Đặt $t=\frac{2}{\cos x}-\cos x\Rightarrow t^2=\left(\frac{2}{\cos x}-\cos x\right)^2\Rightarrow \cos^2 x+\frac{4}{\cos^2 x}=t^2+4$

$(1)\Leftrightarrow 2\left(t^2+4\right)+9t-1=0\Leftrightarrow 2t^2+9t+7=0\Leftrightarrow t=-1\vee t=-\frac{7}{2}$

Với $t=-1\Leftrightarrow \frac{2}{\cos x}-\cos x=-1\Leftrightarrow \cos^2 x-\cos x-2=0\Leftrightarrow \cos x=-1\vee \cos x=2$ (loại).

- $\cos x=-1\Leftrightarrow x=\pi+k2\pi,\left(k\in\mathbb{Z}\right)$.

Với $t=-\frac{7}{2}\Leftrightarrow \frac{2}{\cos x}-\cos x=-\frac{7}{2}\Leftrightarrow 2\cos^2 x-7\cos x-4=0\Leftrightarrow \cos x=-\frac{1}{2}$ hoặc $\cos x=4$ (loại).

- $\cos x=-\frac{1}{2}\Leftrightarrow x=\pm\frac{2\pi}{3}+k2\pi,\left(k\in\mathbb{Z}\right)$.

Vậy nghiệm của phương trình: $x=\pi+k2\pi,\ x=\pm\frac{2\pi}{3}+k2\pi,\left(k\in\mathbb{Z}\right)$

3). $3\left(\tan^2 x+\cot^2 x\right)+4\left(\tan x+\cot x\right)+2=0\quad (1)$

Đặt $t=\tan x+\cot x\Rightarrow t^2=\left(\tan x+\cot x\right)^2\Rightarrow \tan^2 x+\cot^2 x=t^2-2$

$(1)\Leftrightarrow 3\left(t^2-2\right)+4t+2=0\Leftrightarrow 3t^2+4t-4=0\Leftrightarrow t=-2\vee t=\frac{2}{3}$

- Với $t=-2\Leftrightarrow \tan x+\cot x=-2\Leftrightarrow \tan x+\frac{1}{\tan x}=-2\Leftrightarrow \tan^2 x+2\tan x+1=0$

$\Leftrightarrow \tan x=-1\Leftrightarrow \tan x=\tan\left(-\frac{\pi}{4}\right)\Leftrightarrow x=-\frac{\pi}{4}+k\pi,\left(k\in\mathbb{Z}\right)$.

- Với $t=\frac{2}{3}\Leftrightarrow \tan x+\cot x=\frac{2}{3}$

$\Leftrightarrow \tan x+\frac{1}{\tan x}=\frac{2}{3}\Leftrightarrow 3\tan^2 x+2\tan x+3=0$ (phương trình vô nghiệm).

Vậy nghiệm của phương trình: $x=-\frac{\pi}{4}+k\pi,\left(k\in\mathbb{Z}\right)$.

4). $\tan x+\tan^2 x+\tan^3 x+\cot x+\cot^2 x+\cot^3 x=6\quad (1)$

Đặt $t=\tan x+\cot x$.

Có: $\tan^2 x+\cot^2 x=\left(\tan x+\cot x\right)^2-2\tan x\cot x=t^2-2$.

Có: $\tan^3 x+\cot^3 x=\left(\tan x+\cot x\right)^3-3\tan x.\cot x\left(\tan x+\cot x\right)=t^3-3t$.

$\left(1\right)\Leftrightarrow\left(\tan x+\cot x\right)+\left(\tan^2 x+\cot^2 x\right)+\left(\tan^3 x+\cot^3 x\right)=6$.

$\Leftrightarrow t+t^2-2+t^3-3t=6\Leftrightarrow t^3+t^2-2t-8=0\Leftrightarrow t=2$.

Với $t=2\Leftrightarrow\tan x+\cot x=2\Leftrightarrow\tan x+\dfrac{1}{\tan x}=2\Leftrightarrow\tan^2 x-2\tan x+1=0$

$\Leftrightarrow\tan x=1\Leftrightarrow\tan x=\tan\dfrac{\pi}{4}\Leftrightarrow x=\dfrac{\pi}{4}+k\pi,\left(k\in\mathbb{Z}\right)$.

Vậy nghiệm của phương trình: $x=\dfrac{\pi}{4}+k\pi,\left(k\in\mathbb{Z}\right)$.

Câu 6: Giải các phương trình lượng giác sau:

1). $\sin^4+\cos^4 x-2\sin 2x+\dfrac{3}{4}\sin^2 2x=0$ 2). $\cos^6 2x+\sin^6 2x=\dfrac{15}{8}\cos 4x-\dfrac{1}{2}$

3). $\sin^4 x+\dfrac{5}{3}\cos^4 x=1$ 4). $\sin^4 x+\cos^4 x+\cos\left(x-\dfrac{\pi}{4}\right)\sin\left(x-\dfrac{3\pi}{4}\right)=\dfrac{1}{2}$ 5). $\sin^4 x+\cos 2x+4\sin^6 x=0$ 6). $\cos 8x+\sin^3 x\cos x-\cos^3 x\sin x-1=0$

LỜI GIẢI

1). $\sin^4+\cos^4 x-2\sin 2x+\dfrac{3}{4}\sin^2 2x=0\ \left(1\right)$

$\Leftrightarrow\left(1-\dfrac{1}{2}\sin^2 2x\right)-2\sin 2x+\dfrac{3}{4}\sin^2 2x=0\Leftrightarrow\dfrac{1}{4}\sin^2 2x-2\sin 2x+1=0\ \left(1'\right)$

Đặt $\sin 2x=t, t\in[-1;1]$. Phương trình (1') trở thành: $\dfrac{1}{4}t^2-2t+1=0$

$\Leftrightarrow t=4+2\sqrt{3}\vee t=4-2\sqrt{3}$. So với điều kiện nhận $t=4-2\sqrt{3}\Rightarrow\sin 2x=4-2\sqrt{3}$

$$\Leftrightarrow\left[\begin{array}{l}2x=\arcsin\left(4-2\sqrt{3}\right)+k2\pi\\2x=\pi-\arcsin\left(4-2\sqrt{3}\right)+k2\pi\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=\dfrac{\arcsin\left(4-2\sqrt{3}\right)}{2}+k\pi,\left(k\in\mathbb{Z}\right)\\x=\dfrac{\pi-\arcsin\left(4-2\sqrt{3}\right)}{2}+k\pi,\left(k\in\mathbb{Z}\right)\end{array}\right.$$

Vậy nghiệm của phương trình: $x=\dfrac{\arcsin\left(4-2\sqrt{3}\right)}{2}+k\pi, x=\dfrac{\pi-\arcsin\left(4-2\sqrt{3}\right)}{2}+k\pi,\left(k\in\mathbb{Z}\right)$.

2). $\cos^6 2x+\sin^6 2x=\dfrac{15}{8}\cos 4x-\dfrac{1}{2}\quad(1)$

$(1)\Leftrightarrow 1-\dfrac{3}{4}\sin^2 2x=\dfrac{15}{8}\cos 4x-\dfrac{1}{2}\Leftrightarrow 1-\dfrac{3}{4}\cdot\dfrac{1-\cos 4x}{2}=\dfrac{15}{8}\cos 4x-\dfrac{1}{2}$

$\Leftrightarrow 8-3\left(1-\cos 4x\right)=15\cos 4x-4\Leftrightarrow \cos 4x=\dfrac{3}{4}$

$\Leftrightarrow 4x=\pm\arccos\dfrac{3}{4}+k2\pi\Leftrightarrow x=\pm\dfrac{1}{4}\arccos\dfrac{3}{4}+\dfrac{k\pi}{2}\left(k\in\mathbb{Z}\right)$

Vậy nghiệm của phương trình: $x=\pm\dfrac{1}{4}\arccos\dfrac{3}{4}+\dfrac{k\pi}{2}\left(k\in\mathbb{Z}\right)$.

3). $\sin^4 x+\dfrac{5}{3}\cos^4 x=1\Leftrightarrow\left(\sin^2 x\right)^2+\dfrac{5}{3}\left(\cos^2 x\right)^2=1$

$\Leftrightarrow\left(\dfrac{1-\cos 2x}{2}\right)^2+\dfrac{5}{3}\left(\dfrac{1+\cos 2x}{2}\right)^2=1\quad(1)$. Đặt $\cos 2x=t, t\in[-1;1]$

$(1)\Leftrightarrow\dfrac{1-2t+t^2}{4}+\dfrac{5}{3}\cdot\dfrac{1+2t+t^2}{4}=1\Leftrightarrow 8t^2+4t-4=0\Leftrightarrow t=-1\vee t=\dfrac{1}{2}$.

Với $t=-1\Leftrightarrow\cos 2x=-1\Leftrightarrow 2x=\pi+k2\pi\Leftrightarrow x=\dfrac{\pi}{2}+k\pi,\left(k\in\mathbb{Z}\right)$

Với $t=\dfrac{1}{2}\Leftrightarrow\cos 2x=\dfrac{1}{2}\Leftrightarrow\left[\begin{array}{l}2x=\dfrac{\pi}{3}+k2\pi\\2x=-\dfrac{\pi}{3}+k2\pi\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=\dfrac{\pi}{6}+k\pi\\x=-\dfrac{\pi}{6}+k\pi\end{array}\right.,\left(k\in\mathbb{Z}\right)$

Vậy nghiệm của phương trình: $x=\dfrac{\pi}{2}+k\pi,\ x=\dfrac{\pi}{6}+k\pi, x=-\dfrac{\pi}{6}+k\pi,\left(k\in\mathbb{Z}\right)$

4). $\sin^4 x+\cos^4 x+\cos\left(x-\dfrac{\pi}{4}\right)\sin\left(x-\dfrac{3\pi}{4}\right)=\dfrac{1}{2}$

$\Leftrightarrow\left(1-\dfrac{1}{2}\sin^2 2x\right)+\dfrac{1}{2}\left[\sin\left(-\dfrac{\pi}{2}\right)+\sin\left(2x-\pi\right)\right]=\dfrac{1}{2}\Leftrightarrow 1-\dfrac{1}{2}\sin^2 2x+\dfrac{1}{2}\left(-1-\sin 2x\right)=\dfrac{1}{2}$

$\Leftrightarrow\sin^2 2x+\sin 2x=0\Leftrightarrow\sin 2x=0\vee\sin 2x=-1$

Với $\sin 2x = 0 \Leftrightarrow 2x = k\pi \Leftrightarrow x = \frac{k\pi}{2},(k \in \mathbb{Z})$

Với $\sin 2x = -1 \Leftrightarrow 2x = -\frac{\pi}{2} + k2\pi \Leftrightarrow x = -\frac{\pi}{4} + k\pi,(k \in \mathbb{Z})$.

Vậy nghiệm của phương trình: $x = \frac{k\pi}{2}$, $x = -\frac{\pi}{4} + k\pi,(k \in \mathbb{Z})$

5). $\sin^4 x + \cos 2x + 4\sin^6 x = 0 \Leftrightarrow \left(\sin^2 x\right)^2 + \cos 2x + 4\left(\sin^2 x\right)^3 = 0$

$\Leftrightarrow \left(\frac{1-\cos 2x}{2}\right)^2 + \cos 2x + 4\left(\frac{1-\cos 2x}{2}\right)^3 = 0$ (1). Đặt $\cos 2x = t, t \in [-1;1]$. Phương trình (1) trở thành: $\left(\frac{1-t}{2}\right)^2 + t + 4\left(\frac{1-t}{2}\right)^3 = 0 \Leftrightarrow 2t^3 - 7t^2 + 4t - 3 = 0 \Leftrightarrow t = 3$ (loại).

Kết luận phương trình vô nghiệm.

6). $\cos 8x + \sin^3 x\cos x - \cos^3 x\sin x - 1 = 0 \quad (1)$

$(1) \Leftrightarrow \cos 8x + \sin x\cos x\left(\sin^2 x - \cos^2 x\right) - 1 = 0 \Leftrightarrow \cos 8x - \frac{1}{2}\sin 2x\cos 2x - 1 = 0$

$\Leftrightarrow 1 - 2\sin^2 4x - \frac{1}{4}\sin 4x - 1 = 0 \Leftrightarrow 2\sin^2 4x + \frac{1}{4}\sin 4x = 0 \Leftrightarrow \sin 4x = 0 \vee \sin 4x = -\frac{1}{8}$

Với $\sin 4x = 0 \Leftrightarrow 4x = k\pi \Leftrightarrow x = \frac{k\pi}{4},(k \in \mathbb{Z})$.

Với $\sin 4x = -\frac{1}{8} \Leftrightarrow \left[\begin{array}{l} 4x = \arcsin\left(-\frac{1}{8}\right) + k2\pi \\ 4x = \pi - \arcsin\left(-\frac{1}{8}\right) + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{1}{4}\arcsin\left(-\frac{1}{8}\right) + \frac{k\pi}{2} \\ x = \frac{\pi}{4} - \frac{1}{4}\arcsin\left(-\frac{1}{8}\right) + \frac{k\pi}{2} \end{array}\right.,(k \in \mathbb{Z})$

Vậy nghiệm của phương trình: $x = \frac{k\pi}{4}$, $x = \frac{1}{4}\arcsin\left(-\frac{1}{8}\right) + \frac{k\pi}{2}$,

$x = \frac{\pi}{4} - \frac{1}{4}\arcsin\left(-\frac{1}{8}\right) + \frac{k\pi}{2},(k \in \mathbb{Z})$

Câu 7: Giải các phương trình lượng giác sau:

1). $\left(\sin 2x + \sqrt{3}\cos 2x\right)^2 = 2\cos\left(2x - \frac{\pi}{6}\right)$

2). $\cot x+\left(1+\tan x\tan\frac{x}{2}\right)\sin x=4$

3). $\cot x-\tan x=\frac{2}{\sin 2x}-4\sin 2x$

4). $\sqrt{2}\left(2\sin x-1\right)=4\left(\sin x-1\right)-\cos\left(2x+\frac{\pi}{4}\right)-\sin\left(2x+\frac{\pi}{4}\right)$

5). $\cot x-1=\frac{\cos 2x}{1+\tan x}+\sin^2 x-\frac{1}{2}\sin 2x$ (ĐH khối A 2003)

6). $5\sin x-2=3\left(1-\sin x\right)\tan^2 x$ (ĐH khối B 2004)

7). $\cos^2 3x\cos 2x-\cos^2 x=0$ (ĐH khối A 2005).

8). $\sin^4 x+\cos^4 x+\cos\left(x-\frac{\pi}{4}\right)\sin\left(3x-\frac{\pi}{4}\right)-\frac{3}{2}=0$ (ĐH khối D 2005).

9). $\frac{\left(1+\sin x+\cos 2x\right)\sin\left(x+\frac{\pi}{4}\right)}{1+\tan x}=\frac{1}{\sqrt{2}}\cos x$ (ĐH khối A 2010).

10). $\frac{\sin^4 x+\cos^4 x}{5\sin 2x}=\frac{1}{2}\cot 2x-\frac{1}{8\sin 2x}$ (1) [Dự bị 2 ĐH02]

11). $\cot x-\tan x+4\sin 2x=\frac{2}{\sin 2x}$.[ĐH B03]

12). $3\cos 4x-8\cos^6 x+2\cos^2 x+3=0$ (1) [Dự bị 1 ĐH B03]

13). $\cot x=\tan x+\frac{2\cos 4x}{\sin 2x}$ [Dự bị 2 ĐH D03]

LỜI GIẢI

1). $\left(\sin 2x+\sqrt{3}\cos 2x\right)^2=2\cos\left(2x-\frac{\pi}{6}\right)$ (1)

$$(1)\Leftrightarrow 4\left(\frac{1}{2}\sin 2x+\frac{\sqrt{3}}{2}\cos 2x\right)^2=2\cos\left(2x-\frac{\pi}{6}\right)$$

$$\Leftrightarrow 4\left(\cos 2x\cos\frac{\pi}{6}+\sin 2x\sin\frac{\pi}{6}\right)^2=2\cos\left(2x-\frac{\pi}{6}\right)$$