

Dạng 1. (3 điểm)

1. Cho $\sin \alpha = \frac{7}{25}$ ($\frac{\pi}{2} < \alpha < \pi$). Tính $\sin 2\alpha$, $\cos 2\alpha$, $\tan 4\alpha$.

ĐS: $\sin 2\alpha = -\frac{168}{625}$; $\cos 2\alpha = \frac{527}{625}$; $\sin 4\alpha = -\frac{177072}{390625}$; $\cos 4\alpha = \frac{164833}{390625}$; $\tan 4\alpha = -\frac{177072}{164833}$

2. Cho $\cos x = \frac{4}{5}$ ($-\frac{\pi}{2} < x < 0$). Tính $\sin 2x$, $\cos 2x$, $\cot 4x$

ĐS: $\sin 2x = -\frac{24}{25}$; $\cos 2x = \frac{7}{25}$; $\sin 4x = -\frac{336}{625}$; $\cos 4x = \frac{-527}{625}$; $\cot 4x = \frac{527}{336}$

Dạng 2. (2 điểm)

3. Cho $\sin 2x = -\frac{1}{4}$ ($\pi < 2x < \frac{3\pi}{2}$). Tính $\sin x$, $\cos x$, $\tan x$, $\cot x$

ĐS: $\cos 2x = -\frac{\sqrt{15}}{4}$; $\sin x = \frac{\sqrt{3} + \sqrt{5}}{4}$; $\cos x = \frac{\sqrt{3} - \sqrt{5}}{4}$; $\tan x = -4 - \sqrt{15}$; $\cot x = -4 + \sqrt{15}$

4. Cho $\sin 2x = -\frac{4}{5}$ ($-\frac{\pi}{4} < x < -\frac{\pi}{2}$). Tính $\sin x$, $\cos x$, $\tan x$, $\cot x$.

ĐS: $\cos 2x = \frac{3}{5}$; $\sin x = -\frac{2\sqrt{5}}{5}$; $\cos x = \frac{\sqrt{5}}{5}$; $\tan x = -2$; $\cot x = -\frac{1}{2}$

Dạng 3. (1 điểm)

5. $\cos 2B - (\cos 2A + \cos 2C) = 4 \sin A \cdot \cos B \cdot \sin C - 1$

6. $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \cdot \sin \frac{B}{2} \cdot \sin \frac{C}{2}$

7. $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

8. $\sin A + \sin B - \sin C = 4 \sin \frac{A}{2} \sin \frac{B}{2} \cos \frac{C}{2}$

9. $\tan \frac{A}{2} \cdot \tan \frac{B}{2} + \tan \frac{A}{2} \cdot \tan \frac{C}{2} + \tan \frac{B}{2} \cdot \tan \frac{C}{2} = 1$

10. $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} \cdot \cot \frac{B}{2} \cdot \cot \frac{C}{2}$

11. $\tan \frac{3A}{2} \cdot \tan \frac{3B}{2} + \tan \frac{3A}{2} \cdot \tan \frac{3C}{2} + \tan \frac{3B}{2} \cdot \tan \frac{3C}{2} = 1$

12. $\cot \frac{3A}{2} + \cot \frac{3B}{2} + \cot \frac{3C}{2} = \cot \frac{3A}{2} \cdot \cot \frac{3B}{2} \cdot \cot \frac{3C}{2}$

13. $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

14. $\cot A \cdot \cot B + \cot B \cdot \cot C + \cot A \cdot \cot C = 1$

15. $\cos \frac{3A}{2} \cdot \cos(\frac{5B}{2} + \frac{C}{2}) + \sin(A + 2B) = \sin(\frac{5B}{2} + \frac{C}{2}) \cdot \cos(\frac{\pi}{2} - \frac{3A}{2})$

16. $\sin \frac{A}{2} = \cos \frac{B}{2} \cdot \cos \frac{C}{2} - \sin \frac{B}{2} \cdot \sin \frac{C}{2}$

17. $\cos A = \sin B \cdot \sin C - \cos B \cdot \cos C$

18. $\cos C - \sin A \cdot \sin B = -\cos A \cdot \cos B$

$$19. \sin 5A \cdot \cos 5B = \sin 5C - \sin 5B \cdot \cos 5A$$

$$20. \cos \frac{7A}{2} \cdot \cos \frac{7B}{2} + \cos \frac{7C}{2} = \sin \frac{7A}{2} \cdot \sin \frac{7B}{2}$$

$$21. \sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A \cdot \cos B \cdot \cos C$$

Hướng dẫn

$$\text{Vế trái} = \frac{1 - \cos 2A}{2} + \frac{1 - \cos 2B}{2} + 1 - \cos^2 C$$

$$= \frac{1}{2} - \frac{\cos 2A}{2} + \frac{1}{2} - \frac{\cos 2B}{2} + 1 - \cos^2 C$$

$$= 2 - \frac{1}{2}(\cos 2A + \cos 2B) - \cos^2 C$$

$$= 2 - \frac{1}{2} \cdot 2 \cdot \cos \frac{2A+2B}{2} \cdot \cos \frac{2A-2B}{2} - \cos^2 C$$

$$= 2 - \cos(A+B) \cdot \cos(A-B) - \cos C \cdot \cos C$$

Ta có $A+B+C = \pi$

$$\rightarrow A+B = \pi - C$$

$$\rightarrow \cos(A+B) = \cos(\pi - C)$$

$$\rightarrow \cos(A+B) = -\cos C$$

$$\text{Thay } \begin{cases} \cos(A+B) = -\cos C \\ \text{và } -\cos C = \cos(A+B) \end{cases}$$

$$\text{Vế trái} = 2 - (-\cos C) \cdot \cos(A-B) + \cos(A+B) \cdot \cos C$$

$$= 2 + \cos C \cdot \cos(A-B) + \cos(A+B) \cdot \cos C$$

$$= 2 + \cos C \cdot [\cos(A-B) + \cos(A+B)]$$

$$= 2 + \cos C \cdot 2 \cdot \cos \frac{A-B+A+B}{2} \cdot \cos \frac{A-B-(A+B)}{2}$$

$$= 2 + 2\cos C \cdot \cos A \cdot \cos(-B)$$

$$= 2 + 2\cos C \cdot \cos A \cdot \cos B$$

$$= 2 + 2\cos A \cdot \cos B \cdot \cos C$$

=Vế phải

$$22. \cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A \cdot \cos B \cdot \cos C$$

Hướng dẫn

Cách 1

$$\text{Vế trái} = 1 - \sin^2 A + 1 - \sin^2 B + 1 - \sin^2 C$$

$$= 3 - (\sin^2 A + \sin^2 B + \sin^2 C) \quad \text{chứng minh tương tự bài 13 ta được}$$

$$= 3 - (2 + 2\cos A \cdot \cos B \cdot \cos C)$$

$$= 1 - 2\cos A \cdot \cos B \cdot \cos C$$

=Vế phải

Cách 2

$$\text{Vế trái} = \frac{1 + \cos 2A}{2} + \frac{1 + \cos 2B}{2} + \cos^2 C$$

$$= \frac{1}{2} + \frac{1}{2} \cos 2A + \frac{1}{2} + \frac{1}{2} \cos 2B + \cos^2 C$$

$$\begin{aligned}
 &= 1 + \frac{1}{2}(\cos 2A + \cos 2B) + \cos^2 C \\
 &= 1 + \frac{1}{2} \cdot 2 \cdot \cos \frac{2A+2B}{2} \cdot \cos \frac{2A-2B}{2} + \cos^2 C \\
 &= 1 + \cos(A+B) \cdot \cos(A-B) + \cos C \cdot \cos C
 \end{aligned}$$

Ta có $A+B+C=\pi$

$$\rightarrow A+B=\pi-C$$

$$\rightarrow \cos(A+B)=\cos(\pi-C)$$

$$\rightarrow \cos(A+B)=-\cos C$$

$$\text{Thay } \begin{cases} \cos(A+B)=-\cos C \\ \text{và } \cos C=-\cos(A+B) \end{cases}$$

$$\text{Vế trái} = 2 - \cos C \cdot \cos(A-B) - \cos(A+B) \cdot \cos C$$

$$= 1 - \cos C \cdot [\cos(A-B) + \cos(A+B)]$$

$$= 1 - \cos C \cdot 2 \cdot \cos \frac{A-B+A+B}{2} \cdot \cos \frac{A-B-(A+B)}{2}$$

$$= 1 - 2 \cos C \cdot \cos A \cdot \cos(-B)$$

$$= 1 - 2 \cos C \cdot \cos A \cdot \cos B$$

$$= 1 - 2 \cos A \cdot \cos B \cdot \cos C$$

=Vế phải

$$23. \sin 3A + \sin 3B + \sin 3C = -4 \cos \frac{3A}{2} \cdot \cos \frac{3B}{2} \cdot \cos \frac{3C}{2}$$

$$\text{Vế trái} = 2 \sin \frac{3A+3B}{2} \cdot \cos \frac{3A-3B}{2} + 2 \sin \frac{3C}{2} \cdot \cos \frac{3C}{2}$$

Ta có $A+B+C=\pi$

$$\Leftrightarrow A+B=\pi-C$$

$$\Leftrightarrow \frac{3A+3B}{2} = \frac{3\pi}{2} - \frac{3C}{2}$$

$$\Rightarrow \begin{cases} \sin\left(\frac{3A+3B}{2}\right) = \sin\left(\frac{3\pi}{2} - \frac{3C}{2}\right) = -\cos \frac{3C}{2} \\ \cos\left(\frac{3A+3B}{2}\right) = \cos\left(\frac{3\pi}{2} - \frac{3C}{2}\right) = -\sin \frac{3C}{2} \end{cases}$$

$$\text{Thay } \begin{cases} \sin\left(\frac{3A+3B}{2}\right) = -\cos \frac{3C}{2} \\ \sin \frac{3C}{2} = -\cos\left(\frac{3A+3B}{2}\right) \end{cases}$$

$$\text{Vế trái} = -2 \cos \frac{3C}{2} \cdot \cos \frac{3A-3B}{2} - 2 \cos \frac{3A+3B}{2} \cdot \cos \frac{3C}{2}$$

$$= -2 \cos \frac{3C}{2} \left[\cos \frac{3A-3B}{2} + \cos \frac{3A+3B}{2} \right]$$

$$= -2 \cos \frac{3C}{2} \cdot 2 \cos \frac{\frac{3A-3B}{2} + \frac{3A+3B}{2}}{2} \cdot \cos \frac{\frac{3A-3B}{2} - \frac{3A+3B}{2}}{2}$$

$$= -4 \cos \frac{3C}{2} \cos \frac{3A}{2} \cos \frac{-3B}{2}$$

$$= -4 \cos \frac{3C}{2} \cos \frac{3A}{2} \cos \frac{3B}{2}$$

$$= -4 \cos \frac{3A}{2} \cdot \cos \frac{3B}{2} \cdot \cos \frac{3C}{2}$$

=Về phải

$$24. \sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C$$

$$\text{Về trái} = 2 \sin \frac{2A+2B}{2} \cdot \cos \frac{2A-2B}{2} - 2 \sin C \cdot \cos C$$

$$= 2 \sin(A+B) \cdot \cos(A-B) - 2 \sin C \cdot \cos C$$

Ta có $A + B + C = \pi$

$$\Leftrightarrow A + B = \pi - C$$

$$\Rightarrow \begin{cases} \sin(A+B) = \sin(\pi - C) = \sin C \\ \cos(A+B) = \cos(\pi - C) = -\cos C \end{cases}$$

$$\text{Thay } \begin{cases} \sin(A+B) = \sin C \\ \cos C = -\cos(A+B) \end{cases}$$

$$\text{Về trái} = 2 \sin C \cdot \cos(A-B) + 2 \sin C \cdot \cos(A+B)$$

$$= 2 \sin C [\cos(A-B) + \cos(A+B)]$$

$$= 2 \sin C \cdot 2 \cdot \cos \frac{A-B+A+B}{2} \cdot \cos \frac{A-B-(A+B)}{2}$$

$$= 4 \sin C \cdot \cos A \cdot \cos(-B)$$

$$= 4 \sin C \cdot \cos A \cdot \cos B$$

$$= 4 \cos A \cos B \sin C$$

=Về phải

Dạng 4. Tính giá trị biểu thức (1 điểm)

$$25. A = \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ$$

$$\text{ĐS: } A = \frac{1}{8}$$

$$26. A = \sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ$$

$$\text{ĐS: } A = \frac{1}{16}$$

$$27. A = \sin \frac{\pi}{30} \cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15}$$

$$\text{ĐS: } A = \frac{1}{16}$$

$$28. B = \sin \frac{\pi}{9} \sin \frac{2\pi}{9} \sin \frac{3\pi}{9} \sin \frac{4\pi}{9}$$

$$\text{ĐS: } A = \frac{\sqrt{3}}{8}$$

$$29. A = \cos 10^\circ \cdot \cos 50^\circ - \cos 5^\circ \cdot \cos 25^\circ + \frac{1}{2} \sin 10^\circ$$

$$\text{ĐS: } A = \frac{1}{4} - \frac{\sqrt{3}}{4}$$

$$30. A = 2(\cos 14^\circ + \cos 28^\circ) - \frac{\sin 35^\circ}{\sin 7^\circ}$$

$$\text{ĐS: } A = -1$$

$$31. A = \tan 189^\circ - \tan 27^\circ - \tan 63^\circ + \tan 441^\circ$$

$$\text{ĐS: } A = 4$$

$$32. A = \cos \frac{\pi}{7} - \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7}$$

$$\text{ĐS: } A = \frac{1}{2}$$

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$$33. B = \frac{\tan 30^\circ + \tan 40^\circ + \tan 50^\circ + \tan 60^\circ}{\cos 20^\circ}$$

$$\text{ĐS: } B = \frac{8\sqrt{3}}{3}$$

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