or all even numbered. So required number of ways are $^{11}C_9 + {}^{10}C_9$ = **65**.

74. In a group of people, there are 19 who like apples, 13 who like bananas, 17 who like cherries, and 4 who like dates. (A person can like more than 1 kind of fruit.) Each person who likes bananas also likes exactly one of apples and cherries. Each person who likes cherries also likes exactly one of bananas and dates. Find the minimum possible number of people in the group.

As each of 17 person who likes cherries also like exactly one of bananas and dates, so 13 persons who like bananas all like cherries and do not like apples. Also all 4 persons who like dates also like cherries. Now out of 19 people who like apples, 4 can be common with date likers. So minimum number of people in the group is 17 + (19 - 4) = 17 + 15 = **32**.

75. Let M and P be the points on sides AC and BC of ΔABC respectively such that AM : MC = 3 : 1 and BP : PC = 1 : 2. If Q is the intersection point of AP and BM and area of ΔBPQ is 1 square unit, find the area of ΔABC.



Join QC. As BP : PC = 1 : 2, area of ΔQPC = 2. Let area of ΔCQM = a, then area of ΔAQM = 3a because AM : CM = 3 : 1. Also area of ΔAQB = 9 because of same reason. As area of ΔABP = 10 units so area of ΔABC = 3 × area of ΔABP = **30 units**.

76. How many pairs of non-negative integers (x, y) satisfy $(xy - 7)^2 = x^2 + y^2$?

$(xy - 7)^2 = x^2y^2 - 14xy + 49 = x^2 + y^2$

=> $x^2y^2 - 2(xy)(6) + 6^2 + 13 = x^2 + y^2 + 2xy$

=> $(xy - 6)^2 + 13 = (x + y)^2$

=> $(x + y + xy - 6)(x + y - xy + 6) = 13$

=> (x, y) = (0, 7), (7, 0), (3, 4) or (4, 3). So there are **four** such pairs.

77. What is the 50^{th} digit after decimal for: $\sqrt{\dfrac{2009 \times 2010 \times 2011 \times 2012 + 1}{4}}$?

$x(x + 1)(x + 2)(x + 3) + 1 = (x^2 + 3x + 1 - 1)(x^2 + 3x + 1 + 1) + 1 = (x^2 + 3x + 1)^2$

So $2009 \times 2010 \times 2011 \times 2012 + 1$ is an odd perfect square and the given expression is half of an odd integer which will give only one digit i.e. 5 after decimal. So 50^{th} digit after decimal is **zero**.

78.Year is 2051 and there is a strange game being played by 2051 inhabitants of TG Land. All 2051 inhabitants are standing in a circle. Now TG appears and randomly selects a person who shouts loudly IN, then person standing next clockwise say OUT, as must be the rule of the game, and get out of the circle. Again next person says IN and remain in his position and next says OUT and go out of circle. This process continues for a long time and in the end there is only one person remaining in the original circle. What is the position of the last survivor in the original circle, if first person selected by TG is numbered as 1 and numbers increases clockwise?

Check for smaller number of persons, when total number of persons is a power of 2, then the survivor is person1 other wise it increases to next odd number as 3, 5, 7, ... and so on as can be observed from below table:

Total Persons	Survivor		Total Persons	Survivor		Total Persons	Survivor		Total Persons	Survivor
1	1		2	1		4	1		8	1
			3	3		5	3		9	3
						6	5		10	5
						7	7		11	7
									12	9
									13	11
									14	13
									15	15

Also, if you observe from above table the binary representation of the numbers, you can notice if leading digit 1 of total persons if switched to unit's place then we get the last survivor.

So if total persons are 2051 which is $2048 + 1 + 1 + 1$, the last survivor is **7**.

A better solution using recursion is that, if we have total number of persons as n, then let the survivor be S(n). Now total number of persons may be 2n or 2n + 1.

If we have total 2n persons, then in first round of game all even numbered n persons will be eliminated leaving n persons 1, 3, 5, ..., 2n − 1. So we can comfortably write, $S(2n) = 2S(n) - 1$.

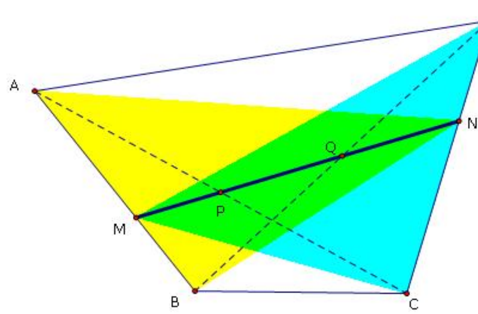
Similarly it can be easily proved for 2n + 1 persons that $S(2n + 1) = 2S(n) + 1$.

Now because: $2051 = 2 \times 1025 + 1$
and $1025 = 2 \times 512 + 1$
and $512 = 2 \times 256$
and $256 = 2 \times 128$
and $128 = 2 \times 64$
and $64 = 2 \times 32$
and $32 = 2 \times 16$
and $16 = 2 \times 8$
and $8 = 2 \times 4$
and $4 = 2 \times 2$
and $2 = 2 \times 1$
Now moving back, we know that $S(1) = 1$
so $S(2) = 2S(1) - 1 = 1$
so $S(4) = 2S(2) - 1 = 1$
so $S(8) = 2S(4) - 1 = 1$
so $S(16) = 2S(8) - 1 = 1$
so $S(32) = 2S(16) - 1 = 1$
so $S(64) = 2S(32) - 1 = 1$
so $S(128) = 2S(64) - 1 = 1$

so S(256) = 2S(128) – 1 = 1
so S(512) = 2S(256) – 1 = 1
so S(1025) = 2S(512) + 1 = 3
so S(2051) = 2S(1025) + 1 = **7**.

79. DaGny bought a rare earring set for $700, sold it for $800, bought it back for $900 and sold it again for $1000. How much profit did she make?
She made a profit of $100 in both transactions so total profit is **$200**.

80. ABCD is a convex quadrilateral that is not parallelogram. P and Q are the midpoints of diagonals AC and BD respectively. PQ extended meets AB and CD at M and N respectively. Find the ratio of area(ΔANB) : area(ΔCMD).



Let P and Q be the mid points of diagonals AC and BD as shown.
Area of triangle (APN) = Area of triangle (CPN)
Area of triangle (APM) = Area of triangle (CPM)
Area of triangle (BQN) = Area of triangle (DQN)
Area of triangle (BQM) = Area of triangle (DQM)
Adding all the four equations we have,
Area of triangle (ABN) = Area of triangle (CDM) as required. So the required ratio is **1 : 1**.

81. How many positive integers N are there such that 3 × N is a three digit number and 4 × N is a four digit number?
N varies from 250 to 333 inclusive, so total numbers of different values are: 333 – 249 = **84**.

82. Lara is deciding whether to visit Kullu or Cherapunji for the holidays. She makes her decision by rolling a regular 6-sided die. If she gets a 1 or 2, she goes to Kullu. If she rolls a 3, 4, or 5, she goes to Cherapunji. If she rolls a 6, she rolls again. What is the probability that she goes to Cherapunji?

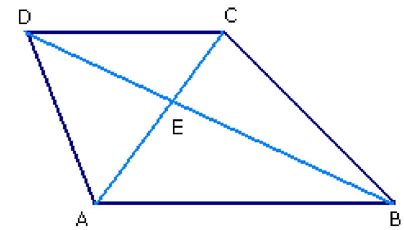
The required probability is = $\frac{1}{2}+\left(\frac{1}{6}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{6}\right)^2\left(\frac{1}{2}\right)+\left(\frac{1}{6}\right)^3\left(\frac{1}{2}\right)+\ldots = \frac{\frac{1}{2}}{1-\frac{1}{6}} = \boxed{\frac{3}{5}}$.

83. The numbers 201, 204, 209, 216, 225, … are of the form $a_n = 200 + n^2$ where n = 1, 2, 3, 4, 5, …. For each n, let D_n be the greatest common divisor of a_n and a_{n+1}. What is the maximum value of D_n?
$HCF(a_n, a_{n+1}) = HCF(200 + n^2, 200 + (n + 1)^2) = HCF(200 + n^2, 200 + n^2 + 2n + 1) = HCF(200 + n^2, 2n + 1) = HCF(800 + 4n^2, 2n + 1) = HCF(800 - 2n, 2n + 1) = HCF(801, 2n + 1) =$ **801** when n = 400.

84. Messers Baker, Cooper, Parson and Smith are a baker, a cooper, a parson and a smith. However, no one has the same name as his vocation. The cooper is not the namesake of Mr. Smith's vocation; the baker is neither Mr. Parson nor is he the namesake of Mr. Baker's vocation. What is Mr. Baker's vocation?
There are two possible cases:

Mr. Baker	parson	parson
Mr. Cooper	baker	smith
Mr. Parson	smith	cooper
Mr. Smith	cooper	baker

So in both the cases: Mr. Baker is **a parson**.



85. In trapezium ABCD, AB||CD. If area(ΔABE) = $\log_a 11$, area(ΔCDE) = $\log_{11} a$, and area(ΔABC) = 11, find the area of ABCD.

$[\text{area}(\Delta BEC)]^2 = [\text{area}(\Delta ABE)][\text{area}(\Delta CDE)] = 1$

=> area(ΔBEC) = 1 and area(ΔABE) = 10 and area(ΔCDE) = 1/10

=> area(ABCD) = $\left(\sqrt{10} + \frac{1}{\sqrt{10}}\right)^2 = \left(\frac{11}{\sqrt{10}}\right)^2 = \boxed{12.1}$.

86. Let P(x) be a polynomial such that $P(x) = x^{19} - 2011x^{18} + 2011x^{17} - ... - 2011x^2 + 2011x$. Calculate P(2010).

$P(x) = (x^{19} - 2010x^{18}) - (x^{18} - 2010x^{17}) + (x^{17} - 2010x^{16}) - ... - (x^2 - 2010x) + x$

So P(2010) = **2010**.

87. How many 9-digit numbers (in decimal system) divisible by 11 are there in which every digit occurs except zero?

Using divisibility rule of 11; difference between sums of alternate digits of the nine digit number must be divisible by 11. As there are 9 digits from 1 to 9, sum of all the digits is 45 and one group will be having 4 digits and other 5 digits. Also there difference will be 11 only. So sum of one group is 28 and other is 17. Now there are two cases: four digits a + b + c + d = 17 or 28.

For a + b + c + d = 17, we have (1, 2, 5, 9); (1, 3, 4, 9); (1, 2, 6, 8); (1, 3, 5, 8); (1, 3, 6, 7); (1, 4, 5, 7); (2, 3, 4, 8); (2, 3, 5, 7); (2, 4, 5, 6) i.e. 9 cases, and for a + b + c + d = 28, we have (4, 7, 8, 9); (5, 6, 8, 9) i.e. 2 cases. So total numbers are 11 • 4! • 5! = **31680**.

88. There are four unit spheres inside a larger sphere, such that each of them touches the large sphere and the other three unit spheres. What is the radius of larger sphere?

Analyze the structure as three unit spheres touching each other and the fourth one is now lying over them such that when centers of all the four unit spheres are joined they form a regular tetrahedron of edge length two units. Also observe that altitude from vertex of this tetrahedron on the opposite triangular face falls on its centroid.

So height of tetrahedron can be calculated as $\sqrt{2^2 - \left(\frac{2}{3}\sqrt{3}\right)^2} = 2\sqrt{\frac{2}{3}}$.

[*Remember length of altitude in an equilateral triangle of side 'a' is* $\frac{\sqrt{3}}{2}a$ *and centroid divides the altitude in the ratio 2 : 1 and also barycenter of a regular tetrahedron divides the height in the ratio 3 : 1*]

So radius of circumsphere of the tetrahedron is = $\frac{3}{4}\left(2\sqrt{\frac{2}{3}}\right) + 1 = \boxed{\sqrt{\frac{3}{2}} + 1}$.

89. For the real numbers a, b and c, it is known that

$$\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = 1, \text{ and}$$

$$a + b + c = 1.$$

Find the value of the expression, M = $\frac{1}{1+a+ab}+\frac{1}{1+b+bc}+\frac{1}{1+c+ca}$.

By given two equation we find that a + b + c = abc = 1

Now M = $\frac{1}{1+a+ab}+\frac{1}{1+b+bc}+\frac{1}{1+c+ca}$ (1)

=> M = $\frac{1}{1+a+(1/c)}+\frac{1}{1+b+(1/a)}+\frac{1}{1+c+(1/b)}$

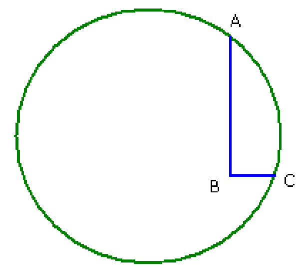
=> M = $\frac{c}{1+c+ac}+\frac{a}{1+a+ab}+\frac{b}{1+b+bc}$ (2)

=> M = $\frac{c}{1+c+(1/b)}+\frac{a}{1+a+(1/c)}+\frac{b}{1+b+(1/a)}$

=> M = $\frac{bc}{1+b+bc}+\frac{ca}{1+c+ca}+\frac{ab}{1+a+ab}$ (3)

Adding (1), (2) and (3), we get

3M = 3 => M = **1**.



90. In the circle shown, radius = $\sqrt{50}$, AB = 6, BC = 2, ∠ABC = 90°. Find the distance from B to the centre of the circle.

Extend AB to D and CB to E as shown.

Using AB × BD = BC × BE we have BE = 3 × BD

Let BD = 2x, then BE = 6x.

Let O be the centre of circle then drop perpendiculars from O to AD and CE to meet at their midpoints P and Q respectively as shown.

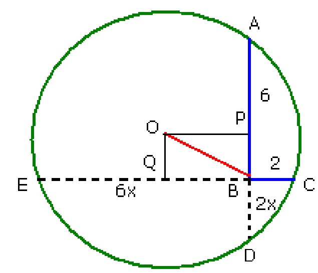
Using Pythagoras theorem in triangle OPA, we get

$OA^2 = OP^2 + AP^2$

=> $50 = (3x - 1)^2 + (3 + x)^2 = (9x^2 - 6x + 1) + (x^2 + 6x + 9) = 10x^2 + 10$

=> $10x^2 = 40$ and x = 2.

So OB = $\sqrt{OP^2+BP^2} = \sqrt{5^2+1^2} = \boxed{\sqrt{26}}$.



91. Today is Friday. What day will it be after 4^{2010} days?

$4^{2010} \equiv 4^{3 \times 670}$ mod7 $\equiv 1^{670}$ mod7 ≡ 1 mod7. So it will be next to Friday i.e. **Saturday** after 4^{2010} days.

92. Solve the congruence cryptarithm *LIFE* ≡ *SIZE* (mod *ELS*) in base 6 with E, L and S nonzero, all alphabets representing different numerals and Z > L > S. Write the 6 letter-word denoting the digits 012345 as answer.

As *LIFE* – *SIZE* ends in zero that means S must be 2 or 3 only and the quotient will be 3 or 2 respectively.

If S = 3, then L = 4 and Z = 5 are only values. It can be easily verified that it doesn't satisfy.

If S = 2, L = 3 that also doesn't satisfy. Now only case left is: S = 2, L = 4 and Z = 5 and it satisfies with the values as – 4103 ≡ 2153 mod342 i.e. 012345 ≡ ***FISELZ***.

93. Find the sum $1+\frac{3}{2}+\frac{5}{4}+\frac{7}{8}+\ldots$

Let S = $1+\frac{3}{2}+\frac{5}{4}+\frac{7}{8}+\ldots$

So S/2 = $\frac{1}{2}+\frac{3}{4}+\frac{5}{8}+\frac{7}{16}+\ldots$ Subtracting the two equations we get, S – S/2 = S/2 = 1 + 1 + ½ + ¼ + 1/8 + ...

=> S/2 = 1 + $\frac{1}{1-1/2}$ = 3 and S = **6**.

94. Two boats start at same instant from opposite ends of the river traveling across the water perpendicular to shores. Each travels at a constant but different speed. They pass at a point 720 meters from the nearest shore. Both boats remain at their slips for 15 minutes before starting back. On the return trip, they pass 400 meters from the other shore. Find the width of the river.

When they meet for the first time, sum of the distances traveled = d (width of the river)
And when they meet in the return trip, sum of the distances traveled = 3d.
Because they are traveling with constant speeds and for equal time; total distance traveled by each of the boat till second meeting = 3 × distance traveled by that boat till first meeting
So d + 400 = 3 × 720 = 2160 => d = **1760 meters**.

95. Larry, Curly, and Moe had an unusual combination of ages. The sum of any two of the three ages was the reverse of the third age (*e.g.*, 16 + 52 = 68, the reverse of 86). All were under 100 years old. What was the sum of the ages?

Let the three numbers be ab, cd and ef where a, b, c, d, e, f are the single digit positive integers.
As cd + ef = ba, sum of the three numbers is: ab + ba = 11(a + b) = 11(c + d) = 11(e + f)
Also ab + cd = fe

- ⇨ 10a + b + 10c + d = 10f + e
- ⇨ 10(a + c - f) = e – b – d
- ⇨ a + c = f and e = b + d OR a + c = f – 1 and e + 10 = b + d

From equation above we have, (a + b) + (c + d) = 2(e + f)

- ⇨ a + c = 2(e + f) – (b + d)

So if a + c = f, then using e = b + d, we get f = e + 2f i.e. e + f = 0 which is not possible.
If a + c = f – 1, then using e + 10 = b + d, we get f – 1 = e + 2f – 10 i.e. e + f = 9.
(a) Thus a + b = c + d = e + f = 9 and sum of all the three ages = 11 × 9 = **99**.

96. Find the sum of all four-digit numbers N whose sum of digits is equal to 2010 – N.

N has to be greater than 2010 – 28 = 1982 because maximum sum of digits less than 2010 is 1 + 9 + 9 + 9 i.e. 28. Now checking for all the decades there are only two possible values of N i.e. 1986 and 2004. So the required sum is **3990**.

97. DaGny has 11 different colors of fingernail polish. Find the number of ways she can paint the five fingernails on her left hand by using at least three colors such that no two consecutive finger nails have same color. Also she is to apply only one color at one fingernail which is quite unusual for her.

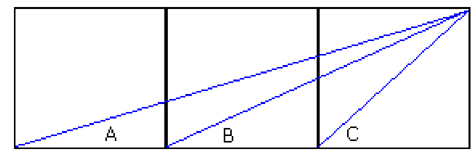
First fingernail can be painted in 11 ways and every next fingernail can be painted in 10 ways each such that no two consecutive fingernails have same color i.e. total 11 × 10^4 ways. In this number of ways atleast two colors have been used. So we need to subtract the ways in which exactly two colors have been used which are 11 × 10.
Thus required numbers of ways are 110000 – 110 = **109890**.

98. A box contains 300 matches. Kamal and Sandeep take turns removing no more than half the matches in the box. The player who cannot move loses. What should be Kamal's first move to ensure his win if he is starting the game?

Kamal must always leave N matches for Sandeep to continue where N is a natural number of the form 2^a – 1. So Kamal must remove **45 matches** in his first move so as to leave 255 i.e. 2^8 – 1 matches for Sandeep to continue with. Now whatever number of matches Sandeep removes Kamal will make Sandeep to continue with 127 matches. (Notice that Sandeep cannot remove more than 127 matches from 255.)

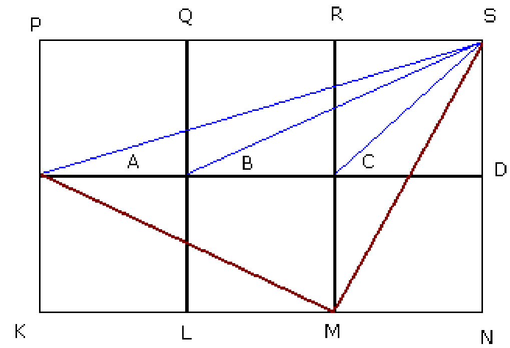
99. Let N be an integer such that $2N^2$ has exactly 28 distinct positive divisors and $3N^2$ has exactly 24 distinct positive divisors. How many distinct positive divisors does $6N^2$ have?

$N = 2 \times 3^3$, so $6N^2 = 2^3 \times 3^7$ and required number of divisors: $4 \times 8 =$ **32**.



100. Three unit squares are joined as shown. Find the measure of ∠A + ∠B + ∠C.

Construct three more identical squares below the three given ones and draw two lines as shown on the right.



Now observe that triangle ASM is an isosceles right triangle, right angled at M. So ∠SAM = 45°.

Also triangle ACM ≅ BDS, so ∠B = ∠MAC.

=> ∠A + ∠B = 45°

As triangle CDS is also isosceles right triangle, ∠C = 45°.

Hence, ∠A + ∠B + ∠C = **90°**.

101. What is the 625th term of the series where each term is made up of even digits only?

2, 4, 6, 8, 20, 22, 24, 26, 28, 40, 42, …

625th term of the series is the smallest 5-digit number of the series i.e. 20000.

102. The houses in a street are spaced so that each house of one lane is directly opposite to a house of other lane. The houses are numbered 1, 2, 3, … and so on up one side, continuing the order back down the other side. Number 39 is opposite to 66. How many houses are there?

Total number of houses = 39 + 66 − 1 = 104.

103. In a polygon, internal angles have the measures of 90° and 270° only. If there are 18 angles of measure 270°, then what is the number of angles with measure of 90°?

Let there be n angles of measure 90°, then sum of all the angles = $(90n) + (18 \times 270) = (n + 16)180$

So n = 22.

104. How many pair of positive integers (a, b) are there such that their LCM is 2012?

$2012 = 2^2 \times 503$. Among a and b atleast one must have 2^2 and 503. So the required number of ordered pairs (a, b) = $(3^2 - 2^2)(2^2 - 1^2)$ = 15.

105. What is the sum of all natural numbers which are less than 2012 and co-prime to it?

It is simply $\frac{1}{2} \times 2012 \times \varnothing(2012)$ = 1010024.

106. How many positive integers N satisfy: (i) N < 1000 and (ii) $N^2 - N$ is divisible by 1000?

$N \equiv 0$ or 1 mod2^3 and also $N \equiv 0$ or 1 mod5^3. So there are 3 positive integers i.e. 1, 376, and 625 which satisfy the given conditions.

107. 25 is a square number and can be written as average of two different square numbers i.e. 1 and 49. How many other square numbers from 1 to 625 inclusive can be written as average of two different square numbers?

Only square of largest number of a Pythagorean triplet can be written in the desired form, as if $c^2 = a^2 + b^2$, then only we can write $2c^2 = (a + b)^2 + (a - b)^2$. So from 1^2 to 25^2, c^2 can take 6 values other than 5^2 i.e. 10^2, 13^2, 15^2, 17^2, 20^2 and 25^2.

108. I can break a block of 7 kg in smaller blocks of integral weights in four ways i.e. {1, 2, 4}, {1, 2, 2, 2}, {1, 1, 1, 4}, {1, 1, 1, 1, 1, 1, 1} such that I can measure each weight from 1