77. What is the 50^{th} digit after decimal for:

$$\sqrt{\frac{2009 \times 2010 \times 2011 \times 2012 + 1}{4}} ?$$

78. Year is 2051 and there is a strange game being played by 2051 inhabitants of TG Land. All 2051 inhabitants are standing in a circle. Now TG appears and randomly selects a person who shouts loudly IN, then person standing next clockwise say OUT, as must be the rule of the game, and get out of the circle. Again next person says IN and remain in his position and next says OUT and go out of circle. This process continues for a long time and in the end there is only one person remaining in the original circle. What is the position of the last survivor in the original circle, if first person selected by TG is numbered as 1 and numbers increases clockwise?

79. DaGny bought a rare earring set for \$700, sold it for \$800, bought it back for \$900 and sold it again for \$1000. How much profit did she make?

80. ABCD is a convex quadrilateral that is not parallelogram. P and Q are the midpoints of diagonals AC and BD respectively. PQ extended meets AB and CD at M and N respectively. Find the ratio of area(ΔANB) : area(ΔCMD).

81. How many positive integers N are there such that $3 \times N$ is a three digit number and $4 \times N$ is a four digit number?

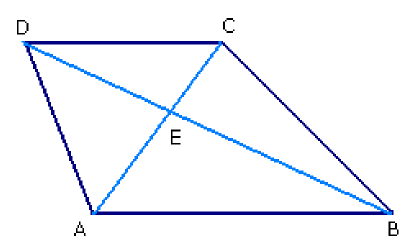
82. Lara is deciding whether to visit Kullu or Cherapunji for the holidays. She makes her decision by rolling a regular 6-sided die. If she gets a 1 or 2, she goes to Kullu. If she rolls a 3, 4, or 5, she goes to Cherapunji. If she rolls a 6, she rolls again. What is the probability that she goes to Cherapunji?

83. The numbers 201, 204, 209, 216, 225, ... are of the form $a_n = 200 + n^2$ where n = 1, 2, 3, 4, 5, ... For each n, let D_n be the greatest common divisor of a_n and a_{n+1}. What is the maximum value of D_n?

84. Messrs Baker, Cooper, Parson and Smith are a baker, a cooper, a parson and a smith. However, no one has the same name as his vocation. The cooper is not the namesake of Mr. Smith's vocation; the baker is neither Mr. Parson nor is he the namesake of Mr. Baker's vocation.

What is Mr. Baker's vocation?

85. In trapezium ABCD, AB||CD. If area(ΔABE) = $\log_a 11$, area(ΔCDE) = $\log_{11} a$, and area(ΔABC) = 11, find the area of ABCD.



86. Let P(x) be a polynomial such that:

$P(x) = x^{19} - 2011x^{18} + 2011x^{17} - ... - 2011x^2 + 2011x$. Calculate P(2010).

87. How many 9-digit numbers (in decimal system) divisible by 11 are there in which every digit occurs except zero?

88. There are four unit spheres inside a larger sphere, such that each of them touches the large sphere and the other three unit spheres. What is the radius of large sphere?

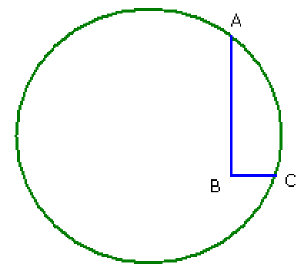
89. For the real numbers a, b and c, it is known that

$$\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}=1\text{, and}$$

$$a + b + c = 1.$$

Find the value of the expression, M = $\frac{1}{1+a+ab}+\frac{1}{1+b+bc}+\frac{1}{1+c+ca}$.

90. In the circle shown, radius = $\sqrt{50}$, AB = 6, BC = 2, ∠ABC = 90°. Find the distance from B to the centre of the circle.



91. Today is Friday. What day will it be after 4^{2010} days?

92. Solve the congruence cryptarithm *LIFE* ≡ *SIZE* (mod *ELS*) in base 6 with E, L and S nonzero, all alphabets representing different numerals and Z > L > S.

Write the 6 letter-word denoting the digits 012345 as answer.

93. Find the sum $1+\frac{3}{2}+\frac{5}{4}+\frac{7}{8}+\ldots$ up to infinity.

94. Two boats start at same instant from opposite ends of the river traveling across the water perpendicular to shores. Each travels at a constant but different speed. They pass at a point 720 meters from the nearest shore. Both boats remain at their slips for 15 minutes before starting back. On the return trip, they pass 400 meters from the other shore. Find the width of the river.

95. Larry, Curly, and Moe had an unusual combination of ages. The sum of any two of the three ages was the reverse of the third age (*e.g.*, 16 + 52 = 68, the reverse of 86). All were under 100 years old. What was the sum of the ages?

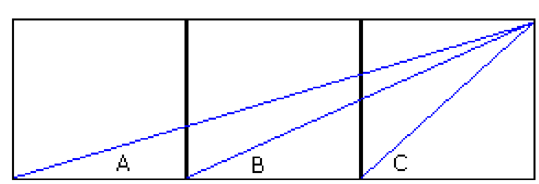
96. Find the sum of all four-digit numbers N whose sum of digits is equal to 2010 – N.

97. DaGny has 11 different colors of fingernail polish. Find the number of ways she can paint the five fingernails on her left hand by using at least three colors such that no two consecutive finger nails have same color. Also she is to apply only one color at one fingernail which is quite unusual for her.

98. A box contains 300 matches. Kamal and Sandeep take turns removing no more than half the matches in the box. The player who cannot move loses. What should be Kamal's first move to ensure his win if he is starting the game?

99. Let N be an integer such that $2N^2$ has exactly 28 distinct positive divisors and $3N^2$ has exactly 24 distinct positive divisors. How many distinct positive divisors does $6N^2$ have?

100. Three unit squares are joined as shown. Find the measure of ∠A + ∠B + ∠C.



101. What is the 625^{th} term of the series where each term is made up of even digits only?

2, 4, 6, 8, 20, 22, 24, 26, 28, 40, 42, ...

102. The houses in a street are spaced so that each house of one lane is directly opposite to a house of other lane. The houses are numbered 1, 2, 3, … and so on up one side, continuing the order back down the other side. Number 39 is opposite to 66. How many houses are there?

103. In a polygon, internal angles have the measures of 90° and 270° only. If there are 18 angles of measure 270°, then what is the number of angles with measure of 90°?

104. How many pair of positive integers (a, b) are there such that their LCM is 2012?

105. What is the sum of all natural numbers which are less than 2012 and co-prime to it?

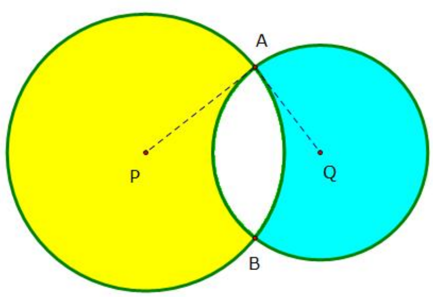
106. How many positive integers N satisfy: (i) $N < 1000$ and (ii) $N^2 - N$ is divisible by 1000?

107. 25 is a square number and can be written as average of two different square numbers i.e. 1 and 49. How many other square numbers from 1 to
625 inclusive can be written as average of two different square numbers?

108. I can break a block of 7 kg in smaller blocks of integral weights in four ways i.e. {1, 2, 4}, {1, 2, 2, 2}, {1, 1, 1, 4}, {1, 1, 1, 1, 1, 1, 1} such that I can measure each weight from 1 kg to 7 kg in exactly one way in either case.
For example, using 1st case this is the only possible combination of weights to measure 1 kg to 7 kg: 1 = 1, 2 = 2, 3 = 1 + 2, 4 = 4, 5 = 1 + 4, 6 = 2 + 4, and 7 = 1 + 2 + 4. So find the number of ways a block of 14 kg can be broken under similar conditions e.g. {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1} is a valid case but {1, 2, 3, 4, 4} is not.

109. I am twice as old as you were when I was as old as you are. What is the ratio of ages of mine and yours?

110. Circles with centers P and Q have radii 20 and 15 cm respectively and intersect at two points A, B such that $\angle PAQ = 90°$. What is the difference in the area of two shaded regions?



111. What is the largest integer that is a divisor of

$$(n + 1)(n + 3)(n + 5)(n + 7)(n + 9)$$

for all positive even integers n?

112. For how many ordered pairs of positive integers (x, y), $xy/(x+y) = 9$?

113. Amu, Bebe, Chanda and Dori played with a deck of 52 cards. In one game, Dori was dealing out the cards one by one to the players, starting with Amu, followed by Bebe, Chanda and Dori in this order, when suddenly some of the cards she had not dealt out yet slipped out of her hands and fell on the floor. The girls noticed that the number of cards on the floor was 2/3 of the number of cards Amu had already got, and the

number of cards that Chanda had got was 2/3 of those in the remaining part of the deck in Dori's hand that she had not dealt out yet. How many cards had Dori dealt out altogether?

114. In a city, 2/3 of the men and 3/5 of the women are married. (Everyone has one spouse and the spouses live in the same city.) What fraction of the inhabitants of the city is married?

115. The sum of all interior angles of eight polygons is 3240°. What is the total number of sides of polygons?

116. Consider a triangle ABC with BC = 3. Choose a point D on BC such that BD = 2. Find the value of

$$AB^2 + 2AC^2 - 3AD^2.$$

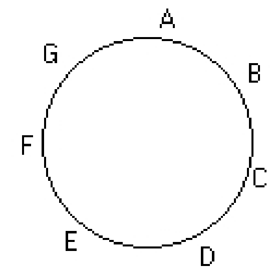
117. Determine the number of divisors of 2012^8 that are less than 2012^4.

118. How many numbers in the following sequence are prime numbers?

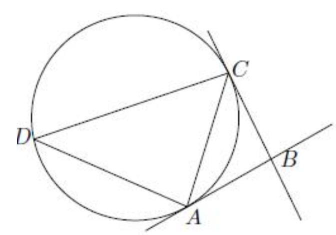
$$\{1, 101, 10101, 1010101, 101010101, \ldots\}$$

119. Find all triples of natural numbers (a, b, c) such that a, b and c are in geometric progression, and a + b + c = 111.

120. What is the smallest integer n for which $\sqrt{n} - \sqrt{n-1} < 0.01$?



121. Seven people, A, B, C, D, E, F and G can sit down for a meal at a round table as shown. Each person has two neighbours at the table: for example, A's neighbours are B and G. There are other ways in which the people can be seated round the table. Last month they dined together on a number of occasions, and no two of the people were neighbours more than once. How many meals could they have had together during the month?



122. Points A, D and C lie on the circumference of a circle. The tangents to the circle at points A and C meet at the point B. If ∠DAC = 83° and ∠DCA = 54°. Find ∠ABC.

123. How many 4-digit numbers uses exactly three different digits?

124. The ratio of two six digit numbers abcabc and ababab is 55 : 54. Find the value of a + b + c.

125. Find the infinite sum:

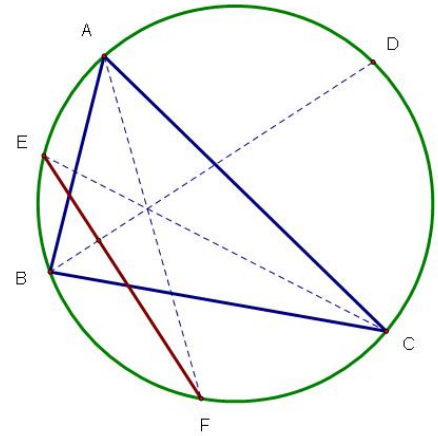
$$\frac{1}{4}+\frac{1}{8}+\frac{2}{16}+\frac{3}{32}+\frac{5}{64}+\frac{8}{128}+\frac{13}{256}+\frac{21}{512}+\ldots$$

126. What is the probability of tossing a coin 6 times such that no two consecutive throws result in a head?

127. In how many ways 3 letters can be selected from 3 identical A's, 3 identical B's and 3 identical C's?

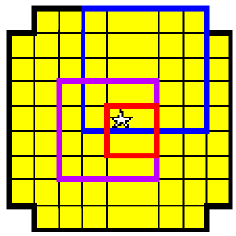
128. For how many pairs of positive integers (x, y) both $x^2 + 4y$ and $y^2 + 4x$ are perfect squares?

129. Jar X contains six liters of a 46% milk solution; Jar Y contains three liters of a 43% milk solution and Jar Z contains one liter of p% milk solution. q/r liters of solution from Jar Z is transferred to Jar X and remaining solution from Jar Z is transferred to Jar Y such that resulting two solutions both contain 50% milk solution. Also q and r are positive integers co-prime to each other. Find the value of p + q + r.



130. ABC is an isosceles right triangle inscribed in a circle such that $\angle B = 90°$. BD, CE and AF are angle bisectors of triangle ABC as shown. What is the measure of smaller angle of intersection of BD and EF?

131. In the diagram, three squares are shown, all containing the star. Altogether, how many squares containing the star can be found in the diagram?



132. Find all integers x, y, z (such that $x \le y \le z$) greater than 1 for which xy - 1 is divisible by z, yz - 1 is divisible by x, and zx - 1 is divisible by y.

133. In a rectangle ABCD, AB = 13 and BC = 8. PQ lies inside the rectangle such that BP = 11, DQ = 6 and AB||PQ, BP||DQ. Find the length of PQ.

134. What are the last two digits of the sum obtained by adding all the possible remainders of numbers of the form 2^n, n being a non-negative integer, when divided by 100?

135. On planet LOGIKA, there live two kinds of inhabitants; black and white ones and they answer every question posed to them in a Yes or No. Black inhabitants of northern hemisphere always lie while white inhabitants of northern hemisphere always tell the truth. Also white inhabitants of southern hemisphere always lie while black inhabitants of southern hemisphere always tell the truth. On a dark night, there is an electricity failure and you meet an inhabitant without knowing your location on the planet. What single yes/no question can you ask the inhabitant to determine color of the inhabitant?

136. N is product of first 50 prime numbers. A is a factor of N and B is a factor of A. How many ordered pairs (A, B) exist?

137. For how many positive integers, N > 2, (N - 2)! + (N + 2)! is a perfect square?

138. Let $P = \frac{1}{1\times2}+\frac{1}{3\times4}+\frac{1}{5\times6}+\dots+\frac{1}{2013\times2014}$ and
$Q = \frac{1}{1008\times2014}+\frac{1}{1009\times2013}+\frac{1}{1010\times2012}+\dots.+\frac{1}{2014\times1008}$.
Find P/Q.

139. C and D are two points on a semicircle with AB as diameter such that AC – BC = 7 and AD – BD = 13. AD and BC intersect at P. Find the difference in area of triangles ACP and BDP.

140. TG *fashions* hold its annual sale on the eve of Pi-day (14^{th} March) and offered a discount of 90% on all its apparels. But this month it is offering the usual 80% discount. How much percent more I need to pay now than that on the annual sale's eve for purchase of similar clothing?

141. In how many ways 1,000,000 can be expressed as sum of a square number and a prime number?

142. How many ordered triples of three positive integers (a, b, c) exist such that $a^3 + b^3 + c^3 = 2011$?

143. In a quadrilateral ABCD, sides AD and BC are parallel but not equal and sides AB = DC = x. The area of the quadrilateral is 676 cm^2. A circle with centre O and radius 13 cm is inscribed in the quadrilateral such that it is tangent to each of the four sides of the quadrilateral. Determine the length of x.

144. Kiran, Shashi and Rajni are Kiran's spouse, Shashi's sibling and Rajni's sister-in-law in no particular order. Also Kiran's spouse and Shashi's sibling are of same sex. Who among the three is a married male?

145. A and B start running from two opposite ends of a 1000m racing track. A and B travel with a speed of 8m/s and 5m/s respectively. How many times they meet, while running, in first 1000s after start?

146. According to death-will of Mr. Ranjan, all of this money was to be divided among his children in the following manner: `N to the first born plus 1/17 of what remains, `2N to the second born plus 1/17 of what then remains, `3N to the third born plus 1/17 of what then remains, and so on. When the distribution of the money was complete, each child received the same amount and no money was left over. Determine the number of children.

147. One number is removed from the set of integers from 1 to n. The average of the remaining numbers is 40.75. Which integer was removed?

148. What is the 2037^{th} positive integer that can be expressed as the sum of two or more consecutive positive integers? (The first three are 3 = 1+2, 5 = 2+3, and 6 = 1+2+3.)

149. Determine the number of ordered triplets (A, B, C) of sets which have the property that
(i) $A \cup B \cup C = \{1, 2, 3, \ldots, 1000\}$, and
(ii) $A \cap B \cap C = \varnothing$.

150. In a parallelogram ABCD, let M be the midpoint of the side AB and N the midpoint of BC. Let P be the intersection point of the lines MC and ND. Find the ratio of area of ▲s APB: BPC: CPD: DPA.

151. Kali-Jot is a game played by two players each of them having some number of marbles with her. One of the two players has to determine whether the number of marbles with other player is even or odd. A particular game of Kali-Jot has seven players and starts with players P_1 and P_2 on field and the other players P_3, P_4, P_5, P_6, P_7 waiting in a queue for their turn in order. After each game is played, the loser goes to the end of the queue; the winner adds 1 point to her score and stays on the field; and the player at the head of the queue comes on to contest the next point. Game continues until someone has scored 11 points. At that moment, it was found out that a total of 43 points have been scored by all seven players together. Who is the winner?

152. For positive real numbers; A, B, C, D such that A + B + C + D = 8, find the minimum possible value of $\frac{1}{A} + \frac{1}{B} + \frac{1}{C} + \frac{1}{D}$.

153. In two equilateral triangles ABC and BMN, $\angle ABM = 120°$. AN & CM intersects at O. Find $\angle MON$.

FOR ROUGH USE

PROBLEMS with SOLUTIONS

1. The number of persons who booked ticket for the New Year's concert is a perfect square. If 100 more persons booked ticket then the number of spectators would be a perfect square plus 1. If still 100 more persons booked ticket then the number of spectators would be again a perfect square. How many persons booked ticket for the concert?

Let a^2 be the number of tickets booked initially which when increased by 200 becomes another perfect square, say b^2. So we know that $b^2 - a^2 = (b - a)(b + a) = 200 = (2)(100) = (4)(50) = (10)(20)$
Solving, we get $(a, b) \equiv (49, 51), (23, 27)$ or $(5, 15)$
But according to given conditions $a^2 + 100$ is just one more than a perfect square so only pair (49, 51) satisfies. Hence **number of persons who booked ticket for the concert = 49^2 = 2401**.

2. If the last digits of the products 1·2, 2·3, 3·4, ..., *n*(*n*+1) are added, the result is 2010. How many products are used?

Unit digits of the numbers repeat after every 10 terms as 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, … So unit digits of the product of two consecutive numbers as required can be found as:

1	×	2	=	2
2	×	3	=	6
3	×	4	=	2
4	×	5	=	0
5	×	6	=	0
6	×	7	=	2
7	×	8	=	6
8	×	9	=	2
9	×	0	=	0
0	×	1	=	0

2, 6, 2, 0, 0, 2, 6, 2, 0, 0, … that repeats after every fifth term. Adding we get sum of unit digits of first five products as 2 + 6 + 2 + 0 + 0 = 10 which is same for next five numbers and so on. So to get a sum of 2010, 10 needs to be added 201 times or total 201 × 5 = 1005 products must have been used. But if we see the unit digit of 1004th and 1005th product are not contributing to the sum as they are zeroes only. Hence the **number of products used can be: 1003, 1004 or 1005**.

3. What is the remainder obtained when 2^{32} is divided by 641?

$2^8 \equiv 256 \bmod 641$
$2^{16} = 2^8 2^8 \equiv 256^2 \bmod 641 \equiv 65536 \bmod 641 \equiv 154 \bmod 641$
$2^{32} = 2^{16} 2^{16} \equiv 154^2 \bmod 641 \equiv 23716 \bmod 641 \equiv \mathbf{640} \bmod 641$.

4. How many integers may be the measure, in degrees, of the angles of a regular polygon?

Each interior angle of a regular polygon of 'n' sides is given by $\left(180 - \frac{360}{n}\right)^\circ$.

For this angle to be an integer, 'n' must be a positive integral divisor of 360 greater than 2.
Number of positive integral divisors of 360 i.e. $(2^3 3^2 5) = 4 \times 3 \times 2 = 24$. Leaving 1 and 2 we have **22** different integers which can be the measure of each angle of a regular polygon.

5. Several sets of prime numbers, such as {7, 83, 421, 659}, use each of the nine nonzero digits exactly once. What is the smallest possible sum such a set of primes could have?

For smallest possible sum, we must try to make single digit and two digit prime numbers only. For two digit prime numbers, even digits (i.e. 2, 4, 6, 8) and 5 cannot appear on the units place so they must be at 10's place for sure (if used in two digit prime numbers). Also 1 is not a prime so must be used somewhere in two digit primes. But remember that 2 and 5 can be treated as single digit prime numbers.
So minimum we must have three two-digit prime numbers starting with, 4, 6 and 8 and remaining three numbers must be single digit primes for the smallest possible sum as desired. Hence the required sum is: 40 + 60 + 80 + 1 + 2 + 3 + 5 + 7 + 9 = **207**.
Required set of prime numbers can be {2, 3, 5, 41, 67, 89}, {2, 3, 5, 47, 61, 89}, {2, 5, 7, 43, 61, 89}