

GIỚI HẠN HÀM SỐ LƯỢNG GIÁC

Dạng 4: $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

Câu 1: Tìm các giới hạn sau:

1). $\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$

2). $\lim_{x \rightarrow 0} \frac{\tan 2x}{3x}$

3). $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x}$

4). $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

5). $\lim_{x \rightarrow 0} \frac{\sin 5x \cdot \sin 3x \cdot \sin x}{45x^3}$

6). $\lim_{x \rightarrow 0} \frac{\sin 7x - \sin 5x}{\sin x}$

7). $\lim_{x \rightarrow 0} \frac{1 - \cos 5x}{1 - \cos 3x}$

8). $\lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{x \cdot \sin x}$

9). $L = \lim_{x \rightarrow 0} \frac{x \cdot \sin ax}{1 - \cos ax}$

LỜI GIẢI

1). $\lim_{x \rightarrow 0} \frac{\sin 5x}{x} = \lim_{x \rightarrow 0} \frac{1}{5} \cdot \frac{\sin 5x}{5x} = \frac{1}{5}$

2). $\lim_{x \rightarrow 0} \frac{\tan 2x}{3x} = \lim_{x \rightarrow 0} \frac{2}{3} \cdot \frac{\tan 2x}{2x} = \frac{2}{3}$

3). $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \lim_{x \rightarrow 0} \tan \frac{x}{2} = 0$

4). $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{x^2} = \lim_{x \rightarrow 0} \frac{1}{2} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 = \frac{1}{2}$

5). $\lim_{x \rightarrow 0} \frac{\sin 5x \cdot \sin 3x \cdot \sin x}{45x^3} = \lim_{x \rightarrow 0} \frac{1}{3} \cdot \frac{\sin 5x}{5x} \cdot \frac{\sin 3x}{3x} \cdot \frac{\sin x}{x} = \frac{1}{3}$

6). $\lim_{x \rightarrow 0} \frac{\sin 7x - \sin 5x}{\sin x} = \lim_{x \rightarrow 0} \frac{2 \cos 6x \sin x}{\sin x} = \lim_{x \rightarrow 0} 2 \cos 6x = 2$

7). $\lim_{x \rightarrow 0} \frac{1 - \cos 5x}{1 - \cos 3x} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{5x}{2}}{2 \sin^2 \frac{3x}{2}} = \lim_{x \rightarrow 0} \frac{25}{9} \cdot \left(\frac{\sin \frac{5x}{2}}{\frac{5x}{2}} \right)^2 \cdot \left(\frac{\frac{3x}{2}}{\sin \frac{3x}{2}} \right)^2 = \frac{25}{9}$

$$\left(\text{Vì } \lim_{x \rightarrow 0} \frac{\sin \frac{5x}{2}}{\frac{5x}{2}} = 1, \lim_{x \rightarrow 0} \frac{\frac{3x}{2}}{\sin \frac{3x}{2}} = 1 \right)$$

$$\begin{aligned} 8). \lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{x \cdot \sin x} &= \lim_{x \rightarrow 0} \frac{(1 - \cos 2x)(1 + \cos 2x)}{x \cdot \sin x} \\ &= \lim_{x \rightarrow 0} \frac{2 \sin^2 x (1 + \cos 2x)}{x \cdot \sin x} = \lim_{x \rightarrow 0} 2(1 + \cos 2x) \frac{\sin x}{x} = 4 \end{aligned}$$

$$9). L = \lim_{x \rightarrow 0} \frac{x \cdot \sin ax}{1 - \cos ax} = \lim_{x \rightarrow 0} \frac{x \cdot 2 \sin \frac{ax}{2} \cos \frac{ax}{2}}{2 \sin^2 \frac{ax}{2}} = \lim_{x \rightarrow 0} \frac{x}{\sin \frac{ax}{2}} \cdot \cos \frac{ax}{2} = \lim_{x \rightarrow 0} \frac{\frac{ax}{2}}{\sin \frac{ax}{2}} \cdot \frac{\cos \frac{ax}{2}}{\frac{a}{2}}$$

$$\left(\text{Vì } \lim_{x \rightarrow 0} \frac{\frac{ax}{2}}{\sin \frac{ax}{2}} = 1 \text{ và } \lim_{x \rightarrow 0} \frac{\cos \frac{ax}{2}}{\frac{a}{2}} = \frac{2}{a} \right). \text{ Vậy } L = \frac{2}{a}.$$

Câu 2: Tìm các giới hạn sau:

$$1). \lim_{x \rightarrow 0} \frac{1 - \cos ax}{1 - \cos bx}$$

$$2). \lim_{x \rightarrow 0} \frac{\sin x \cdot \sin 2x \dots \sin nx}{n! x^n}$$

$$3). \lim_{x \rightarrow 0} \frac{1 - \cos ax}{x^2} \quad (a \neq 0)$$

$$4). \lim_{x \rightarrow 0} \frac{\sin x - \tan x}{x^3}$$

$$5). \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x}$$

$$6). \lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a}$$

$$7). \lim_{x \rightarrow b} \frac{\cos x - \cos b}{x - b}$$

$$8). \lim_{x \rightarrow 0} \frac{1 - \sqrt{2x+1}}{\sin 2x}$$

$$9). \lim_{x \rightarrow 0} \frac{\cos(a+x) - \cos(a-x)}{x}$$

LỜI GIẢI

$$1). L = \lim_{x \rightarrow 0} \frac{1 - \cos ax}{1 - \cos bx} = \frac{2 \sin^2 \frac{ax}{2}}{2 \sin^2 \frac{bx}{2}} = \lim_{x \rightarrow 0} \left(\frac{a}{b} \cdot \frac{\sin \frac{ax}{2}}{\frac{ax}{2}} \cdot \frac{\frac{bx}{2}}{\sin \frac{bx}{2}} \right)$$

$$\text{Vì } \lim_{x \rightarrow 0} \frac{\sin \frac{ax}{2}}{\frac{ax}{2}} = 1, \lim_{x \rightarrow 0} \frac{\frac{bx}{2}}{\sin \frac{bx}{2}} = 1. \text{ Vậy } L = \frac{a}{b}$$

$$2). L = \lim_{x \rightarrow 0} \frac{\sin x \cdot \sin 2x \dots \sin nx}{n! x^n} = \lim_{x \rightarrow 0} \frac{\sin x \cdot \sin 2x \dots \sin nx}{1 \cdot 2 \cdot 3 \dots nx^n} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{\sin 2x}{2x} \dots \frac{\sin nx}{nx}$$

$$\text{Vì } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1, \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} = 1, \dots, \lim_{x \rightarrow 0} \frac{\sin nx}{nx} = 1$$

Vậy $L = 1$.

$$3). L = \lim_{x \rightarrow 0} \frac{1 - \cos ax}{x^2} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{ax}{2}}{x^2} = \lim_{x \rightarrow 0} \frac{a^2}{4} \left(\frac{\sin \frac{ax}{2}}{\frac{ax}{2}} \right)^2 \quad (\text{vì } \lim_{x \rightarrow 0} \frac{\sin \frac{ax}{2}}{\frac{ax}{2}} = 1).$$

Vậy $L = \frac{a^2}{4}$.

$$4). L = \lim_{x \rightarrow 0} \frac{\sin x - \tan x}{x^3} = \lim_{x \rightarrow 0} \frac{\sin x - \frac{\sin x}{\cos x}}{x^3} = \frac{\sin x (\cos x - 1)}{x^3 \cos x}$$

$$= \lim_{x \rightarrow 0} \frac{-2 \sin^2 \frac{x}{2} \sin x}{x^3 \cos x} = \lim_{x \rightarrow 0} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 \cdot \frac{\sin x}{x} \cdot \frac{-1}{2 \cos x}$$

$$\text{Vì } \lim_{x \rightarrow 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}} = 1, \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1, \lim_{x \rightarrow 0} \frac{-1}{2 \cos x} = -\frac{1}{2}.$$

Vậy $L = -\frac{1}{2}$

$$5). \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x} = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x} - \sin x}{\sin^3 x} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{\cos x \sin^2 x} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{\cos x \sin^2 x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{2} \cdot \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2}{\cos x \cdot \left(\frac{\sin x}{x} \right)^2} = \frac{1}{2}$$

$$6). \lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} = \lim_{x \rightarrow a} \frac{2 \sin \frac{x+a}{2} \sin \frac{x-a}{2}}{x - a} = \lim_{x \rightarrow a} \sin \frac{x+a}{2} \cdot \frac{\sin \frac{x-a}{2}}{\frac{x-a}{2}} = \sin a$$

$$7). \lim_{x \rightarrow b} \frac{\cos x - \cos b}{x - b} = \lim_{x \rightarrow b} \frac{-2 \sin \frac{x+b}{2} \sin \frac{x-b}{2}}{x - b} = \lim_{x \rightarrow b} \left(-\sin \frac{x+b}{2} \right) \cdot \frac{\sin \frac{x-b}{2}}{\frac{x-b}{2}} = -\sin b$$

$$8). \lim_{x \rightarrow 0} \frac{1 - \sqrt{2x+1}}{\sin 2x} = \lim_{x \rightarrow 0} \frac{2x}{\sin 2x} \cdot \frac{-1}{1 + \sqrt{2x+1}} = -\frac{1}{2}$$

$$9). L = \lim_{x \rightarrow 0} \frac{\cos(a+x) - \cos(a-x)}{x} = \lim_{x \rightarrow 0} \frac{-2 \sin a \sin x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot (-2 \sin a)$$

(Vì $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$). Vậy $L = -2 \sin a$

Câu 2: Tìm các giới hạn sau:

1). $\lim_{x \rightarrow c} \frac{\tan x - \tan c}{x - c}$

2). $\lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{x \sin x}$

3). $\lim_{x \rightarrow a} \frac{\sin^2 x - \sin^2 a}{x^2 - a^2}$

4). $\lim_{x \rightarrow 0} \frac{\cos \alpha x - \cos \beta x}{x^2}$

5). $\lim_{x \rightarrow 0} \frac{\sin 5x - \sin 3x}{\sin x}$

6). $\lim_{x \rightarrow 1} (1-x) \tan \frac{\pi x}{2}$

7). $\lim_{x \rightarrow 2} \frac{x^3 + 8}{\tan(x+2)}$

8). $\lim_{x \rightarrow 0} \frac{1 - \cos x \cdot \cos 2x \cdot \cos 3x}{1 - \cos x}$

9). $\lim_{x \rightarrow 0} \frac{\sin(a+2x) - 2 \sin(a+x) + \sin a}{x^2}$

10). $\lim_{x \rightarrow 0} \frac{\tan(a+2x) - 2 \tan(a+x) + \tan a}{x^2}$

LỜI GIẢI

1). $\lim_{x \rightarrow c} \frac{\tan x - \tan c}{x - c} = \lim_{x \rightarrow c} \frac{\sin(x-c)}{x-c} \cdot \frac{1}{\cos x \cos c} = \frac{1}{\cos^2 c}$ (vì $\lim_{x \rightarrow c} \frac{\sin(x-c)}{x-c} = 1$).

2). $\lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{x \sin x} = \lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \cos x + \cos^2 x)}{x \sin x}$

$$= \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{x \cdot 2 \sin \frac{x}{2} \cos \frac{x}{2}} (1 + \cos x + \cos^2 x) = \lim_{x \rightarrow 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}} \cdot \frac{1 + \cos x + \cos^2 x}{2 \cos \frac{x}{2}} = \frac{3}{2}$$

3). $\lim_{x \rightarrow a} \frac{\sin^2 x - \sin^2 a}{x^2 - a^2} = \lim_{x \rightarrow a} \frac{(\sin x - \sin a)(\sin x + \sin a)}{(x-a)(x+a)}$

$$= \lim_{x \rightarrow a} \frac{2 \cos \frac{x+a}{2} \sin \frac{x-a}{2}}{2 \cdot \frac{x-a}{2}} \cdot \frac{\sin x + \sin a}{x+a} = \lim_{x \rightarrow a} \frac{\sin \frac{x-a}{2}}{\frac{x-a}{2}} \cdot \frac{\cos \frac{x+a}{2} (\sin x + \sin a)}{x+a}$$

$$= \frac{2 \cos a \cdot \sin a}{2a} = \frac{\sin 2a}{2a}.$$

$$4). \lim_{x \rightarrow 0} \frac{\cos \alpha x - \cos \beta x}{x^2} = \lim_{x \rightarrow 0} \frac{-2 \sin \frac{x(\alpha+\beta)}{2} \cdot \sin \frac{x(\alpha-\beta)}{2}}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{\sin \frac{x(\alpha+\beta)}{2}}{\frac{x(\alpha+\beta)}{2}} \cdot \lim_{x \rightarrow 0} \frac{\sin \frac{x(\alpha-\beta)}{2}}{\frac{x(\alpha-\beta)}{2}} \cdot \lim_{x \rightarrow 0} \frac{(\alpha+\beta)(\alpha-\beta)}{2 \cdot 2} (-2) = \frac{\beta^2 - \alpha^2}{2}.$$

$$5). \lim_{x \rightarrow 0} \frac{\sin 5x - \sin 3x}{\sin x} = \lim_{x \rightarrow 0} \frac{2 \cos 4x \sin x}{\sin x} = \lim_{x \rightarrow 0} (2 \cos 4x) = 2$$

$$6). L = \lim_{x \rightarrow 1} (1-x) \tan \frac{\pi x}{2}. \text{ Đặt } t = x-1, \text{ vì } x \rightarrow 1 \Rightarrow t \rightarrow 0$$

$$L = \lim_{t \rightarrow 0} (-t) \tan \frac{\pi}{2} (t+1) = \lim_{t \rightarrow 0} (-t) \tan \left(\frac{\pi}{2} + \frac{\pi}{2} t \right) = \lim_{t \rightarrow 0} t \cot \frac{\pi}{2} t$$

$$= \lim_{t \rightarrow 0} t \cdot \frac{\cos \frac{\pi}{2} t}{\sin \frac{\pi}{2} t} = \lim_{t \rightarrow 0} \frac{\frac{\pi}{2} t}{\sin \frac{\pi}{2} t} \cdot \frac{\cos \frac{\pi}{2} t}{\frac{\pi}{2}} = \frac{2}{\pi}$$

$$7). \lim_{x \rightarrow -2} \frac{x^3 + 8}{\tan(x+2)} = \lim_{x \rightarrow -2} \frac{(x+2)(x^2 - 2x + 4)}{\tan(x+2)} = \lim_{x \rightarrow -2} \frac{x+2}{\tan(x+2)} (x^2 - 2x + 4) = 12$$

$$(\forall \lim_{x \rightarrow -2} \frac{x+2}{\tan(x+2)} = 1).$$

$$8). \lim_{x \rightarrow 0} \frac{1 - \cos x \cdot \cos 2x \cdot \cos 3x}{1 - \cos x}$$

$$\lim_{x \rightarrow 0} \frac{(1 - \cos x) \cos 2x \cdot \cos 3x + (1 - \cos 2x) \cos 3x + (1 - \cos 3x)}{1 - \cos x}$$

$$= \lim_{x \rightarrow 0} \frac{(1 - \cos x) \cos 2x \cdot \cos 3x}{1 - \cos x} + \lim_{x \rightarrow 0} \frac{(1 - \cos 2x) \cos 3x}{1 - \cos x} + \lim_{x \rightarrow 0} \frac{1 - \cos 3x}{1 - \cos x}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \cos 2x \cdot \cos 3x + \lim_{x \rightarrow 0} \frac{2 \sin^2 x \cos 3x}{2 \sin^2 \frac{x}{2}} + \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{3x}{2}}{2 \sin^2 \frac{x}{2}} \\
 &= 1 + \lim_{x \rightarrow 0} \frac{4 \sin^2 \frac{x}{2} \cos^2 \frac{x}{2} \cos 3x}{\sin^2 \frac{x}{2}} + \lim_{x \rightarrow 0} 9 \cdot \frac{\left(\frac{\sin \frac{3x}{2}}{\frac{x}{2}} \right)^2}{\left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2} = 1 + 4 + 9 = 14
 \end{aligned}$$

9). $\lim_{x \rightarrow 0} \frac{\sin(a+2x) - 2\sin(a+x) + \sin a}{x^2}$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{\sin(a+2x) - \sin(a+x) + \sin a - \sin(a+x)}{x^2} \\
 &= \lim_{x \rightarrow 0} \frac{2 \cos\left(a + \frac{3x}{2}\right) \sin \frac{x}{2} - 2 \cos\left(a + \frac{x}{2}\right) \sin \frac{x}{2}}{x^2} \\
 &= \lim_{x \rightarrow 0} \frac{2 \sin \frac{x}{2} \left[\cos\left(a + \frac{3x}{2}\right) - \cos\left(a + \frac{x}{2}\right) \right]}{x^2} = \lim_{x \rightarrow 0} \frac{-4 \sin \frac{x}{2} \sin(a+x) \sin \frac{x}{2}}{x^2} \\
 &= \lim_{x \rightarrow 0} (-1) \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 \sin(a+x) = -\sin a
 \end{aligned}$$

10). $\lim_{x \rightarrow 0} \frac{\tan(a+2x) - 2\tan(a+x) + \tan a}{x^2}$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{\tan(a+2x) - \tan(a+x) - (\tan(a+x) - \tan a)}{x^2} \\
 &= \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos(a+2x)\cos(a+x)} - \frac{\sin x}{\cos(a+x)\cos a}}{x^2} \\
 &= \lim_{x \rightarrow 0} \frac{\sin x}{x^2} \left(\frac{\cos a - \cos(a+2x)}{\cos(a+2x)\cos(a+x)\cos a} \right) = \lim_{x \rightarrow 0} \frac{\sin x}{x^2} \left(\frac{2 \sin x \sin(a+x)}{\cos(a+2x)\cos(a+x)\cos a} \right)
 \end{aligned}$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 \left(\frac{2 \sin(a+x)}{\cos(a+2x) \cos(a+x) \cos a} \right) = \frac{2 \sin a}{\cos^3 a}.$$

Câu 3: Tìm các giới hạn sau:

1). $\lim_{x \rightarrow 0} \frac{\sin ax + \tan bx}{(a+b)x} \quad (a+b \neq 0)$

2). $\lim_{x \rightarrow 0} \frac{\cos 3x - \cos 5x \cdot \cos 7x}{x^2}$

3). $\lim_{x \rightarrow 0} \frac{\cos ax - \cos bx \cdot \cos cx}{x^2}$

4). $\lim_{x \rightarrow 0} \frac{\sin(a+x) - \sin(a-x)}{\tan(a+x) - \tan(a-x)}$

5). $\lim_{x \rightarrow 0} \frac{\sqrt{2x+1} - \sqrt[3]{x^2+1}}{\sin x}$

6). $\lim_{x \rightarrow 0} \frac{\sin^2 2x - \sin x \cdot \sin 4x}{x^4}$

7). $\lim_{x \rightarrow 0} \frac{1 - \cos 5x \cdot \cos 7x}{\sin^2 11x}$

8). $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{\tan x} \right)$

9). $\lim_{x \rightarrow 0} \frac{\sin x - \sin 2x}{x \left(1 - 2 \sin^2 \frac{x}{2} \right)}$

10). $\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \cos x}{x^2}$

LỜI GIẢI

1). $\lim_{x \rightarrow 0} \frac{\sin ax + \tan bx}{(a+b)x} = \lim_{x \rightarrow 0} \frac{\sin ax + \frac{\sin bx}{\cos bx}}{(a+b)x} = \lim_{x \rightarrow 0} \frac{\sin ax}{(a+b)x} + \lim_{x \rightarrow 0} \frac{\sin bx}{(a+b)x \cdot \cos bx}$

$$= \lim_{x \rightarrow 0} \frac{a}{a+b} \cdot \frac{\sin ax}{ax} + \lim_{x \rightarrow 0} \frac{b}{(a+b) \cos bx} \cdot \frac{\sin bx}{bx} = \frac{a}{a+b} + \frac{b}{a+b} = 1$$

2). $\lim_{x \rightarrow 0} \frac{\cos 3x - \cos 5x \cdot \cos 7x}{x^2} = \lim_{x \rightarrow 0} \frac{\cos 3x - 1 + (1 - \cos 5x) \cos 7x + 1 - \cos 7x}{x^2}$

$$= \lim_{x \rightarrow 0} \frac{\cos 3x - 1}{x^2} + \lim_{x \rightarrow 0} \frac{(1 - \cos 5x) \cos 7x}{x^2} + \lim_{x \rightarrow 0} \frac{1 - \cos 7x}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{-2 \sin^2 \frac{3x}{2}}{x^2} + \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{5x}{2} \cos 7x}{x^2} + \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{7x}{2}}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{-9}{2} \cdot \left(\frac{\sin \frac{3x}{2}}{\frac{3x}{2}} \right)^2 + \lim_{x \rightarrow 0} \frac{25 \cos 7x}{2} \left(\frac{\sin \frac{5x}{2}}{\frac{5x}{2}} \right)^2 + \lim_{x \rightarrow 0} \frac{49}{2} \left(\frac{\sin \frac{7x}{2}}{\frac{7x}{2}} \right)^2 = -\frac{9}{2} + \frac{25}{2} + \frac{49}{2} = \frac{65}{2}$$

$$3). \lim_{x \rightarrow 0} \frac{\cos ax - \cos bx \cdot \cos cx}{x^2} = \lim_{x \rightarrow 0} \frac{\cos ax - 1 - (\cos bx - 1) \cos cx + 1 - \cos cx}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{-2 \sin^2 \frac{ax}{2}}{x^2} + \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{bx}{2} \cos cx}{x^2} + \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{cx}{2}}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{-a^2}{2} \left(\frac{\sin \frac{ax}{2}}{\frac{ax}{2}} \right)^2 + \lim_{x \rightarrow 0} \frac{b^2 \cos cx}{2} \cdot \left(\frac{\sin \frac{bx}{2}}{\frac{bx}{2}} \right)^2 + \lim_{x \rightarrow 0} \frac{c^2}{2} \cdot \left(\frac{\sin \frac{cx}{2}}{\frac{cx}{2}} \right)^2 = \frac{-a^2 + b^2 + c^2}{2}$$

$$4). \lim_{x \rightarrow 0} \frac{\sin(a+x) - \sin(a-x)}{\tan(a+x) - \tan(a-x)} = \lim_{x \rightarrow 0} \frac{2 \cos a \sin x}{\frac{\sin 2x}{\cos(a+x) \cos(a-x)}}$$

$$= \lim_{x \rightarrow 0} \frac{\cos a \cos(a+x) \cos(a-x)}{\cos x} = \cos^3 a$$

$$5). \lim_{x \rightarrow 0} \frac{\sqrt{2x+1} - \sqrt[3]{x^2+1}}{\sin x}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{2x+1} - 1 + 1 - \sqrt[3]{x^2+1}}{\sin x} = \lim_{x \rightarrow 0} \frac{\sqrt{2x+1} - 1}{\sin x} + \lim_{x \rightarrow 0} \frac{1 - \sqrt[3]{x^2+1}}{\sin x}$$

$$= \lim_{x \rightarrow 0} \frac{2x}{\sin x (\sqrt{2x+1} + 1)} + \lim_{x \rightarrow 0} \frac{-x^2}{\sin x \left[1 + \sqrt[3]{x^2+1} + (\sqrt[3]{x^2+1})^2 \right]}$$

$$= \lim_{x \rightarrow 0} \frac{x}{\sin x} \cdot \frac{2}{\sqrt{2x+1} + 1} + \lim_{x \rightarrow 0} \frac{x}{\sin x} \cdot \frac{-x}{1 + \sqrt[3]{x^2+1} + (\sqrt[3]{x^2+1})^2} = \frac{2}{1+1} + 0 = 1$$

$$6). \lim_{x \rightarrow 0} \frac{\sin^2 2x - \sin x \cdot \sin 4x}{x^4} = \lim_{x \rightarrow 0} \frac{\sin^2 2x - 2 \sin x \sin 2x \cos 2x}{x^4}$$

$$= \lim_{x \rightarrow 0} \frac{\sin 2x (2 \sin x \cos x - 2 \sin x \cos 2x)}{x^4}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin 2x \cdot \sin x (\cos x - \cos 2x)}{x^4} = \lim_{x \rightarrow 0} \frac{4 \sin 2x \cdot \sin x \cdot \sin \frac{3x}{2} \cdot \sin \frac{x}{2}}{x^4}$$

$$= \lim_{x \rightarrow 0} 6 \cdot \left(\frac{\sin 2x}{2x} \right) \cdot \left(\frac{\sin x}{x} \right) \cdot \left(\frac{\sin \frac{3x}{2}}{\frac{3x}{2}} \right) \cdot \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right) = 6$$

7). $\lim_{x \rightarrow 0} \frac{1 - \cos 5x \cdot \cos 7x}{\sin^2 11x}$

$$= \lim_{x \rightarrow 0} \frac{(1 - \cos 5x) \cos 7x + 1 - \cos 7x}{\sin^2 11x} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{5x}{2} \cos 7x}{\sin^2 11x} + \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{7x}{2}}{\sin^2 11x}$$

$$= \lim_{x \rightarrow 0} \frac{\left(\frac{\sin \frac{5x}{2}}{\frac{5x}{2}} \right)^2 \cos 7x}{\left(\frac{\sin 11x}{11x} \right)^2} \cdot \frac{25}{484} + \lim_{x \rightarrow 0} \frac{\left(\frac{\sin \frac{7x}{2}}{\frac{7x}{2}} \right)^2}{\left(\frac{\sin 11x}{11x} \right)^2} \cdot \frac{49}{484} = \frac{25}{484} + \frac{49}{484} = \frac{37}{242}$$

8). $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{\tan x} \right) = \lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right) = \lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$

$$= \lim_{x \rightarrow 0} \tan \frac{x}{2} = 0.$$

9). $\lim_{x \rightarrow 0} \frac{\sin x - \sin 2x}{x(1 - 2 \sin^2 \frac{x}{2})} = \lim_{x \rightarrow 0} \frac{2 \cos \frac{3x}{2} \sin \frac{-x}{2}}{x \cos x} = \lim_{x \rightarrow 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}} \cdot \frac{-\cos \frac{3x}{2}}{\cos x} = -1$

10). $\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \cos x}{x^2} = \lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - 1 + 1 - \cos x}{x^2} = \lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - 1}{x^2} + \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

$$= \lim_{x \rightarrow 0} \frac{x^2}{x^2(\sqrt{1+x^2} + 1)} + \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{x^2} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{1+x^2} + 1} + \lim_{x \rightarrow 0} \frac{1}{2} \cdot \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 = \frac{1}{2} + \frac{1}{2} = 1.$$

Câu 3: Tìm các giới hạn sau:

1). $\lim_{x \rightarrow \frac{\pi}{4}} \tan 2x \cdot \tan \left(\frac{\pi}{4} - x \right)$ 2). $\lim_{x \rightarrow 0} \frac{\sqrt{1 + \tan x} - \sqrt{1 + \sin x}}{x^3}$ 3). $\lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{\tan(x-1)}$

4). $\lim_{x \rightarrow -\frac{\pi}{2}} \frac{\cos x}{x + \frac{\pi}{2}}$

5). $\lim_{x \rightarrow \pi} \frac{1 + \cos x}{(x - \pi)^2}$

6). $\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2 - 4x + 3}$

7). $\lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin x - 1}{4 \cos^2 x - 3}$

8). $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \sin x - 1}{2 \cos^2 x - 1}$

9). $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin\left(\frac{\pi}{6} - x\right)}{1 - 2 \sin x}$

LỜI GIẢI

1). $L = \lim_{x \rightarrow \frac{\pi}{4}} \tan 2x \cdot \tan\left(\frac{\pi}{4} - x\right)$. Đặt $t = x - \frac{\pi}{4}$, vì $x \rightarrow \frac{\pi}{4} \Rightarrow t \rightarrow 0$

$$L = \lim_{t \rightarrow 0} \left[\tan\left(2t + \frac{\pi}{2}\right) (-1) \tan t \right] = \lim_{t \rightarrow 0} (\cot 2t \cdot \tan t)$$

$$= \lim_{t \rightarrow 0} \frac{\cos 2t \sin t}{\sin 2t \cos t} = \lim_{t \rightarrow 0} \frac{\cos 2t}{2 \sin t \cos t} \cdot \frac{\sin t}{\cos t} = \lim_{t \rightarrow 0} \frac{\cos 2t}{2 \cos^2 t} = \frac{1}{2}$$

2). $\lim_{x \rightarrow 0} \frac{\sqrt{1 + \tan x} - \sqrt{1 + \sin x}}{x^3} = \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3 \left(\underbrace{\sqrt{1 + \tan x} + \sqrt{1 + \sin x}}_A \right)} = \lim_{x \rightarrow 0} \frac{\sin x (x - \cos x)}{x^3 \cdot A \cdot \cos x}$

$$= \lim_{x \rightarrow 0} \frac{2 \sin x \sin^2 \frac{x}{2}}{x^3 \cdot A \cdot \cos x} = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \cdot \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 \cdot \frac{1}{2A \cdot \cos x} = \frac{1}{4}$$

3). $\lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{\tan(x-1)} = \lim_{x \rightarrow 1} \frac{x+3-4}{\left(\sqrt{x+3}+2\right) \tan(x-1)} = \lim_{x \rightarrow 1} \frac{x-1}{\tan(x-1)} \cdot \frac{1}{\sqrt{x+3}+2}$

(Vì $\lim_{x \rightarrow 1} \frac{x-1}{\tan(x-1)} = 1$, $\lim_{x \rightarrow 1} \frac{1}{\sqrt{x+3}+2} = \frac{1}{4}$)

Vậy $L = \frac{1}{4}$.

4). $L = \lim_{x \rightarrow -\frac{\pi}{2}} \frac{\cos x}{x + \frac{\pi}{2}}$. Đặt $t = x + \frac{\pi}{2}$, vì $x \rightarrow -\frac{\pi}{2} \Rightarrow t \rightarrow 0$

$$L = \lim_{t \rightarrow 0} \frac{\cos\left(t - \frac{\pi}{2}\right)}{t} = \lim_{t \rightarrow 0} \frac{\sin t}{t} = 1.$$

5). $L = \lim_{x \rightarrow \pi} \frac{1 + \cos x}{(x - \pi)^2}$. Đặt $t = x - \pi$, vì $x \rightarrow \pi \Rightarrow t \rightarrow 0$

$$L = \lim_{t \rightarrow 0} \frac{1 + \cos(t + \pi)}{t^2} = \lim_{t \rightarrow 0} \frac{1 - \cos t}{t^2} = \lim_{t \rightarrow 0} \frac{2 \sin^2 \frac{t}{2}}{t^2} = \lim_{t \rightarrow 0} \frac{1}{2} \left(\frac{\sin \frac{t}{2}}{\frac{t}{2}} \right)^2 = \frac{1}{2}.$$

6). $L = \lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2 - 4x + 3} = \lim_{x \rightarrow 1} \frac{\sin(x-1)}{(x-1)(x-3)}$. Đặt $t = x - 1$, vì $x \rightarrow 1 \Rightarrow t \rightarrow 0$

$$L = \lim_{t \rightarrow 0} \frac{\sin t}{t(t-2)} = \lim_{t \rightarrow 0} \frac{\sin t}{t} \cdot \frac{1}{t-2} = -\frac{1}{2}.$$

7). $L = \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin x - 1}{4 \cos^2 x - 3} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin x - 1}{4(1 - \sin^2 x) - 3} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin x - 1}{1 - 4 \sin^2 x}$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin x - 1}{(1 - 2 \sin x)(1 + 2 \sin x)} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{-1}{1 + 2 \sin x} = -\frac{1}{2}$$

8). $L = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \sin x - 1}{2 \cos^2 x - 1} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \sin x - 1}{2(1 - \sin^2 x) - 1} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \sin x - 1}{1 - 2 \sin^2 x}$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \sin x - 1}{(1 - \sqrt{2} \sin x)(1 + \sqrt{2} \sin x)} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{-1}{1 + \sqrt{2} \sin x} = -\frac{1}{2}.$$

9). $= \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin\left(\frac{\pi}{6} - x\right)}{1 - 2 \sin x} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin\left(\frac{\pi}{6} - x\right)}{-2\left(\sin x - \frac{1}{2}\right)} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin\left(x - \frac{\pi}{6}\right)}{2\left(\sin x - \sin \frac{\pi}{6}\right)}$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin\left(\frac{x}{2} - \frac{\pi}{12}\right) \cos\left(\frac{x}{2} - \frac{\pi}{12}\right)}{4 \cos\left(\frac{x}{2} + \frac{\pi}{12}\right) \sin\left(\frac{x}{2} - \frac{\pi}{12}\right)} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{1}{2} \frac{\cos\left(\frac{x}{2} - \frac{\pi}{12}\right)}{\cos\left(\frac{x}{2} + \frac{\pi}{12}\right)} = \frac{\sqrt{3}}{3}$$

Câu 4: Tìm các giới hạn sau:

$$\begin{array}{lll}
 1). \lim_{x \rightarrow \frac{\pi}{6}} \frac{1-2\sin x}{\frac{\pi}{6}-x} & 2). \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin\left(\frac{\pi}{4}-x\right)}{1-\sqrt{2}\sin x} & 3). \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2}-2\cos x}{\sin\left(x-\frac{\pi}{4}\right)} \\
 4). \lim_{x \rightarrow 0} \frac{\sqrt{x^2+1}-\cos 2x}{x^2} & 5). \lim_{x \rightarrow 0} \frac{\sqrt{1+2x}-\cos x-x}{x^2} & 6). \lim_{x \rightarrow 0} \frac{\sqrt[3]{2x+1}-\sqrt{1-x}}{\sin 2x} \\
 7). \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin x - \sqrt{3}\cos x}{\sin 3x} & 8). \lim_{x \rightarrow 0} \frac{1-\cos x\sqrt{\cos 2x}}{x^2} & 9). \lim_{x \rightarrow 0} \frac{1-\sqrt[3]{\cos x}}{\tan^2 x}
 \end{array}$$

LỜI GIẢI

$$\begin{aligned}
 1). \lim_{x \rightarrow \frac{\pi}{6}} \frac{1-2\sin x}{\frac{\pi}{6}-x} &= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2\sin x-1}{x-\frac{\pi}{6}} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{2\left(\sin x-\frac{1}{2}\right)}{x-\frac{\pi}{6}} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{2\left(\sin x-\sin\frac{\pi}{6}\right)}{x-\frac{\pi}{6}} \\
 &= \lim_{x \rightarrow \frac{\pi}{6}} \frac{4\cos\left(\frac{x}{2}+\frac{\pi}{12}\right)\sin\left(\frac{x}{2}-\frac{\pi}{12}\right)}{2\left(\frac{x}{2}-\frac{\pi}{12}\right)} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\left(\frac{x}{2}-\frac{\pi}{12}\right)} 2\cos\left(\frac{x}{2}+\frac{\pi}{12}\right) = \sqrt{3} \\
 2). \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin\left(\frac{\pi}{4}-x\right)}{1-\sqrt{2}\sin x} &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin\left(x-\frac{\pi}{4}\right)}{\sqrt{2}\sin x-1} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin\left(x-\frac{\pi}{4}\right)}{\sqrt{2}\left(\sin x-\frac{\sqrt{2}}{2}\right)} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin\left(x-\frac{\pi}{4}\right)}{\sqrt{2}\left(\sin x-\sin\frac{\pi}{4}\right)} \\
 &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{2\sin\left(\frac{x}{2}-\frac{\pi}{8}\right)\cos\left(\frac{x}{2}-\frac{\pi}{8}\right)}{\sqrt{2}\cos\left(\frac{x}{2}+\frac{\pi}{8}\right)\sin\left(\frac{x}{2}-\frac{\pi}{8}\right)} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2}\cos\left(\frac{x}{2}-\frac{\pi}{8}\right)}{\cos\left(\frac{x}{2}+\frac{\pi}{8}\right)} = 2 \\
 3). \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2}-2\cos x}{\sin\left(x-\frac{\pi}{4}\right)} &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{-2\left(\cos x-\frac{\sqrt{2}}{2}\right)}{\sin\left(x-\frac{\pi}{4}\right)} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{-2\left(\cos x-\cos\frac{\pi}{4}\right)}{\sin\left(x-\frac{\pi}{4}\right)} \\
 &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{4\sin\left(\frac{x}{2}+\frac{\pi}{8}\right)\sin\left(\frac{x}{2}-\frac{\pi}{8}\right)}{2\sin\left(\frac{x}{2}-\frac{\pi}{8}\right)\cos\left(\frac{x}{2}-\frac{\pi}{8}\right)} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{2\sin\left(\frac{x}{2}+\frac{\pi}{8}\right)}{\cos\left(\frac{x}{2}-\frac{\pi}{8}\right)} = \sqrt{2}
 \end{aligned}$$

4). $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+1} - \cos 2x}{x^2}$. Đặt $f(x) = \frac{\sqrt{x^2+1} - \cos 2x}{x^2}$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2+1} - 1 + 1 - \cos 2x}{x^2} = \lim_{x \rightarrow 0} \frac{\sqrt{x^2+1} - 1}{x^2} + \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$$

• Tính $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+1} - 1}{x^2} = \lim_{x \rightarrow 0} \frac{x^2 + 1 - 1}{x^2(\sqrt{x^2+1} + 1)} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{x^2+1} + 1} = \frac{1}{2}$

• Tính $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} = \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 = 2$

Vậy $\lim_{x \rightarrow 0} f(x) = \frac{1}{2} + 2 = \frac{5}{2}$

5). $L = \lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \cos x - x}{x^2}$. Đặt $f(x) = \frac{\sqrt{1+2x} - \cos x - x}{x^2}$

$$L = \lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - (1+x) + 1 - \cos x}{x^2} = \lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - (1+x)}{x^2} + \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

• Tính $\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - (1+x)}{x^2} = \lim_{x \rightarrow 0} \frac{1+2x - (1+x)^2}{x^2(\sqrt{1+2x} - (1+x))}$

$$= \lim_{x \rightarrow 0} \frac{-x^2}{x^2(\sqrt{1+2x} - (1+x))} = \lim_{x \rightarrow 0} \frac{-1}{(\sqrt{1+2x} + (1+x))} = -\frac{1}{2}$$

• Tính $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{x^2} = \lim_{x \rightarrow 0} \frac{1}{2} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 = \frac{1}{2}$

Vậy $\lim_{x \rightarrow 0} f(x) = -\frac{1}{2} + \frac{1}{2} = 0$.

6). $L = \lim_{x \rightarrow 0} \frac{\sqrt[3]{2x+1} - \sqrt{1-x}}{\sin 2x}$. Đặt $f(x) = \frac{\sqrt[3]{2x+1} - \sqrt{1-x}}{\sin 2x}$

$$L = \lim_{x \rightarrow 0} \frac{\sqrt[3]{2x+1} - 1 + 1 - \sqrt{1-x}}{\sin 2x} = \lim_{x \rightarrow 0} \frac{\sqrt[3]{2x+1} - 1}{\sin 2x} + \lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{\sin 2x}$$

• Tính $\lim_{x \rightarrow 0} \frac{\sqrt[3]{2x+1}-1}{\sin 2x} = \lim_{x \rightarrow 0} \frac{2x+1-1}{\sin 2x \left[(\sqrt[3]{2x+1})^2 + \sqrt[3]{2x+1} + 1 \right]}$

$$= \lim_{x \rightarrow 0} \frac{2x}{2 \sin x \cos x \left[(\sqrt[3]{2x+1})^2 + \sqrt[3]{2x+1} + 1 \right]} = \lim_{x \rightarrow 0} \frac{x}{\sin x} \cdot \frac{1}{\cos x \left[(\sqrt[3]{2x+1})^2 + \sqrt[3]{2x+1} + 1 \right]} = \frac{1}{3}$$

• Tính $\lim_{x \rightarrow 0} \frac{1-\sqrt{1-x}}{\sin 2x} = \lim_{x \rightarrow 0} \frac{x}{2 \sin x \cos x (1+\sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{x}{\sin x} \cdot \frac{1}{2 \cos x (1+\sqrt{1-x})} = \frac{1}{4}$

Vậy $\lim_{x \rightarrow 0} f(x) = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

7). $\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin x - \sqrt{3} \cos x}{\sin 3x} = \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin x - \sqrt{3} \cos x}{3 \sin x - 4 \sin^3 x} = \frac{\sin^2 x - 3 \cos^2 x}{\sin x (3 - 4 \sin^2 x) (\sin x + \sqrt{3} \cos x)}$

$$= \frac{4 \sin^2 x - 3}{\sin x (3 - 4 \sin^2 x) (\sin x + \sqrt{3} \cos x)} = \frac{-1}{\sin x (\sin x + \sqrt{3} \cos x)} = \frac{-2}{3}$$

8). $L = \lim_{x \rightarrow 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2}$

$$L = \lim_{x \rightarrow 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2} = \lim_{x \rightarrow 0} \frac{1 - \cos^2 x \cos 2x}{x^2 (1 + \cos x \sqrt{\cos 2x})}$$

$$= \lim_{x \rightarrow 0} \frac{\cos^2 x + \sin^2 x - \cos^2 x \cos 2x}{x^2 (1 + \cos x \sqrt{\cos 2x})} = \lim_{x \rightarrow 0} \frac{\cos^2 x (1 - \cos 2x) + \sin^2 x}{x^2 (1 + \cos x \sqrt{\cos 2x})}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin^2 x \cos^2 x + \sin^2 x}{x^2 (1 + \cos x \sqrt{\cos 2x})} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \cdot \frac{2 \cos^2 x + 1}{1 + \cos x \sqrt{\cos 2x}} = \frac{3}{2}$$

9). $L = \lim_{x \rightarrow 0} \frac{1 - \sqrt[3]{\cos x}}{\tan^2 x}$

$$L = \lim_{x \rightarrow 0} \frac{1 - \cos x}{\tan^2 x \left[1 + \sqrt[3]{\cos x} + (\sqrt[3]{\cos x})^2 \right]} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2} \cos^2 x}{4 \sin^2 \frac{x}{2} \cos^2 \frac{x}{2} \left[1 + \sqrt[3]{\cos x} + (\sqrt[3]{\cos x})^2 \right]}$$

$$= \lim_{x \rightarrow 0} \frac{\cos^2 x}{2 \cos^2 \frac{x}{2} \left[1 + \sqrt[3]{\cos x} + \left(\sqrt[3]{\cos x} \right)^2 \right]} = \frac{1}{6}.$$

Câu 5: Tìm các giới hạn sau:

1). $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt[3]{\tan x} - 1}{2 \sin^2 x - 1}$

2). $\lim_{x \rightarrow 0} \frac{1 - \cos x \cos 2x}{x^2}$

3). $\lim_{x \rightarrow 0} \left(\frac{2}{\sin 2x} - \cot x \right)$

4). $\lim_{x \rightarrow 0} \frac{1 - \sqrt{2x+1} + \sin x}{\sqrt{3x+4} - 2 - x}$

5). $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+1} - \cos x}{x^2}$

6). $L = \lim_{x \rightarrow 0} \frac{1 - \sin 2x - \cos 2x}{1 + \sin 2x - \cos 2x}$

7). $\lim_{x \rightarrow \frac{\pi}{3}} \frac{\cos 3x + 2 \cos 2x + 2}{\sin 3x}$

8). $\lim_{x \rightarrow 0} \frac{\cos\left(\frac{\pi}{2} \cos x\right)}{\sin^2 \frac{x}{2}}$

9). $\lim_{x \rightarrow 0} \frac{1 - \cos x}{(1 - \sqrt{1-x})^2}$

LỜI GIẢI

1). $L = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt[3]{\tan x} - 1}{2 \sin^2 x - 1}$

$$L = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{(\sin^2 x - \cos^2 x) \left[\left(\sqrt[3]{\tan x} \right)^2 + \sqrt[3]{\tan x} + 1 \right]}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\cos x (\sin x - \cos x) (\sin x + \cos x) \left[\left(\sqrt[3]{\tan x} \right)^2 + \sqrt[3]{\tan x} + 1 \right]}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{1}{\cos x (\sin x + \cos x) \left[\left(\sqrt[3]{\tan x} \right)^2 + \sqrt[3]{\tan x} + 1 \right]} = \frac{1}{3}$$

2). $\lim_{x \rightarrow 0} \frac{1 - \cos x \cos 2x}{x^2}$

$$= \lim_{x \rightarrow 0} \frac{\sin^2 x + \cos^2 x - \cos x \cos 2x}{x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} + \lim_{x \rightarrow 0} \frac{\cos x (\cos x - \cos 2x)}{x^2}$$

$$= 1 + \lim_{x \rightarrow 0} \frac{2 \cos x \sin \frac{3x}{2} \sin \frac{x}{2}}{x^2} = 1 + \lim_{x \rightarrow 0} \left(\frac{\sin \frac{3x}{2}}{\frac{3x}{2}} \right) \cdot \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right) \cdot \frac{9}{8} \cdot \cos x = 1 + \frac{9}{8} = \frac{17}{8}$$

$$3). L = \lim_{x \rightarrow 0} \left(\frac{2}{\sin 2x} - \cot x \right)$$

$$L = \lim_{x \rightarrow 0} \left(\frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x} \right) = \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{\sin x \cos x} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sin x \cos x} = \lim_{x \rightarrow 0} \tan x = 0$$

$$4). L = \lim_{x \rightarrow 0} \frac{1 - \sqrt{2x+1} + \sin x}{\sqrt{3x+4} - 2 - x}$$

$$L = \lim_{x \rightarrow 0} \frac{1 - \sqrt{2x+1} + \sin x}{\sqrt{3x+4} - 2 - x} = \lim_{x \rightarrow 0} \frac{1 - \sqrt{2x+1}}{\sqrt{3x+4} - 2 - x} + \lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{3x+4} - 2 - x}$$

$$= \lim_{x \rightarrow 0} \frac{-2x(\sqrt{3x+4} + 2 + x)}{(-x^2 - x)(1 + \sqrt{2x+1})} + \lim_{x \rightarrow 0} \frac{(\sqrt{3x+4} + 2 + x)\sin x}{-x^2 - x}$$

$$= \lim_{x \rightarrow 0} \frac{2(\sqrt{3x+4} + 2 + x)}{(x+1)(1 + \sqrt{2x+1})} + \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{\sqrt{3x+4} + 2 + x}{-x - 1} = 4 - 4 = 0$$

$$5). \lim_{x \rightarrow 0} \frac{\sqrt{x^2+1} - \cos x}{x^2}$$

$$I = \lim_{x \rightarrow 0} \frac{(\sqrt{1+x^2} - 1) + (1 - \cos x)}{x^2} = \lim_{x \rightarrow 0} \left[\frac{(\sqrt{1+x^2} - 1)}{x^2} + \frac{1 - \cos x}{x^2} \right]$$

$$= \lim_{x \rightarrow 0} \left(\frac{x^2}{x^2(\sqrt{1+x^2} + 1)} + \frac{2 \sin^2 \frac{x}{2}}{x^2} \right) = \lim_{x \rightarrow 0} \left(\frac{1}{\sqrt{1+x^2} + 1} + \frac{\sin^2 \frac{x}{2}}{2 \frac{x^2}{4}} \right) = 1$$

$$6). L = \lim_{x \rightarrow 0} \frac{1 - \sin 2x - \cos 2x}{1 + \sin 2x - \cos 2x}$$

$$L = \lim_{x \rightarrow 0} \frac{1 - \sin 2x - \cos 2x}{1 + \sin 2x - \cos 2x} = \lim_{x \rightarrow 0} \frac{1 - \cos 2x - \sin 2x}{1 - \cos 2x + \sin 2x} = \lim_{x \rightarrow 0} \frac{2 \sin^2 x - 2 \sin x \cos x}{2 \sin^2 x + 2 \sin x \cos x}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin x (\sin x - \cos x)}{2 \sin x (\sin x + \cos x)} = \lim_{x \rightarrow 0} \frac{\sin x - \cos x}{\sin x + \cos x} = -1$$

$$7). \lim_{x \rightarrow \frac{\pi}{3}} \frac{\cos 3x + 2 \cos 2x + 2}{\sin 3x} = \lim_{x \rightarrow \frac{\pi}{3}} \frac{4 \cos^3 x + 4 \cos^2 x - 3 \cos x}{3 \sin x - 4 \sin^3 x}$$

$$= \lim_{x \rightarrow \frac{\pi}{3}} \frac{\cos x (2 \cos x + 3)(2 \cos x - 1)}{\sin x (2 \cos x - 1)(2 \cos x + 1)} =$$

$$8). \lim_{x \rightarrow 0} \frac{\cos\left(\frac{\pi}{2} \cos x\right)}{\sin^2 \frac{x}{2}}$$

$$= \lim_{x \rightarrow 0} \frac{\sin\left(\frac{\pi}{2} - \frac{\pi}{2} \cos x\right)}{\sin^2 \frac{x}{2}} = \lim_{x \rightarrow 0} \frac{\sin\left(\frac{\pi}{2} 2 \sin^2 \frac{x}{2}\right)}{\sin^2 \frac{x}{2}} = \lim_{x \rightarrow 0} \left[\frac{\sin\left(\pi \sin^2 \frac{x}{2}\right)}{\pi \sin^2 \frac{x}{2}} \right] = \pi$$

$$9). \lim_{x \rightarrow 0} \frac{1 - \cos x}{(1 - \sqrt{1-x})^2}$$

$$= \lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \sqrt{1-x})^2}{(1 - \sqrt{1-x})^2 (1 + \sqrt{1-x})^2} = \lim_{x \rightarrow 0} \frac{2 \sin^2\left(\frac{x}{2}\right) (1 + \sqrt{1-x})^2}{4 \left(\frac{x}{2}\right)^2} = \lim_{x \rightarrow 0} \frac{(1 + \sqrt{1-x})^2}{2} = 2$$