PHƯƠNG TRÌNH BẬC NHẤT VỚI SINX VÀ COSX

I. LÝ THUYẾT

PHƯƠNG TRÌNH BẬC NHẤT:

DẠNG: $\begin{cases} \circ\ a\sin u + b\cos u = c \\ \circ\ a\sin u - b\cos u = c \\ \circ\ a\cos u - b\sin u = c \end{cases}$

Điều kiện để phương trình có nghiệm là : $a^2 + b^2 \ge c^2$

Giả sử giải phương trình: $a\sin u + b\cos u = c\ (*)$

Cách giải chia hai vế của (*) cho $\sqrt{a^2+b^2}$

Ta được : $\frac{a}{\sqrt{a^2+b^2}}\sin u + \frac{b}{\sqrt{a^2+b^2}}\cos u = \frac{c}{\sqrt{a^2+b^2}}$

Đặt $\frac{a}{\sqrt{a^2+b^2}} = \cos\varphi \Rightarrow \frac{b}{\sqrt{a^2+b^2}} = \sin\varphi$.

$\Leftrightarrow \sin u.\cos\varphi + \sin\varphi.\cos u = \frac{c}{\sqrt{a^2+b^2}} \Leftrightarrow \sin(u+\varphi) = \frac{c}{\sqrt{a^2+b^2}}$ (**)

Đặt $\frac{c}{\sqrt{a^2+b^2}} = \sin\alpha$.

(**) $\Leftrightarrow \sin(u+\varphi) = \sin\alpha$. Giải phương trình cơ bản.

II. BÀI TẬP MẪU

Câu 1: Giải các phương trình sau:

1). $\cos x + \sqrt{3}\sin x = \sqrt{2}$ **2).** $3\sin x - 4\cos x = 5$

3). $\sqrt{3}\sin x - \cos x = \sqrt{2}$ **4).** $\sin x - \cos x = 1$

5). $\sin x + \cos x = \frac{\sqrt{6}}{2}$ **6).** $5\sin 2x + 12\cos 2x = 13$

7). $\sin 8x - \cos 6x = \sqrt{3}\left(\sin 6x + \cos 8x\right)$ **8).** $\sin x + \cos x = 2\sqrt{2}\sin x.\cos x$

9). $2\sin^2 x + \sqrt{3}\sin 2x = 3$ **10).** $3\cos x - 4\sin x + \dfrac{2}{3\cos x - 4\sin x - 6} = 3$

11). $2\sin\left(\dfrac{\pi}{4} + x\right) + \sin\left(x - \dfrac{\pi}{4}\right) = \dfrac{3\sqrt{2}}{2}$

LỜI GIẢI

1). $\cos x + \sqrt{3}\sin x = \sqrt{2}$ **(1)**

Ta có $a = 1, b = \sqrt{3}, c = \sqrt{2} \Rightarrow \sqrt{a^2 + b^2} = 2$. Chia hai vế của (1) cho 2 được:

$$(1) \Leftrightarrow \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = \frac{\sqrt{2}}{2} \quad \Leftrightarrow \cos x.\cos\frac{\pi}{3} + \sin x.\sin\frac{\pi}{3} = \frac{\sqrt{2}}{2}$$

$$\Leftrightarrow \cos\left(x - \frac{\pi}{3}\right) = \cos\frac{\pi}{4} \Leftrightarrow \left[\begin{array}{l} x - \frac{\pi}{3} = \frac{\pi}{4} + k2\pi \\ x - \frac{\pi}{3} = -\frac{\pi}{4} + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{7\pi}{12} + k2\pi \\ x = \frac{\pi}{12} + k2\pi \end{array}\right., \left(k \in \mathbb{Z}\right)$$

Kết luận nghiệm của phương trình: $x = \dfrac{7\pi}{12} + k2\pi, x = \dfrac{\pi}{12} + k2\pi, \left(k \in \mathbb{Z}\right)$

2). $3\sin x - 4\cos x = 5 \ \ (1)$. Ta có $a = 3, b = 4, c = 5 \Rightarrow \sqrt{a^2 + b^2} = 5$. Chia hai vế của (1) cho 5 được:

$(1) \Leftrightarrow \dfrac{3}{5}\sin x - \dfrac{4}{5}\cos x = 1$. Đặt $\dfrac{3}{5} = \cos\alpha \ \Rightarrow \dfrac{4}{5} = \sin\alpha$

$\Leftrightarrow \sin x.\cos\alpha - \cos x.\sin\alpha = 1 \ \Leftrightarrow \ \sin\left(x - \alpha\right) = 1 \ \Leftrightarrow \ x - \alpha = \dfrac{\pi}{2} + k2\pi \ \Leftrightarrow \ x = \alpha + \dfrac{\pi}{2} + k2\pi$ Vậy nghiệm của phương trình: $x = \alpha + \dfrac{\pi}{2} + k2\pi, \left(k \in \mathbb{Z}\right)$

3). $\sqrt{3}\sin x - \cos x = \sqrt{2} \ \ (1)$.

Ta có $a = \sqrt{3}, b = 1, c = \sqrt{2} \Rightarrow \sqrt{a^2 + b^2} = 2$. Chia hai vế của (1) cho 2 được:

$$(1) \Leftrightarrow \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = \frac{\sqrt{2}}{2} \quad \Leftrightarrow \sin x.\cos\frac{\pi}{6} - \cos x.\sin\frac{\pi}{6} = \frac{\sqrt{2}}{2}$$

$$\Leftrightarrow \sin\left(x-\frac{\pi}{6}\right)=\sin\frac{\pi}{4} \Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{6}=\frac{\pi}{4}+k2\pi \\ x-\frac{\pi}{6}=\pi-\frac{\pi}{4}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{5\pi}{12}+k2\pi \\ x=\frac{11\pi}{12}+k2\pi \end{array}\right.,\left(k\in\mathbb{Z}\right)$$

Vậy nghiệm của phương trình đã cho là $x=\frac{5\pi}{12}+k2\pi, x=\frac{11\pi}{12}+k2\pi,\left(k\in\mathbb{Z}\right)$

4). $\sin x-\cos x=1$ (1)

Ta có $a=1,b=1,c=1 \Rightarrow \sqrt{a^2+b^2}=\sqrt{2}$. Chia hai vế của (1) cho $\sqrt{2}$ được:

$$\left(1\right)\Leftrightarrow \frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x=\frac{1}{\sqrt{2}} \quad \Leftrightarrow \sin x.\cos\frac{\pi}{4}-\cos x.\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$$

$$\Leftrightarrow \sin\left(x-\frac{\pi}{4}\right)=\sin\frac{\pi}{4} \quad \Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{4}=\frac{\pi}{4}+k2\pi \\ x-\frac{\pi}{4}=\pi-\frac{\pi}{4}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{2}+k2\pi \\ x=\pi+k2\pi \end{array}\right.,\left(k\in\mathbb{Z}\right)$$

Vậy nghiệm của phương trình: $x=\frac{\pi}{2}+k2\pi, x=\pi+k2\pi,\left(k\in\mathbb{Z}\right)$

5). $\sin x+\cos x=\frac{\sqrt{6}}{2}$ (1) Ta có $a=1,b=1,c=\frac{\sqrt{6}}{2} \Rightarrow \sqrt{a^2+b^2}=\sqrt{2}$. Chia hai vế của (1) cho $\sqrt{2}$ được: $\left(1\right)\Leftrightarrow \frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x=\frac{\sqrt{3}}{2} \Leftrightarrow \sin x.\cos\frac{\pi}{4}+\cos x.\sin\frac{\pi}{4}=\frac{\sqrt{3}}{2}$

$$\Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=\sin\frac{\pi}{3} \Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{4}=\frac{\pi}{3}+k2\pi \\ x+\frac{\pi}{4}=\pi-\frac{\pi}{3}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{12}+k2\pi \\ x=\frac{5\pi}{12}+k2\pi \end{array}\right.,\left(k\in\mathbb{Z}\right)$$

Vậy nghiệm của phương trình: $x=\frac{\pi}{12}+k2\pi, x=\frac{5\pi}{12}+k2\pi,\left(k\in\mathbb{Z}\right)$

6). $5\sin 2x+12\cos 2x=13$ (1) .Ta có $a=5,b=12,c=13 \Rightarrow \sqrt{a^2+b^2}=13$. Chia hai vế của (1) cho 13 được: $\left(1\right)\Leftrightarrow \frac{5}{13}\sin 2x+\frac{12}{13}\cos 2x=1$. Đặt $\cos\alpha=\frac{5}{13} \Rightarrow \sin\alpha=\frac{12}{13}$.

$$\Leftrightarrow \sin 2x\cos\alpha+\sin\alpha\cos 2x=1 \Leftrightarrow \sin\left(2x+\alpha\right)=1 \Leftrightarrow 2x+\alpha=\frac{\pi}{2}+k2\pi \Leftrightarrow x=-\frac{\alpha}{2}+\frac{\pi}{4}+k\pi.$$

Vậy nghiệm của phương trình: $x=-\frac{\alpha}{2}+\frac{\pi}{4}+k\pi,\left(k\in\mathbb{Z}\right)$

7). $\sin 8x - \cos 6x = \sqrt{3}\left(\sin 6x + \cos 8x\right)\ \ (1)$

$(1) \Leftrightarrow \sin 8x - \sqrt{3}\cos 8x = \sqrt{3}\sin 6x + \cos 6x$

$\Leftrightarrow \frac{1}{2}\sin 8x - \frac{\sqrt{3}}{2}\cos 8x = \frac{\sqrt{3}}{2}\sin 6x + \frac{1}{2}\cos 6x$

$\Leftrightarrow \sin 8x.\cos\frac{\pi}{3} - \cos 8x.\sin\frac{\pi}{3} = \sin 6x.\cos\frac{\pi}{6} + \cos 6x.\sin\frac{\pi}{6}$

$$\Leftrightarrow \sin\left(8x - \frac{\pi}{3}\right) = \sin\left(6x + \frac{\pi}{6}\right) \Leftrightarrow \left[\begin{array}{l} 8x - \frac{\pi}{3} = 6x + \frac{\pi}{6} + k2\pi \\ 8x - \frac{\pi}{3} = \pi - \left(6x + \frac{\pi}{6}\right) + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{4} + k\pi \\ x = \frac{\pi}{12} + \frac{k\pi}{7} \end{array}\right.$$

8). $\sin x + \cos x = 2\sqrt{2}\sin x.\cos x\ \ (1)$

$(1) \Leftrightarrow \sin x + \cos x = \sqrt{2}\sin 2x \quad \Leftrightarrow \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \sin 2x$

$\Leftrightarrow \sin x.\cos\frac{\pi}{4} + \cos x.\sin\frac{\pi}{4} = \sin 2x$

$$\Leftrightarrow \sin\left(x + \frac{\pi}{4}\right) = \sin 2x \Leftrightarrow \left[\begin{array}{l} 2x = x + \frac{\pi}{4} + k2\pi \\ 2x = \pi - \left(x + \frac{\pi}{4}\right) + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{4} + k2\pi \\ x = \frac{\pi}{4} + \frac{k2\pi}{3} \end{array}\right.,\left(k \in \mathbb{Z}\right)$$

Vậy nghiệm của phương trình: $x = \frac{\pi}{4} + k2\pi, x = \frac{\pi}{4} + \frac{k2\pi}{3}, \left(k \in \mathbb{Z}\right)$

9). $2\sin^2 x + \sqrt{3}\sin 2x = 3 \Leftrightarrow 1 - \cos 2x + \sqrt{3}\sin 2x = 3 \Leftrightarrow \sqrt{3}\sin 2x - \cos 2x = 2$

$\Leftrightarrow \frac{\sqrt{3}}{2}\sin 2x - \frac{1}{2}\cos 2x = 1 \Leftrightarrow \sin 2x.\cos\frac{\pi}{6} - \cos 2x.\sin\frac{\pi}{6} = 1$

$\Leftrightarrow \sin\left(2x - \frac{\pi}{6}\right) = 1 \Leftrightarrow 2x - \frac{\pi}{6} = \frac{\pi}{2} + k2\pi \Leftrightarrow x = \frac{\pi}{3} + k\pi$

Vậy nghiệm của phương trình: $x = \frac{\pi}{3} + k\pi, \left(k \in \mathbb{Z}\right)$

10). $3\cos x - 4\sin x + \frac{2}{3\cos x - 4\sin x - 6} = 3\ \ (1)$

Đặt $t = 3\cos x - 4\sin x - 6 \Rightarrow 3\cos x - 4\sin x = t + 6$

$(1) \Leftrightarrow t + 6 + \frac{2}{t} = 3 \Leftrightarrow t^2 + 3t + 2 = 0 \Leftrightarrow \left[\begin{array}{l} t = -1 \\ t = -2 \end{array}\right.$

Với $t = -1 \Leftrightarrow 3\cos x - 4\sin x = 5 \Leftrightarrow \frac{3}{5}\cos x - \frac{4}{5}\sin x = 1$. Đặt $\cos\alpha = \frac{3}{5} \Rightarrow \sin\alpha = \frac{4}{5}$.

$\Leftrightarrow \cos x.\cos\alpha - \sin x.\sin\alpha = 1$

$\Leftrightarrow \cos(x + \alpha) = 1 \Leftrightarrow x + \alpha = \frac{\pi}{2} + k2\pi \Leftrightarrow x = -\alpha + \frac{\pi}{2} + k2\pi$.

Với $t = -2 \Leftrightarrow 3\cos x - 4\sin x = 4 \Leftrightarrow \frac{3}{5}\cos x - \frac{4}{5}\sin x = \frac{4}{5}$. Đặt $\cos\alpha = \frac{3}{5} \Rightarrow \sin\alpha = \frac{4}{5}$.

$\Leftrightarrow \cos x.\cos\alpha - \sin x.\sin\alpha = \sin\alpha \quad \Leftrightarrow \cos(x + \alpha) = \sin\alpha$

$$\Leftrightarrow \cos(x - \alpha) = \cos\left(\frac{\pi}{2} - \alpha\right) \Leftrightarrow \left[\begin{array}{l} x + \alpha = \frac{\pi}{2} - \alpha + k2\pi \\ x + \alpha = -\left(\frac{\pi}{2} - \alpha\right) + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{2} - 2\alpha + k2\pi \\ x = -\frac{\pi}{2} + k2\pi \end{array}\right., (k \in \mathbb{Z}).$$

Nghiệm phương trình: $x = -\alpha + \frac{\pi}{2} + k2\pi$, $x = \frac{\pi}{2} - 2\alpha + k2\pi, x = -\frac{\pi}{2} + k2\pi, (k \in \mathbb{Z})$

11). $2\sin\left(\frac{\pi}{4} + x\right) + \sin\left(x - \frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2}$.

$\Leftrightarrow 2.\frac{\sin x + \cos x}{\sqrt{2}} + \frac{\sin x - \cos x}{\sqrt{2}} = \frac{3\sqrt{2}}{2} \Leftrightarrow 3\sin x + \cos x = 3$

$\Leftrightarrow \frac{3}{\sqrt{10}}\sin x + \frac{1}{\sqrt{10}}\cos x = \frac{3}{\sqrt{10}}$. Đặt $\frac{3}{\sqrt{10}} = \cos\alpha \Rightarrow \frac{1}{\sqrt{10}} = \sin\alpha$

$\Leftrightarrow \sin x.\cos\alpha + \cos x.\sin\alpha = \cos\alpha \Leftrightarrow \sin(x + \alpha) = \sin\left(\frac{\pi}{2} - \alpha\right)$

$$\Leftrightarrow \left[\begin{array}{l} x + \alpha = \frac{\pi}{2} - \alpha + k2\pi \\ x + \alpha = \pi - \left(\frac{\pi}{2} - \alpha\right) + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{2} - 2\alpha + k2\pi \\ x = \frac{\pi}{2} + k2\pi \end{array}\right., (k \in \mathbb{Z})$$

Nghiệm phương trình: $x = \frac{\pi}{2} - 2\alpha + k2\pi, x = \frac{\pi}{2} + k2\pi, (k \in \mathbb{Z})$

Câu 2: Giải các phương trình sau:

1). $\dfrac{\sqrt{3}-\sqrt{3}\cos 2x}{2\sin x}=\cos x$.

2). $\tan\dfrac{\pi}{7}.\sin x+2\cos^2\dfrac{x}{2}=2$

3). $\sqrt{2}\left(\cos^4 x-\sin^4 x\right)=\sin x+\cos x$

4). $\sqrt{3}\cos 2x+\sin 2x+2\sin\left(2x-\dfrac{\pi}{6}\right)=2\sqrt{2}$

5). $\sqrt{3}\sin 7x-\cos 7x=2\sin\left(5x-\dfrac{\pi}{6}\right)$

6). $\sin\left(\dfrac{\pi}{2}+2x\right)+\sqrt{3}\sin\left(\pi-2x\right)=2$

7). $\cos x+\sqrt{3}\sin x+2\cos\left(2x+\dfrac{\pi}{3}\right)=0$

8). $2\cos 2x=\left(1+\sqrt{3}\right)\left(\cos x-\sin x\right)$

9). $\left(\sqrt{3}-1\right)\sin x-\left(\sqrt{3}+1\right)\cos x=1-\sqrt{3}$.

10). $3\sin 3x-\sqrt{3}\cos 9x=1+4\sin^3 3x$

LỜI GIẢI

1). $\dfrac{\sqrt{3}-\sqrt{3}\cos 2x}{2\sin x}=\cos x\quad(1)$. Điều kiện $\sin x\neq 0\Leftrightarrow x\neq k\pi$

$(1)\Leftrightarrow\sqrt{3}-\sqrt{3}\cos 2x=2\sin x\cos x\qquad\Leftrightarrow\sqrt{3}\cos 2x+\sin 2x=\sqrt{3}$

$\Leftrightarrow\dfrac{\sqrt{3}}{2}\cos 2x+\dfrac{1}{2}\sin 2x=\dfrac{\sqrt{3}}{2}\qquad\Leftrightarrow\cos 2x.\cos\dfrac{\pi}{6}+\sin 2x.\sin\dfrac{\pi}{6}=\dfrac{\sqrt{3}}{2}$

$$\Leftrightarrow\cos\left(2x-\frac{\pi}{6}\right)=\cos\frac{\pi}{6}\Leftrightarrow\left[\begin{array}{l}2x-\frac{\pi}{6}=\frac{\pi}{6}+k2\pi\\2x-\frac{\pi}{6}=-\frac{\pi}{6}+k2\pi\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=\frac{\pi}{6}+k\pi\\x=k\pi\end{array}\right.,\left(k\in\mathbb{Z}\right)$$

So với điều kiện thì nghiệm $x=k\pi$ loại.

Vậy nghiệm phương trình: $x=\dfrac{\pi}{6}+k\pi,\left(k\in\mathbb{Z}\right)$

2). $\tan\frac{\pi}{7}.\sin x+2\cos^2\frac{x}{2}=2\Leftrightarrow \tan\frac{\pi}{7}.\sin x+1+\cos x=2$ $\Leftrightarrow \frac{\sin\frac{\pi}{7}}{\cos\frac{\pi}{7}}\sin x+\cos x=1$

$\Leftrightarrow \sin\frac{\pi}{7}\sin x+\cos\frac{\pi}{7}\cos x=\cos\frac{\pi}{7}$ $\Leftrightarrow \cos\left(x-\frac{\pi}{7}\right)=\cos\frac{\pi}{7}\Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{7}=\frac{\pi}{7}+k2\pi \\ x-\frac{\pi}{7}=-\frac{\pi}{7}+k2\pi \end{array}\right.\Leftrightarrow \left[\begin{array}{l} x=\frac{2\pi}{7}+k2\pi \\ x=k2\pi \end{array}\right.$

Nghiệm phương trình: $x=\frac{2\pi}{7}+k2\pi, x=k2\pi, (k\in\mathbb{Z})$

3). $\sqrt{2}\left(\cos^4 x-\sin^4 x\right)=\sin x+\cos x$

$\Leftrightarrow \sqrt{2}\left(\cos^2 x-\sin^2 x\right)\left(\cos^2 x+\sin^2 x\right)=\sin x+\cos x$

$\Leftrightarrow \sqrt{2}\cos 2x=\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$

$\Leftrightarrow \cos 2x=\cos\left(x-\frac{\pi}{4}\right)\Leftrightarrow \left[\begin{array}{l} 2x=x-\frac{\pi}{4}+k2\pi \\ 2x=-x+\frac{\pi}{4}+k2\pi \end{array}\right.\Leftrightarrow \left[\begin{array}{l} x=-\frac{\pi}{4}+k2\pi \\ x=\frac{\pi}{12}+\frac{k2\pi}{3} \end{array}\right., (k\in\mathbb{Z})$

Nghiệm của phương trình: $x=-\frac{\pi}{4}+k2\pi, x=\frac{\pi}{12}+\frac{k2\pi}{3}, (k\in\mathbb{Z})$

4). $\sqrt{3}\cos 2x+\sin 2x+2\sin\left(2x-\frac{\pi}{6}\right)=2\sqrt{2}$

$\Leftrightarrow \frac{\sqrt{3}}{2}\cos 2x+\frac{1}{2}\sin 2x+\sin\left(2x-\frac{\pi}{6}\right)=\sqrt{2}$

$\Leftrightarrow \cos 2x.\cos\frac{\pi}{6}+\sin 2x.\sin\frac{\pi}{6}+\sin\left(2x-\frac{\pi}{6}\right)=\sqrt{2}$

$\Leftrightarrow \cos\left(2x-\frac{\pi}{6}\right)+\sin\left(2x-\frac{\pi}{6}\right)=\sqrt{2}$

$\Leftrightarrow \frac{1}{\sqrt{2}}\cos\left(2x-\frac{\pi}{6}\right)+\frac{1}{\sqrt{2}}\sin\left(2x-\frac{\pi}{6}\right)=1$

$\Leftrightarrow \cos\left(2x-\frac{\pi}{6}\right).\cos\frac{\pi}{4}+\sin\left(2x-\frac{\pi}{6}\right)\sin\frac{\pi}{4}=1$

$\Leftrightarrow \cos\left(2x-\frac{\pi}{6}-\frac{\pi}{4}\right)=1 \Leftrightarrow 2x-\frac{\pi}{6}-\frac{\pi}{4}=k2\pi \Leftrightarrow x=\frac{5\pi}{24}+k\pi$.

Nghiệm của phương trình: $x=\frac{5\pi}{24}+k\pi$.

5). $\sqrt{3}\sin 7x-\cos 7x=2\sin\left(5x-\frac{\pi}{6}\right)$

$\Leftrightarrow \frac{\sqrt{3}}{2}\sin 7x-\frac{1}{2}\cos 7x=\sin\left(5x-\frac{\pi}{6}\right)$

$\Leftrightarrow \sin 7x\cos\frac{\pi}{6}-\cos 7x\sin\frac{\pi}{6}=\sin\left(5x-\frac{\pi}{6}\right) \Leftrightarrow \sin\left(7x-\frac{\pi}{6}\right)=\sin\left(5x-\frac{\pi}{6}\right)$

$\Leftrightarrow \left[\begin{array}{l} 7x-\frac{\pi}{6}=5x-\frac{\pi}{6}+k2\pi \\ 7x-\frac{\pi}{6}=\pi-\left(5x-\frac{\pi}{6}\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=k\pi \\ x=\frac{\pi}{9}+\frac{k\pi}{6} \end{array}\right.$

6). $\sin\left(\frac{\pi}{2}+2x\right)+\sqrt{3}\sin(\pi-2x)=2 \Leftrightarrow \cos 2x+\sqrt{3}\sin 2x=2 \Leftrightarrow \frac{1}{2}\cos 2x+\frac{\sqrt{3}}{2}\sin 2x=1$

$\Leftrightarrow \cos 2x.\cos\frac{\pi}{3}+\sin 2x.\sin\frac{\pi}{3}=1 \Leftrightarrow \cos\left(2x-\frac{\pi}{3}\right)=1$

$\Leftrightarrow 2x-\frac{\pi}{3}=k2\pi \Leftrightarrow x=\frac{\pi}{6}+k\pi,(k\in\mathbb{Z})$

7). $\Leftrightarrow \cos x+\sqrt{3}\sin x+2\cos\left(2x+\frac{\pi}{3}\right)=0$

$\Leftrightarrow \frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x=-\cos\left(2x+\frac{\pi}{3}\right) \Leftrightarrow \cos x\cos\frac{\pi}{3}+\sin x\sin\frac{\pi}{3}=\cos\left(2x+\frac{\pi}{3}+\pi\right)$

$\Leftrightarrow \cos\left(x-\frac{\pi}{3}\right)=\cos\left(2x+\frac{4\pi}{3}\right) \Leftrightarrow \left[\begin{array}{l} 2x+\frac{4\pi}{3}=x-\frac{\pi}{3}+k2\pi \\ 2x+\frac{4\pi}{3}=-\left(x-\frac{\pi}{3}\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\frac{5\pi}{3}+k2\pi \\ x=-\pi+\frac{k2\pi}{3} \end{array}\right.$

8). $2\cos 2x=\left(1+\sqrt{3}\right)(\cos x-\sin x)$

$\Leftrightarrow 2\left(\cos^2 x-\sin^2 x\right)=\left(1+\sqrt{3}\right)(\cos x-\sin x)$

$\Leftrightarrow 2(\cos x-\sin x)(\cos x+\sin x)=(1+\sqrt{3})(\cos x-\sin x)$

$\Leftrightarrow (\cos x-\sin x)\left[2(\cos x+\sin x)-(1+\sqrt{3})\right]=0$

$$\Leftrightarrow \left[\begin{array}{l}\cos x-\sin x=0\\ 2(\cos x+\sin x)-(1+\sqrt{3})=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)=0\\ \sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=\frac{1+\sqrt{3}}{2}\end{array}\right. \Leftrightarrow \left[\begin{array}{ll}\cos\left(x+\frac{\pi}{4}\right)=0 & (1)\\ \cos\left(x-\frac{\pi}{4}\right)=\frac{1+\sqrt{3}}{2\sqrt{2}} & (2)\end{array}\right.$$

Giải (1): $\Leftrightarrow x+\frac{\pi}{4}=\frac{\pi}{2}+k\pi \Leftrightarrow x=\frac{\pi}{4}+k\pi$.

Giải (2): $\cos\left(x-\frac{\pi}{4}\right)=\cos\frac{\pi}{12} \Leftrightarrow \left[\begin{array}{l}x-\frac{\pi}{4}=\frac{\pi}{12}+k2\pi\\ x-\frac{\pi}{4}=-\frac{\pi}{12}+k2\pi\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{3}+k2\pi\\ x=\frac{\pi}{6}+k2\pi\end{array}\right.$

Vậy nghiệm của phương trình: $x=\frac{\pi}{4}+k\pi$, $x=\frac{\pi}{3}+k2\pi$, $x=\frac{\pi}{6}+k2\pi,(k\in\mathbb{Z})$

9). $(\sqrt{3}-1)\sin x-(\sqrt{3}+1)\cos x=1-\sqrt{3}$.

Ta có $a=\sqrt{3}-1, b=\sqrt{3}+1, c=1-\sqrt{3} \Rightarrow \sqrt{a^2+b^2}=2\sqrt{2}$

$\Leftrightarrow \frac{\sqrt{3}-1}{2\sqrt{2}}\sin x-\frac{\sqrt{3}+1}{2\sqrt{2}}\cos x=\frac{1-\sqrt{3}}{2\sqrt{2}} \Leftrightarrow \sin x.\cos\frac{5\pi}{12}-\cos x.\sin\frac{5\pi}{12}=\frac{1-\sqrt{3}}{2\sqrt{2}}$

$\Leftrightarrow \sin\left(x-\frac{5\pi}{12}\right)=\sin\left(-\frac{\pi}{12}\right) \Leftrightarrow \left[\begin{array}{l}x-\frac{5\pi}{12}=-\frac{\pi}{12}+k2\pi\\ x-\frac{5\pi}{12}=\pi+\frac{\pi}{12}+k2\pi\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{3}+k2\pi\\ x=\frac{3\pi}{2}+k2\pi\end{array}\right.$

Vậy nghiệm của phương trình: $x=\frac{\pi}{3}+k2\pi, x=\frac{3\pi}{2}+k2\pi,(k\in\mathbb{Z})$

10). $3\sin 3x-\sqrt{3}\cos 9x=1+4\sin^3 3x$

$\Leftrightarrow 3\sin 3x-4\sin^3 3x-\sqrt{3}\cos 9x=1 \quad \Leftrightarrow \sin 9x-\sqrt{3}\cos 9x=1$

$\Leftrightarrow \frac{1}{2}\sin 9x-\frac{\sqrt{3}}{2}\cos 9x=\frac{1}{2} \quad \Leftrightarrow \sin 9x.\cos\frac{\pi}{3}-\cos 9x.\sin\frac{\pi}{3}=\frac{1}{2}$

$$\Leftrightarrow \sin\left(9x-\frac{\pi}{3}\right)=\sin\frac{\pi}{6} \Leftrightarrow \left[\begin{array}{l} 9x-\frac{\pi}{3}=\frac{\pi}{6}+k2\pi \\ 9x-\frac{\pi}{3}=\pi-\frac{\pi}{6}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{18}+\frac{k2\pi}{9} \\ x=\frac{7\pi}{54}+\frac{k2\pi}{9} \end{array}\right.$$

Vậy nghiệm của phương trình: $x=\frac{\pi}{18}+\frac{k2\pi}{9}, x=\frac{7\pi}{54}+\frac{k2\pi}{9}, (k\in\mathbb{Z})$

Câu 3: Giải các phương trình sau:

1). $2\cos\left(2x+\frac{\pi}{6}\right)+4\sin x\cos x-1=0$

2). $4\sin\left(x+\frac{\pi}{4}\right)+2\cos\left(x-\frac{\pi}{4}\right)-3\sqrt{2}=0$

3). $8\sin x\sin 2x+6\sin\left(x+\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}-2x\right)=5+7\cos x$

4). $2\sqrt{3}\sin\left(x-\frac{\pi}{8}\right)\cos\left(x-\frac{\pi}{8}\right)+2\cos^2\left(x-\frac{\pi}{8}\right)=\sqrt{3}+1$

5). $\frac{1+\cos x+\cos 2x+\cos 3x}{2\cos^2 x+\cos x-1}=\frac{2}{3}\left(3-\sqrt{3}\sin x\right)$

6). $8\sin x=\frac{\sqrt{3}}{\cos x}+\frac{1}{\sin x}$

7). $2\cos^3 x+2\sin^3 x+2\sin^2 x\cos x+2\cos^2 x\sin x-\sqrt{2}=0$

8). $5\left(\cos x+\sin x\right)+\sin 3x-\cos 3x=2\sqrt{2}\left(2+\sin 2x\right)$

LỜI GIẢI

1). $2\cos\left(2x+\frac{\pi}{6}\right)+4\sin x\cos x-1=0$

$$\Leftrightarrow 2\left(\cos 2x\cos\frac{\pi}{6}-\sin 2x\sin\frac{\pi}{6}\right)+2\sin 2x-1=0 \Leftrightarrow \sqrt{3}\cos 2x+\sin 2x=1$$

$$\Leftrightarrow \frac{\sqrt{3}}{2}\cos 2x+\frac{1}{2}\sin 2x=\frac{1}{2} \Leftrightarrow \cos 2x\cos\frac{\pi}{6}+\sin 2x\sin\frac{\pi}{6}=\frac{1}{2}$$

$$\Leftrightarrow \cos\left(2x-\frac{\pi}{6}\right)=\cos\frac{\pi}{3} \Leftrightarrow \left[\begin{array}{l} 2x-\frac{\pi}{6}=\frac{\pi}{3}+k2\pi \\ 2x-\frac{\pi}{6}=-\frac{\pi}{3}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{4}+k\pi \\ x=-\frac{\pi}{12}+k\pi \end{array}\right.,(k\in\mathbb{Z})$$

Vậy nghiệm của phương trình: $x=\frac{\pi}{4}+k\pi, x=-\frac{\pi}{12}+k\pi, (k\in\mathbb{Z})$

2). $4\sin\left(x+\frac{\pi}{4}\right)+2\cos\left(x-\frac{\pi}{4}\right)-3\sqrt{2}=0$

$$\Leftrightarrow 4.\frac{\sin x+\cos x}{\sqrt{2}}+2.\frac{\sin x+\cos x}{\sqrt{2}}=3\sqrt{2} \Leftrightarrow \sin x+\cos x=1 \Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=1$$

$$\Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}} \Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{4}=\frac{\pi}{4}+k2\pi \\ x+\frac{\pi}{4}=\pi-\frac{\pi}{4}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=k2\pi \\ x=\frac{\pi}{2}+k2\pi \end{array}\right.$$

Vậy nghiệm của phương trình: $x=k2\pi, x=\frac{\pi}{2}+k2\pi, (k\in\mathbb{Z})$

3). $8\sin x\sin 2x+6\sin\left(x+\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}-2x\right)=5+7\cos x$

$$\Leftrightarrow 4(\cos x-\cos 3x)+3\left(\sin 3x+\sin\left(\frac{\pi}{2}-x\right)\right)=5+7\cos x$$

$$\Leftrightarrow 4(\cos x-\cos 3x)+3(\sin 3x+\cos x)=5+7\cos x \Leftrightarrow 3\sin 3x-4\cos 3x=5$$

$$\Leftrightarrow \frac{3}{5}\sin 3x-\frac{4}{5}\cos 3x=1 \Leftrightarrow \sin 3x.\cos\alpha-\cos 3x.\sin\alpha=1 \Leftrightarrow \sin(3x-\alpha)=1$$

$\Leftrightarrow 3x-\alpha=\frac{\pi}{2}+k2\pi \Leftrightarrow x=\frac{\alpha}{3}+\frac{\pi}{6}+\frac{k2\pi}{3}$. (Với $\frac{3}{5}=\cos\alpha, \frac{4}{5}=\sin\alpha$)

Vậy nghiệm của phương trình: $x=\frac{\alpha}{3}+\frac{\pi}{6}+\frac{k2\pi}{3}$

4). $2\sqrt{3}\sin\left(x-\frac{\pi}{8}\right)\cos\left(x-\frac{\pi}{8}\right)+2\sin^2\left(x-\frac{\pi}{8}\right)=\sqrt{3}+1$

Áp dụng công thức nhân đôi: $2\sin\left(x-\frac{\pi}{8}\right)\cos\left(x-\frac{\pi}{8}\right)=\sin\left(2x-\frac{\pi}{4}\right)$*, và hạ bậc* $2\sin^2\left(x-\frac{\pi}{8}\right)=1-\cos\left(2x-\frac{\pi}{4}\right)$ *ta được:*

$$\Leftrightarrow \sqrt{3}\sin\left(2x-\frac{\pi}{4}\right)+1-\cos\left(2x-\frac{\pi}{4}\right)=\sqrt{3}+1$$

$$\Leftrightarrow \sqrt{3}\sin\left(2x-\frac{\pi}{4}\right)-\cos\left(2x-\frac{\pi}{4}\right)=\sqrt{3}$$

$$\Leftrightarrow \frac{\sqrt{3}}{2}\sin\left(2x-\frac{\pi}{4}\right)-\frac{1}{2}\cos\left(2x-\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2}$$

$$\Leftrightarrow \sin\left(2x-\frac{\pi}{4}\right)\cos\frac{\pi}{6}-\cos\left(2x-\frac{\pi}{4}\right)\sin\frac{\pi}{6}=\frac{\sqrt{3}}{2}$$

$$\Leftrightarrow \sin\left(2x-\frac{\pi}{4}-\frac{\pi}{6}\right)=\sin\frac{\pi}{3} \Leftrightarrow \left[\begin{array}{l} 2x-\frac{\pi}{4}-\frac{\pi}{6}=\frac{\pi}{3}+k2\pi \\ 2x-\frac{\pi}{4}-\frac{\pi}{6}=\pi-\frac{\pi}{3}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{3\pi}{8}+k\pi \\ x=\frac{13\pi}{24}+k\pi \end{array}\right.$$

Vậy nghiệm của phương trình: $x=\frac{3\pi}{8}+k\pi, x=\frac{13\pi}{24}+k\pi, \left(k\in\mathbb{Z}\right)$

5). $\frac{1+\cos x+\cos 2x+\cos 3x}{2\cos^2 x+\cos x-1}=\frac{2}{3}\left(3-\sqrt{3}\sin x\right)$ $\left(*\right)$. Điều kiện $2\cos^2 x+\cos x-1\neq 0$

$$\Leftrightarrow \frac{\left(1+\cos 2x\right)+\left(\cos 3x+\cos x\right)}{\left(2\cos^2 x-1\right)+\cos x}=\frac{2}{3}\left(3-\sqrt{3}\sin x\right)$$

$$\Leftrightarrow \frac{2\cos^2 x+2\cos 2x.\cos x}{\cos 2x+\cos x}=\frac{2}{3}\left(3-\sqrt{3}\sin x\right)$$

$$\Leftrightarrow \frac{2\cos x\left(\cos 2x+\cos x\right)}{\cos 2x+\cos x}=\frac{2}{3}\left(3-\sqrt{3}\sin x\right)$$

$$\Leftrightarrow 2\cos x=\frac{2}{3}\left(3-\sqrt{3}\sin x\right) \quad \Leftrightarrow \sqrt{3}\cos x+\sin x=\sqrt{3}$$

$$\Leftrightarrow \frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x=\frac{\sqrt{3}}{2} \quad \Leftrightarrow \cos x.\cos\frac{\pi}{6}+\sin x\sin\frac{\pi}{6}=\frac{\sqrt{3}}{2}$$

$$\Leftrightarrow \cos\left(x-\frac{\pi}{6}\right)=\cos\frac{\pi}{6} \Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{6}=\frac{\pi}{6}+k2\pi \\ x-\frac{\pi}{6}=-\frac{\pi}{6}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{3}+k2\pi \\ x=k2\pi \end{array}\right.$$

Thay hai họ nghiệm của x vào điều kiện ta thấy thỏa.

Vậy nghiệm của phương trình: $x=\frac{\pi}{3}+k2\pi$, $x=k2\pi$.

6). $8\sin x=\frac{\sqrt{3}}{\cos x}+\frac{1}{\sin x}$. Điều kiện $\begin{cases}\sin x\neq 0\\ \cos x\neq 0\end{cases}$

Quy đồng mẫu được: $\Leftrightarrow 8\sin^2 x\cos x=\sqrt{3}\sin x+\cos x$

Hạ bậc $\sin^2 x$ được: $\Leftrightarrow 4(1-\cos 2x)\cos x=\sqrt{3}\sin x+\cos x$

$$\Leftrightarrow 4\cos x-4\cos 2x\cos x=\sqrt{3}\sin x+\cos x$$

$$\Leftrightarrow 4\cos x-2(\cos x+\cos 3x)=\sqrt{3}\sin x+\cos x$$

$$\Leftrightarrow \cos x-\sqrt{3}\sin x=2\cos 3x \qquad \Leftrightarrow \frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x=\cos 3x$$

$$\Leftrightarrow \cos x.\cos\frac{\pi}{3}-\sin x.\sin\frac{\pi}{3}=\cos 3x$$

$$\Leftrightarrow \cos\left(x+\frac{\pi}{3}\right)=\cos 3x \Leftrightarrow \left[\begin{array}{l} 3x=x+\frac{\pi}{3}+k2\pi \\ 3x=-\left(x+\frac{\pi}{3}\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{6}+k\pi \\ x=-\frac{\pi}{12}+\frac{k\pi}{2} \end{array}\right.$$

So với điều kiện nghiệm của phương trình: $x=\frac{\pi}{6}+k\pi$, $x=-\frac{\pi}{12}+\frac{k\pi}{2}$.

7). $2\cos^3 x+2\sin^3 x+2\sin^2 x\cos x+2\cos^2 x\sin x-\sqrt{2}=0$

$$\Leftrightarrow 2(\cos x+\sin x)(\sin^2 x-\sin x\cos x+\cos^2 x)+2\sin x\cos x(\sin x+\cos x)-\sqrt{2}=0$$

$$\Leftrightarrow 2(\cos x+\sin x)(1-\sin x\cos x)+2\sin x\cos x(\sin x+\cos x)-\sqrt{2}=0$$

$$\Leftrightarrow 2(\cos x+\sin x)=\sqrt{2} \Leftrightarrow 2\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=\sqrt{2} \Leftrightarrow \cos\left(x-\frac{\pi}{4}\right)=\frac{1}{2}$$

$$\Leftrightarrow \cos\left(x-\frac{\pi}{4}\right)=\cos\frac{\pi}{3} \Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{4}=\frac{\pi}{3}+k2\pi \\ x-\frac{\pi}{4}=-\frac{\pi}{3}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{7\pi}{12}+k2\pi \\ x=-\frac{\pi}{12}+k2\pi \end{array}\right.$$

So với điều kiện nghiệm của phương trình: $x=\frac{7\pi}{12}+k2\pi, x=-\frac{\pi}{12}+k2\pi, \left(k\in\mathbb{Z}\right)$

8). $5\left(\cos x+\sin x\right)+\sin 3x-\cos 3x=2\sqrt{2}\left(2+\sin 2x\right)$

$\Leftrightarrow 5\left(\cos x+\sin x\right)+3\sin x-4\sin^3 x-\left(4\cos^3 x-3\cos x\right)=2\sqrt{2}\left(2+\sin 2x\right)$

$\Leftrightarrow 8\left(\cos x+\sin x\right)-4\left(\sin^3 x+\cos^3 x\right)=2\sqrt{2}\left(2+\sin 2x\right)$

$\Leftrightarrow 8\left(\cos x+\sin x\right)-4\left(\sin x+\cos x\right)\left(1-\sin x\cos x\right)=2\sqrt{2}\left(2+\sin 2x\right)$

$\Leftrightarrow 4\left(\sin x+\cos x\right)\left(2-1+\sin x\cos x\right)=2\sqrt{2}\left(2+\sin 2x\right)$

$\Leftrightarrow 4\left(\sin x+\cos x\right)\left(1+\frac{1}{2}\sin 2x\right)=2\sqrt{2}\left(2+\sin 2x\right)$

$\Leftrightarrow 2\left(\sin x+\cos x\right)\left(2+\sin 2x\right)=2\sqrt{2}\left(2+\sin 2x\right)$

$\Leftrightarrow \sin x+\cos x=\sqrt{2}$ (vì $2+\sin 2x>0$).

$\Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=\sqrt{2} \Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=1 \Leftrightarrow x+\frac{\pi}{4}=\frac{\pi}{2}+k2\pi \Leftrightarrow x=\frac{\pi}{4}+k2\pi$

Câu 4: Giải các phương trình sau:

1). $\sin x+\sin 2x=\sqrt{3}\left(\cos x+\text{cox}2x\right)$ [Dự bị 1 ĐH A04]

2). $2\sin\left(2x-\frac{\pi}{6}\right)+4\sin x+1=0$ [Dự bị 2 ĐH A06]

3). $\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2+\sqrt{3}\cos x=2$ [ĐH D07]

4). $2\cos^2 x+2\sqrt{3}\sin x\cos x+1=3(\sin x+\sqrt{3}\cos x)$ [Dự bị 2 ĐH A07]

5). $\sin^3 x-\sqrt{3}\cos^3 x=\sin x\cos^2 x-\sqrt{3}\sin^2 x\cos x$ [ĐH B08]

6). $\sin 3x-\sqrt{3}\cos 3x=2\sin 2x$ [CĐ 08]

7). $2\sin\left(x+\frac{\pi}{3}\right)-\sin\left(2x-\frac{\pi}{6}\right)=\frac{1}{2}$ [Dự bị 1 ĐH B08]

8). $\frac{(1-2\sin x)\cos x}{(1+2\sin x)(1-\sin x)}=\sqrt{3}\ (1)$ [ĐH A09]

9). $\sqrt{3}\cos 5x-2\sin 3x\cos 2x-\sin x=0$ [ĐH D09]

LỜI GIẢI

1). $\sin x+\sin 2x=\sqrt{3}(\cos x+\text{cox}2x)$ [Dự bị 1 ĐH A04]

LỜI GIẢI

$\Leftrightarrow \sin x+\sin 2x=\sqrt{3}\cos x+\sqrt{3}\cos 2x \Leftrightarrow \sin x-\sqrt{3}\cos x=\sqrt{3}\cos 2x-\sin 2x$

$\Leftrightarrow \frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x=\frac{\sqrt{3}}{2}\cos 2x-\frac{1}{2}\sin 2x \Leftrightarrow \sin\left(x-\frac{\pi}{3}\right)=\cos\left(2x+\frac{\pi}{6}\right)$

$\Leftrightarrow \sin\left(x-\frac{\pi}{3}\right)=\sin\left[\frac{\pi}{2}-\left(2x+\frac{\pi}{6}\right)\right] \Leftrightarrow \sin\left(x-\frac{\pi}{3}\right)=\sin\left(\frac{\pi}{3}-2x\right)$

$$\Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{3}=\frac{\pi}{3}-2x+k2\pi \\ x-\frac{\pi}{3}=\pi-\left(\frac{\pi}{3}-2x\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{2\pi}{9}+\frac{k2\pi}{3} \\ x=-\pi-k2\pi \end{array}\right.,(k\in\mathbb{Z})$$

2). $2\sin\left(2x-\frac{\pi}{6}\right)+4\sin x+1=0$ [Dự bị 2 ĐH A06]

LỜI GIẢI

$\Leftrightarrow 2\left[\sin 2x\cos\frac{\pi}{6}-\cos 2x\sin\frac{\pi}{6}\right]+4\sin x+1=0$

$\Leftrightarrow \sqrt{3}\sin 2x-\cos 2x+4\sin x+1=0 \Leftrightarrow \sqrt{3}\sin 2x+(1-\cos 2x)+4\sin x=0$

$\Leftrightarrow 2\sqrt{3}\sin x\cos x+2\sin^2 x+4\sin x=0 \Leftrightarrow 2\sin x\left(\sqrt{3}\cos x+\sin x+2\right)=0$

$$\Leftrightarrow \left[\begin{array}{l} \sin x = 0 \\ \sqrt{3}\cos x + \sin x + 2 = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = k\pi \\ \cos\left(x - \frac{\pi}{6}\right) = -1 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = k\pi \\ x = \frac{7\pi}{6} + k2\pi \end{array}\right., (k \in \mathbb{Z})$$

Kết luận nghiệm của phương trình: $x = k\pi$, $x = \frac{7\pi}{6} + k2\pi\ (k \in \mathbb{Z})$

3). $\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2 + \sqrt{3}\cos x = 2$ [ĐH D07]

LỜI GIẢI

$$\Leftrightarrow 1 + \sin x + \sqrt{3}\cos x = 2 \Leftrightarrow \sin x + \sqrt{3}\cos x = 1 \Leftrightarrow \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{1}{2}$$

$$\Leftrightarrow \sin\left(x + \frac{\pi}{3}\right) = \frac{1}{2} = \sin\frac{\pi}{6} \Leftrightarrow \left[\begin{array}{l} x + \frac{\pi}{3} = \frac{\pi}{6} + k2\pi \\ x + \frac{\pi}{3} = \frac{5\pi}{6} + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = -\frac{\pi}{6} + k2\pi \\ x = \frac{\pi}{2} + k2\pi \end{array}\right., (k \in \mathbb{Z})$$

Kết luận nghiệm của phương trình $x = -\frac{\pi}{6} + k2\pi$, $x = \frac{\pi}{2} + k2\pi$, $(k \in \mathbb{Z})$

4). $2\cos^2 x + 2\sqrt{3}\sin x\cos x + 1 = 3(\sin x + \sqrt{3}\cos x)$ [Dự bị 2 ĐH A07]

LỜI GIẢI

$$\Leftrightarrow 1 + \cos 2x + \sqrt{3}\sin 2x + 1 = 3(\sin x + \sqrt{3}\cos x)$$

$$\Leftrightarrow \cos 2x + \sqrt{3}\sin 2x + 2 = 3(\sin x + \sqrt{3}\cos x)$$

$$\Leftrightarrow \left(\frac{1}{2}\cos 2x + \frac{\sqrt{3}}{2}\sin 2x\right) + 1 = 3\left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right)$$

$$\Leftrightarrow \cos\left(2x - \frac{\pi}{3}\right) + 1 = 3\cos\left(x - \frac{\pi}{6}\right) \Leftrightarrow \cos 2\left(x - \frac{\pi}{6}\right) + 1 = 3\cos\left(x - \frac{\pi}{6}\right)$$

$$\Leftrightarrow 2\cos^2\left(x - \frac{\pi}{6}\right) - 3\cos\left(x - \frac{\pi}{6}\right) = 0 \Leftrightarrow \cos\left(x - \frac{\pi}{6}\right)\left[2\cos\left(x - \frac{\pi}{6}\right) - 3\right] = 0$$

$$\Leftrightarrow \cos\left(x - \frac{\pi}{6}\right) = 0 \vee \cos\left(x - \frac{\pi}{6}\right) = \frac{3}{2}\ \text{(loại)}$$

Với $\cos\left(x-\frac{\pi}{6}\right)=0 \Leftrightarrow x-\frac{\pi}{6}=\frac{\pi}{2}+k\pi \Leftrightarrow x=\frac{2\pi}{3}+k\pi,\left(k\in\mathbb{Z}\right)$

Kết luận nghiệm của phương trình $x=\frac{2\pi}{3}+k\pi,\left(k\in\mathbb{Z}\right)$

5). $\sin^3 x-\sqrt{3}\cos^3 x=\sin x\cos^2 x-\sqrt{3}\sin^2 x\cos x$ [ĐH B08]

LỜI GIẢI

$$\Leftrightarrow \sin x(\cos^2 x-\sin^2 x)+\sqrt{3}\cos x(\cos^2 x-\sin^2 x)=0 \Leftrightarrow \cos 2x(\sin x+\sqrt{3}\cos x)=0$$

$$\Leftrightarrow \left[\begin{array}{l}\cos 2x=0\\ \sin x+\sqrt{3}\cos x=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\cos 2x=0\\ \sin\left(x+\frac{\pi}{3}\right)=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{4}+\frac{k\pi}{2}\\ x=-\frac{\pi}{3}+k\pi\end{array}\right.,\left(k\in\mathbb{Z}\right)$$

6). $\sin 3x-\sqrt{3}\cos 3x=2\sin 2x$ [CĐ 08]

LỜI GIẢI

$$\Leftrightarrow \frac{1}{2}\sin 3x-\frac{\sqrt{3}}{2}\cos 3x=\sin 2x$$

$$\Leftrightarrow \sin\left(3x-\frac{\pi}{3}\right)=\sin 2x \Leftrightarrow \left[\begin{array}{l}3x-\frac{\pi}{3}=2x+k2\pi\\ 3x-\frac{\pi}{3}=\pi-2x+k2\pi\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{3}+k2\pi\\ x=\frac{4\pi}{15}+\frac{k2\pi}{5}\end{array}\right.,\left(k\in\mathbb{Z}\right)$$

7). $2\sin\left(x+\frac{\pi}{3}\right)-\sin\left(2x-\frac{\pi}{6}\right)=\frac{1}{2}$ [Dự bị 1 ĐH B08]

LỜI GIẢI

$$\Leftrightarrow \sin x+\sqrt{3}\cos x-\left(\frac{\sqrt{3}}{2}\sin 2x-\frac{1}{2}\cos 2x\right)=\frac{1}{2}$$

$$\Leftrightarrow \sin x+\sqrt{3}\cos x-\sqrt{3}\sin x\cos x+\frac{1-2\sin^2 x}{2}=\frac{1}{2}$$

$$\Leftrightarrow \sqrt{3}\cos x\left(1-\sin x\right)+\sin x\left(1-\sin x\right)=0 \Leftrightarrow (1-\sin x)(\sqrt{3}\cos x+\sin x)=0$$

$$\Leftrightarrow \left[\begin{array}{l} \sin x = 1 \\ \sqrt{3}\cos x + \sin x = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{2} + k2\pi \\ \sin\left(x + \frac{\pi}{3}\right) = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = -\frac{\pi}{3} + k\pi \\ x = \frac{\pi}{2} + k2\pi \end{array}\right., \left(k \in \mathbb{Z}\right)$$

Kết luận nghiệm của phương trình $x = -\frac{\pi}{3} + k\pi$, $x = \frac{\pi}{2} + k2\pi, \left(k \in \mathbb{Z}\right)$

8). $\frac{\left(1 - 2\sin x\right)\cos x}{\left(1 + 2\sin x\right)\left(1 - \sin x\right)} = \sqrt{3}$ $\left(1\right)$ [ĐH A09]

LỜI GIẢI

Điều kiện : $\begin{cases} \sin x \neq 1 \\ \sin x \neq -\frac{1}{2} \end{cases}$

$(1) \Leftrightarrow \left(1 - 2\sin x\right)\cos x = \sqrt{3}(1 + \sin 2x)(1 - \sin x)$

$\Leftrightarrow \cos x - \sin 2x = \sqrt{3}\left(1 + \sin x - 2\sin^2 x\right) \Leftrightarrow \cos x - \sin 2x = \sqrt{3}\left(\cos 2x + \sin x\right)$

$\Leftrightarrow \cos x - \sqrt{3}\sin x = \sin 2x + \sqrt{3}\cos 2x$

$\Leftrightarrow \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = \frac{1}{2}\sin 2x + \frac{\sqrt{3}}{2}\cos 2x$

$$\Leftrightarrow \cos\left(x + \frac{\pi}{3}\right) = \cos\left(2x - \frac{\pi}{6}\right) \Leftrightarrow \left[\begin{array}{l} 2x - \frac{\pi}{6} = x + \frac{\pi}{3} + k2\pi \\ 2x - \frac{\pi}{6} = -x - \frac{\pi}{3} + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{2} + k2\pi \\ x = -\frac{\pi}{18} + \frac{k2\pi}{3} \end{array}\right., \left(k \in \mathbb{Z}\right)$$

So với điều kiện nghiệm $x = \frac{\pi}{2} + k2\pi$ loại.

Kết luận nghiệm của phương trình: $x = -\frac{\pi}{18} + \frac{k2\pi}{3}, \left(k \in \mathbb{Z}\right)$

9). $\sqrt{3}\cos 5x - 2\sin 3x\cos 2x - \sin x = 0$ [ĐH D09]

LỜI GIẢI

$\Leftrightarrow \sqrt{3}\cos 5x - \left(\sin 5x + \sin x\right) - \sin x = 0$

$\Leftrightarrow \sqrt{3}\cos 5x - \sin 5x = 2\sin x \Leftrightarrow \frac{\sqrt{3}}{2}\cos 5x - \frac{1}{2}\sin 5x = \sin x$

$$\Leftrightarrow \sin\left(\frac{\pi}{3}-5x\right)=\sin x \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{3}-5x+k2\pi \\ x=\frac{2\pi}{3}+5x+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{18}+\frac{k\pi}{3} \\ x=-\frac{\pi}{6}-\frac{k\pi}{2} \end{array}\right.,(k\in\mathbb{Z})$$

Kết luận nghiệm của phương trình: $x=\frac{\pi}{18}+\frac{k\pi}{3}, x=-\frac{\pi}{6}-\frac{k\pi}{2},(k\in\mathbb{Z})$