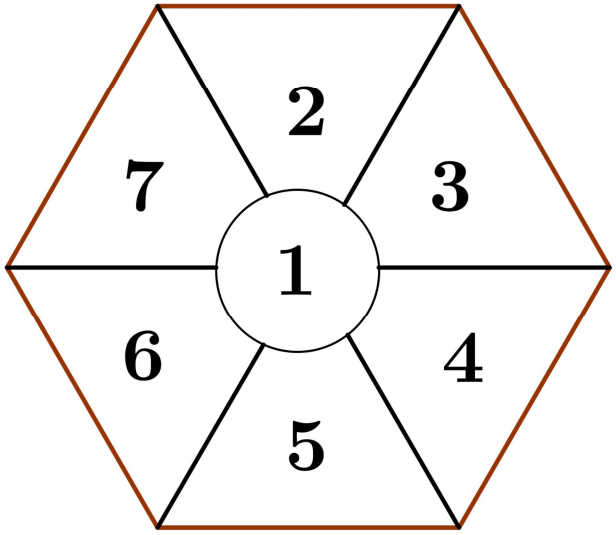
place. In the SECOND guess only TWO letters are in their correct places and these two correct places are not next to each other. In the THIRD guess THREE letters are in their correct places. Each of the 6 letters is in its correct place once. What is the correct combination?

52. A flower plantation has 7 areas as shown in the figure. Plant them with flowers of 4 different colors so that each area has only one colour. In how many ways can we plant the flowers so that the neighbouring regions all have different colors?

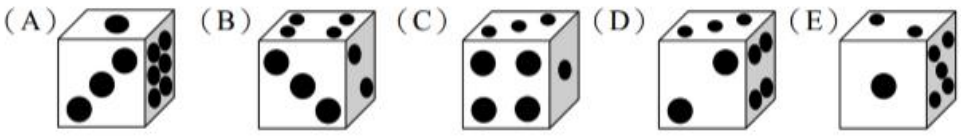


53. Know that $\frac{a}{b} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{131}$ and $\frac{a}{b}$ is an irreducible fraction. Prove that a is composite number.

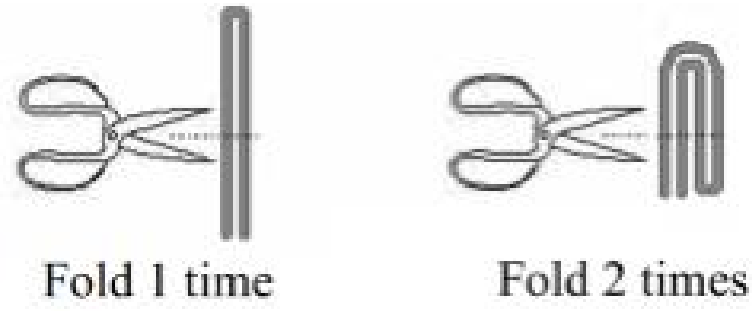
54. Numbers like 1001, 23432, 897798, 3456543 are known as palindromes. If all of the digits $2, 7, 0$ and 4 are used and each digit cannot be used more than twice, find the number of all different palindromes that can be formed.

55. The product $1! \times 2! \times 3! \times \ldots \times 2015! \times 2016!$ is written on the blackboard. Which factor, in terms of factorial of an integer, should be erased so that the remaining product is the square of an integer number? (The factorial sign $n!$ stands for the product of all positive integers less than or equal to n.)

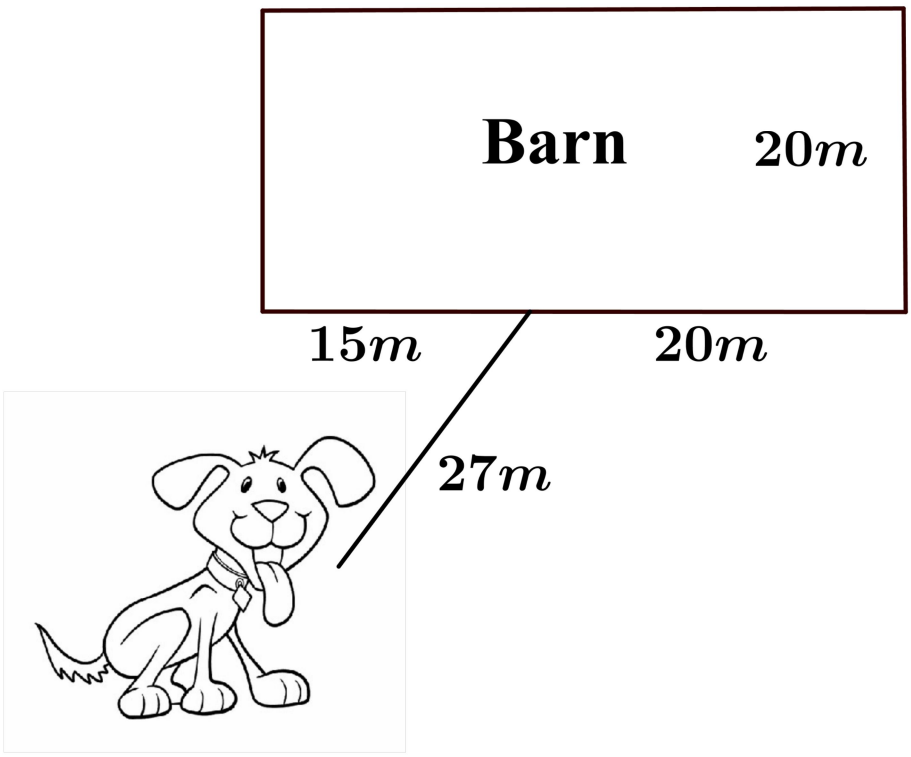
56. Each of the following five six-sided die has 1, 2, 3, 4, 5 and 6 spots on its faces. Which one has a different arrangement of spots from the other four?



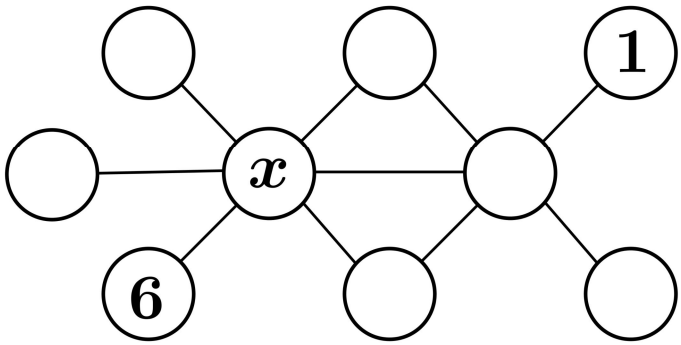
57. The diagram shows that if a rope is folded once and be cut in half, it will separate into 3 pieces; and if it is folded twice instead, it will separate into 5 pieces. If it is folded 6 times instead, into how many pieces will it separate?



58. A farmer fastens the end of his dog's leash to the edge of his barn at a point that is 15m from one corner and $25m$ from the other corner of the barn, as illustrated in the diagram below. The Barn is 20m wide and the leash is 27m long. Calculate the area the dog is able to reach while leashed to the wall, to the nearest whole square metre!



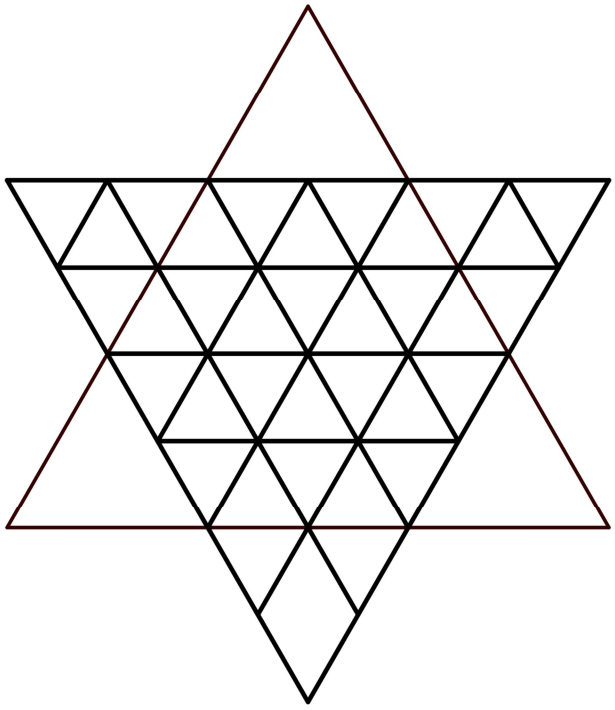
59. In the diagram, each of the integers 1 through 9 is to be placed in one circle so that the integers in every straight row of three joined circles add to 18. The 6 and 1 have been filled in. Find the value of x.



60. A family has seven daughters. Each one after the

eldest is two years younger than the one born before. If the eldest daughter is three times as old as the youngest, how old is the eldest?

61. How many triangles are there in below diagram?



62. The 100000 tickets for an event are numbered from 00000 to 99999. If a number contains two adjacent

digits which differ by exactly 5, it wins a door prize. How many door prizes will be needed if all tickets are sold?

63. 7 out of 8 coins are known to be real and have the same weight. The other one may also be real, but may be a fake coin, which is either heavier or lighter than a real coin. We want to know if there is a fake coin. If so, we wish to know whether it is heavier or lighter, but it is not necessary to identify the fake coin. What is the minimum number of weighing on a scale that would accomplish the task?

64. Find the value of

$$1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{999}{1234}}}}}}}?$$

65. Dates can be written as an 8-digit integer in the format $YYYYMMDD$. For example, 20160125 stands for January 25th 2016. How many days are there in

the year 2016 when the 8-digit representation contains equal numbers of the digits $0, 1, 2$?

66. From 2015 to 6999, how many integers have their sum of digits divisible by 5?

67. Find the sum of all numbers from 1 to 2000, the sum of the digits of which are even?

68. In the correct addition below, each letter stands for a digit. What is the value of the sum $A + 10B + C + D + E + F$?

$$\begin{array}{rccc} & A & 2 & E \\ & 1 & B & D \\ + & F & 2 & C \\ \hline & 6 & 3 & 2 \end{array}$$

69. The numbers $1, 2, \ldots, 25$ are to be placed in a 5×5 table, with one number exactly in each square. Consecutive numbers occupy squares with a common side. Three of the numbers have been placed, as shown in the diagram below. Find the number of different placements of the other 22 numbers.

19		13		
		1		

70. Tom and Jerry play the following game. Tom has some number of coins and Jerry has none. Jerry can take any (non-zero) number of coins from Tom. Then Tom can take some (again, non-zero) number of coins back, but necessarily a different number. Then again, Jerry takes some from Tom, but necessarily a number which did not occur before. And so on. The game stops when someone cannot make a move. What is the largest number of coins Jerry can have at the end if:

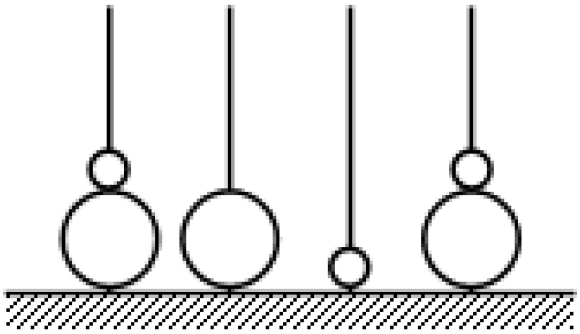
    (a) Tom had 13 coins at the beginning?

    (b) Tom had 50 coins at the beginning?

71. Filll the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 into the boxes below so that the expression will produce the largest product. (Each digit can be used only once)

□□□□□×□□□×□

72. There are 4 fixed steel pins, 3 big balls and plenty of small balls. Consider every small ball as the digit "3" and every big ball as the digit"5" (as shown in Fig. 6, it stands for 8538). Now, these balls are placed on the steel pins. Definitely all pins have balls. Start to read the numbers from left to right. (The sum of the balls representing numbers on every pin is less than 10.) How many different four-digit numbers can be read?



73. Peter and Jane are to take turns to subtract perfect squares from a given whole number and the one who reaches zero first is the winner. If the whole number is 29 and Peter is the first player, what perfect

number must he subtract in order for him to definitely win. (Note: 4; 9 and 16 are examples of perfect squares)

74. Five points lie on a line. Alex finds the distances between every possible pair of points. He obtains, in increasing order, 2, 5, 6, 8, 9, k, 15, 17, 20 and 22. What is the value of k?

75. Eight of the digits $1, 2, 3, \ldots$ and 9 are arranged, one per circle with each circle on one of the eight edges of the cube, on the cube shown so that S, the sum of the numbers on each face of the cube, is the same. Seven of the eight numbers are labelled, but one is not. It is known that S is not divisible by the missing number. What is the missing number x?