$$\Leftrightarrow \cos\left(2x-\frac{\pi}{6}\right)=\cos\frac{\pi}{3} \Leftrightarrow \left[\begin{array}{l} 2x-\frac{\pi}{6}=\frac{\pi}{3}+k2\pi \\ 2x-\frac{\pi}{6}=-\frac{\pi}{3}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{4}+k\pi \\ x=-\frac{\pi}{12}+k\pi \end{array}\right.,(k\in\mathbb{Z})$$

Vậy nghiệm của phương trình: $x=\frac{\pi}{4}+k\pi, x=-\frac{\pi}{12}+k\pi, (k\in\mathbb{Z})$

2). $4\sin\left(x+\frac{\pi}{4}\right)+2\cos\left(x-\frac{\pi}{4}\right)-3\sqrt{2}=0$

$$\Leftrightarrow 4.\frac{\sin x+\cos x}{\sqrt{2}}+2.\frac{\sin x+\cos x}{\sqrt{2}}=3\sqrt{2} \Leftrightarrow \sin x+\cos x=1 \Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=1$$

$$\Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}} \Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{4}=\frac{\pi}{4}+k2\pi \\ x+\frac{\pi}{4}=\pi-\frac{\pi}{4}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=k2\pi \\ x=\frac{\pi}{2}+k2\pi \end{array}\right.$$

Vậy nghiệm của phương trình: $x=k2\pi, x=\frac{\pi}{2}+k2\pi, (k\in\mathbb{Z})$

3). $8\sin x\sin 2x+6\sin\left(x+\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}-2x\right)=5+7\cos x$

$$\Leftrightarrow 4(\cos x-\cos 3x)+3\left(\sin 3x+\sin\left(\frac{\pi}{2}-x\right)\right)=5+7\cos x$$

$$\Leftrightarrow 4(\cos x-\cos 3x)+3(\sin 3x+\cos x)=5+7\cos x \Leftrightarrow 3\sin 3x-4\cos 3x=5$$

$$\Leftrightarrow \frac{3}{5}\sin 3x-\frac{4}{5}\cos 3x=1 \Leftrightarrow \sin 3x.\cos\alpha-\cos 3x.\sin\alpha=1 \Leftrightarrow \sin(3x-\alpha)=1$$

$\Leftrightarrow 3x-\alpha=\frac{\pi}{2}+k2\pi \Leftrightarrow x=\frac{\alpha}{3}+\frac{\pi}{6}+\frac{k2\pi}{3}$. (Với $\frac{3}{5}=\cos\alpha, \frac{4}{5}=\sin\alpha$)

Vậy nghiệm của phương trình: $x=\frac{\alpha}{3}+\frac{\pi}{6}+\frac{k2\pi}{3}$

4). $2\sqrt{3}\sin\left(x-\frac{\pi}{8}\right)\cos\left(x-\frac{\pi}{8}\right)+2\sin^2\left(x-\frac{\pi}{8}\right)=\sqrt{3}+1$

Áp dụng công thức nhân đôi: $2\sin\left(x-\frac{\pi}{8}\right)\cos\left(x-\frac{\pi}{8}\right)=\sin\left(2x-\frac{\pi}{4}\right)$, *và hạ bậc* $2\sin^2\left(x-\frac{\pi}{8}\right)=1-\cos\left(2x-\frac{\pi}{4}\right)$ *ta được:*

$$\Leftrightarrow \sqrt{3}\sin\left(2x-\frac{\pi}{4}\right)+1-\cos\left(2x-\frac{\pi}{4}\right)=\sqrt{3}+1$$

$$\Leftrightarrow \sqrt{3}\sin\left(2x-\frac{\pi}{4}\right)-\cos\left(2x-\frac{\pi}{4}\right)=\sqrt{3}$$

$$\Leftrightarrow \frac{\sqrt{3}}{2}\sin\left(2x-\frac{\pi}{4}\right)-\frac{1}{2}\cos\left(2x-\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2}$$

$$\Leftrightarrow \sin\left(2x-\frac{\pi}{4}\right)\cos\frac{\pi}{6}-\cos\left(2x-\frac{\pi}{4}\right)\sin\frac{\pi}{6}=\frac{\sqrt{3}}{2}$$

$$\Leftrightarrow \sin\left(2x-\frac{\pi}{4}-\frac{\pi}{6}\right)=\sin\frac{\pi}{3}\Leftrightarrow \left[\begin{array}{l}2x-\frac{\pi}{4}-\frac{\pi}{6}=\frac{\pi}{3}+k2\pi\\ 2x-\frac{\pi}{4}-\frac{\pi}{6}=\pi-\frac{\pi}{3}+k2\pi\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=\frac{3\pi}{8}+k\pi\\ x=\frac{13\pi}{24}+k\pi\end{array}\right.$$

Vậy nghiệm của phương trình: $x=\frac{3\pi}{8}+k\pi, x=\frac{13\pi}{24}+k\pi, (k\in\mathbb{Z})$

5). $\frac{1+\cos x+\cos 2x+\cos 3x}{2\cos^2 x+\cos x-1}=\frac{2}{3}\left(3-\sqrt{3}\sin x\right)$ $(*)$. Điều kiện $2\cos^2 x+\cos x-1\neq 0$

$$\Leftrightarrow \frac{(1+\cos 2x)+(\cos 3x+\cos x)}{(2\cos^2 x-1)+\cos x}=\frac{2}{3}\left(3-\sqrt{3}\sin x\right)$$

$$\Leftrightarrow \frac{2\cos^2 x+2\cos 2x.\cos x}{\cos 2x+\cos x}=\frac{2}{3}\left(3-\sqrt{3}\sin x\right)$$

$$\Leftrightarrow \frac{2\cos x(\cos 2x+\cos x)}{\cos 2x+\cos x}=\frac{2}{3}\left(3-\sqrt{3}\sin x\right)$$

$$\Leftrightarrow 2\cos x=\frac{2}{3}\left(3-\sqrt{3}\sin x\right)\quad\Leftrightarrow \sqrt{3}\cos x+\sin x=\sqrt{3}$$

$$\Leftrightarrow \frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x=\frac{\sqrt{3}}{2}\quad\Leftrightarrow \cos x.\cos\frac{\pi}{6}+\sin x\sin\frac{\pi}{6}=\frac{\sqrt{3}}{2}$$

$$\Leftrightarrow \cos\left(x-\frac{\pi}{6}\right)=\cos\frac{\pi}{6} \Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{6}=\frac{\pi}{6}+k2\pi \\ x-\frac{\pi}{6}=-\frac{\pi}{6}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{3}+k2\pi \\ x=k2\pi \end{array}\right.$$

Thay hai họ nghiệm của x vào điều kiện ta thấy thỏa.

Vậy nghiệm của phương trình: $x=\frac{\pi}{3}+k2\pi$, $x=k2\pi$.

6). $8\sin x=\frac{\sqrt{3}}{\cos x}+\frac{1}{\sin x}$. Điều kiện $\begin{cases} \sin x\neq 0 \\ \cos x\neq 0 \end{cases}$

Quy đồng mẫu được: $\Leftrightarrow 8\sin^2 x\cos x=\sqrt{3}\sin x+\cos x$

Hạ bậc $\sin^2 x$ được: $\Leftrightarrow 4\left(1-\cos 2x\right)\cos x=\sqrt{3}\sin x+\cos x$

$$\Leftrightarrow 4\cos x-4\cos 2x\cos x=\sqrt{3}\sin x+\cos x$$

$$\Leftrightarrow 4\cos x-2\left(\cos x+\cos 3x\right)=\sqrt{3}\sin x+\cos x$$

$$\Leftrightarrow \cos x-\sqrt{3}\sin x=2\cos 3x \quad \Leftrightarrow \frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x=\cos 3x$$

$$\Leftrightarrow \cos x.\cos\frac{\pi}{3}-\sin x.\sin\frac{\pi}{3}=\cos 3x$$

$$\Leftrightarrow \cos\left(x+\frac{\pi}{3}\right)=\cos 3x \Leftrightarrow \left[\begin{array}{l} 3x=x+\frac{\pi}{3}+k2\pi \\ 3x=-\left(x+\frac{\pi}{3}\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{6}+k\pi \\ x=-\frac{\pi}{12}+\frac{k\pi}{2} \end{array}\right.$$

So với điều kiện nghiệm của phương trình: $x=\frac{\pi}{6}+k\pi$, $x=-\frac{\pi}{12}+\frac{k\pi}{2}$.

7). $2\cos^3 x+2\sin^3 x+2\sin^2 x\cos x+2\cos^2 x\sin x-\sqrt{2}=0$

$$\Leftrightarrow 2\left(\cos x+\sin x\right)\left(\sin^2 x-\sin x\cos x+\cos^2 x\right)+2\sin x\cos x\left(\sin x+\cos x\right)-\sqrt{2}=0$$

$$\Leftrightarrow 2\left(\cos x+\sin x\right)\left(1-\sin x\cos x\right)+2\sin x\cos x\left(\sin x+\cos x\right)-\sqrt{2}=0$$

$$\Leftrightarrow 2\left(\cos x+\sin x\right)=\sqrt{2} \Leftrightarrow 2\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=\sqrt{2} \Leftrightarrow \cos\left(x-\frac{\pi}{4}\right)=\frac{1}{2}$$

$\Leftrightarrow \cos\left(x-\frac{\pi}{4}\right)=\cos\frac{\pi}{3} \Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{4}=\frac{\pi}{3}+k2\pi \\ x-\frac{\pi}{4}=-\frac{\pi}{3}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{7\pi}{12}+k2\pi \\ x=-\frac{\pi}{12}+k2\pi \end{array}\right.$

So với điều kiện nghiệm của phương trình: $x=\frac{7\pi}{12}+k2\pi, x=-\frac{\pi}{12}+k2\pi,(k\in\mathbb{Z})$

8). $5(\cos x+\sin x)+\sin 3x-\cos 3x=2\sqrt{2}(2+\sin 2x)$

$\Leftrightarrow 5(\cos x+\sin x)+3\sin x-4\sin^3 x-(4\cos^3 x-3\cos x)=2\sqrt{2}(2+\sin 2x)$

$\Leftrightarrow 8(\cos x+\sin x)-4(\sin^3 x+\cos^3 x)=2\sqrt{2}(2+\sin 2x)$

$\Leftrightarrow 8(\cos x+\sin x)-4(\sin x+\cos x)(1-\sin x\cos x)=2\sqrt{2}(2+\sin 2x)$

$\Leftrightarrow 4(\sin x+\cos x)(2-1+\sin x\cos x)=2\sqrt{2}(2+\sin 2x)$

$\Leftrightarrow 4(\sin x+\cos x)\left(1+\frac{1}{2}\sin 2x\right)=2\sqrt{2}(2+\sin 2x)$

$\Leftrightarrow 2(\sin x+\cos x)(2+\sin 2x)=2\sqrt{2}(2+\sin 2x)$

$\Leftrightarrow \sin x+\cos x=\sqrt{2}$ (vì $2+\sin 2x>0$).

$\Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=\sqrt{2} \Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=1 \Leftrightarrow x+\frac{\pi}{4}=\frac{\pi}{2}+k2\pi \Leftrightarrow x=\frac{\pi}{4}+k2\pi$

Câu 4: Giải các phương trình sau:

1). $\sin x+\sin 2x=\sqrt{3}(\cos x+\cos 2x)$ [Dự bị 1 ĐH A04]

2). $2\sin\left(2x-\frac{\pi}{6}\right)+4\sin x+1=0$ [Dự bị 2 ĐH A06]

3). $\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2+\sqrt{3}\cos x=2$ [ĐH D07]

4). $2\cos^2 x+2\sqrt{3}\sin x\cos x+1=3(\sin x+\sqrt{3}\cos x)$ [Dự bị 2 ĐH A07]

5). $\sin^3 x-\sqrt{3}\cos^3 x=\sin x\cos^2 x-\sqrt{3}\sin^2 x\cos x$ [ĐH B08]

6). $\sin 3x-\sqrt{3}\cos 3x=2\sin 2x$ [CĐ 08]

7). $2\sin\left(x+\frac{\pi}{3}\right)-\sin\left(2x-\frac{\pi}{6}\right)=\frac{1}{2}$ [Dự bị 1 ĐH B08]

8). $\frac{(1-2\sin x)\cos x}{(1+2\sin x)(1-\sin x)}=\sqrt{3}\ (1)$ [ĐH A09]

9). $\sqrt{3}\cos 5x-2\sin 3x\cos 2x-\sin x=0$ [ĐH D09]

LỜI GIẢI

1). $\sin x+\sin 2x=\sqrt{3}(\cos x+\text{cox}2x)$ [Dự bị 1 ĐH A04]

LỜI GIẢI

$\Leftrightarrow \sin x+\sin 2x=\sqrt{3}\cos x+\sqrt{3}\cos 2x \Leftrightarrow \sin x-\sqrt{3}\cos x=\sqrt{3}\cos 2x-\sin 2x$

$\Leftrightarrow \frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x=\frac{\sqrt{3}}{2}\cos 2x-\frac{1}{2}\sin 2x \Leftrightarrow \sin\left(x-\frac{\pi}{3}\right)=\cos\left(2x+\frac{\pi}{6}\right)$

$\Leftrightarrow \sin\left(x-\frac{\pi}{3}\right)=\sin\left[\frac{\pi}{2}-\left(2x+\frac{\pi}{6}\right)\right]\Leftrightarrow \sin\left(x-\frac{\pi}{3}\right)=\sin\left(\frac{\pi}{3}-2x\right)$

$\Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{3}=\frac{\pi}{3}-2x+k2\pi \\ x-\frac{\pi}{3}=\pi-\left(\frac{\pi}{3}-2x\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{2\pi}{9}+\frac{k2\pi}{3} \\ x=-\pi-k2\pi \end{array}\right.,(k\in\mathbb{Z})$

2). $2\sin\left(2x-\frac{\pi}{6}\right)+4\sin x+1=0$ [Dự bị 2 ĐH A06]

LỜI GIẢI

$\Leftrightarrow 2\left[\sin 2x\cos\frac{\pi}{6}-\cos 2x\sin\frac{\pi}{6}\right]+4\sin x+1=0$

$\Leftrightarrow \sqrt{3}\sin 2x-\cos 2x+4\sin x+1=0 \Leftrightarrow \sqrt{3}\sin 2x+(1-\cos 2x)+4\sin x=0$

$\Leftrightarrow 2\sqrt{3}\sin x\cos x+2\sin^2 x+4\sin x=0 \Leftrightarrow 2\sin x\left(\sqrt{3}\cos x+\sin x+2\right)=0$