

$$c) C = \sin(a + 30^\circ) \cdot \cos(a - 30^\circ) = \frac{1}{2} (\sin 2a + \sin 60^\circ) = \frac{1}{2} \sin 2a + \frac{\sqrt{3}}{4}$$

$$d) D = \sin\left(x + \frac{\pi}{6}\right) \cdot \sin\left(x - \frac{\pi}{6}\right) \cdot \cos 2x = \frac{1}{2} (\cos \frac{\pi}{3} - \cos 2x) \cdot \cos 2x = \frac{1}{4} \cos 2x - \frac{1}{2} \cos^2 2x$$

$$= \frac{1}{4} \cos 2x - \frac{1}{4} (1 + \cos 4x) = \frac{1}{4} \cos 2x - \frac{1}{4} + \frac{1}{4} \cos 4x$$

Bài 8: Rút gọn các biểu thức sau:

$$a) A = \frac{\sin 3x \cos 5x - \sin 5x \cos 3x}{\cos x}$$

$$b) B = \frac{\sin x + \sin 4x + \sin 7x}{\cos x + \cos 4x + \cos 7x}$$

$$c) C = \frac{2 \sin 2\alpha - \sin 4\alpha}{2 \sin 2\alpha + \sin 4\alpha}$$

$$d) D = \frac{\sin 5\alpha - \sin 3\alpha}{2 \cos 4\alpha}$$

$$e) E = \tan \alpha \left(\frac{1 + \cos^2 \alpha}{\sin \alpha} - \sin \alpha \right)$$

$$f) F = \frac{1 + \cos \alpha - \sin \alpha}{1 - \cos \alpha - \sin \alpha}$$

Giải: a) $A = \frac{\sin 3x \cos 5x - \sin 5x \cos 3x}{\cos x} = \frac{\sin(3x - 5x)}{\cos x} = \frac{\sin(-2x)}{\cos x} = \frac{-\sin 2x}{\cos x}$

$$= \frac{-2 \sin x \cos x}{\cos x} = -2 \sin x$$

$$b) B = \frac{\sin x + \sin 4x + \sin 7x}{\cos x + \cos 4x + \cos 7x} = \frac{(\sin 7x + \sin x) + \sin 4x}{(\cos 7x + \cos x) + \cos 4x} = \frac{2 \sin 4x \cos 3x + \sin 4x}{2 \cos 4x \cos 3x + \cos 4x}$$

$$= \frac{\sin 4x(2 \cos 3x + 1)}{\cos 4x(2 \cos 3x + 1)} = \frac{\sin 4x}{\cos 4x} = \tan 4x$$

$$c) C = \frac{2 \sin 2\alpha - \sin 4\alpha}{2 \sin 2\alpha + \sin 4\alpha} = \frac{2 \sin 2\alpha - 2 \sin 2\alpha \cdot \cos 2\alpha}{2 \sin 2\alpha + 2 \sin 2\alpha \cdot \cos 2\alpha} = \frac{2 \sin 2\alpha(1 - \cos 2\alpha)}{2 \sin 2\alpha(1 + \cos 2\alpha)}$$

$$= \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha} = \frac{2 \sin^2 \alpha}{2 \cos^2 \alpha} = \tan^2 \alpha$$

$$d) D = \frac{\sin 5\alpha - \sin 3\alpha}{2 \cos 4\alpha} = \frac{2 \cos 4\alpha \sin \alpha}{2 \cos 4\alpha} = \sin \alpha$$

$$e) E = \tan \alpha \left(\frac{1 + \cos^2 \alpha}{\sin \alpha} - \sin \alpha \right) = \frac{\sin \alpha}{\cos \alpha} \left(\frac{1 + \cos^2 \alpha}{\sin \alpha} - \sin \alpha \right) = \frac{1 + \cos^2 \alpha}{\cos \alpha} - \frac{\sin^2 \alpha}{\cos \alpha}$$

$$= \frac{1 + \cos^2 \alpha}{\cos \alpha} - \frac{\sin^2 \alpha}{\cos \alpha} = \frac{1 + \cos^2 \alpha - \sin^2 \alpha}{\cos \alpha} = \frac{2 \cos^2 \alpha}{\cos \alpha} = 2 \cos \alpha$$

$$f) F = \frac{1 + \cos a - \sin a}{1 - \cos a - \sin a} = \frac{2 \cos^2 \frac{a}{2} - 2 \sin \frac{a}{2} \cos \frac{a}{2}}{2 \sin^2 \frac{a}{2} - 2 \sin \frac{a}{2} \cos \frac{a}{2}} = \frac{2 \cos \frac{a}{2} (\cos \frac{a}{2} - \sin \frac{a}{2})}{2 \sin \frac{a}{2} (\sin \frac{a}{2} - \cos \frac{a}{2})} = -\cot \frac{a}{2}$$

Bài 9: Chứng minh các đồng nhất thức:

a) $\cot a - \tan 5a + \frac{4 \sin^2 3a}{\sin 6a - \sin 4a} = \frac{1}{\sin a \cdot \cos 5a}$

b) $\frac{1 - \cos x + \cos 2x}{\sin 2x - \sin x} = \cot x$

c) $\frac{\sin^3 a + \cos^3 a}{\sin a + \cos a} = 1 - \sin a \cos a$

d) $\sin^4 x + \cos^4 x - \sin^6 x - \cos^6 x = \sin^2 x \cdot \cos^2 x$

e) $\sin x(1 + 2\cos 2x + 2\cos 4x + 2\cos 6x) = \sin 7x$

f) $\frac{1 - \sin^2 \alpha \cdot \cos^2 \alpha}{\cos^2 \alpha} - \cos^2 \alpha = \tan^2 \alpha$

Giải: a) VT = $\cot a - \tan 5a + \frac{4 \sin^2 3a}{\sin 6a - \sin 4a} = \frac{\cos a}{\sin a} - \frac{\sin 5a}{\cos 5a} + \frac{4 \sin^2 3a}{\sin 6a - \sin 4a}$

$$= \frac{\cos 5a \cdot \cos a - \sin 5a \cdot \sin a}{\cos 5a \cdot \sin a} + \frac{2(1 - \cos 6a)}{2 \cos 5a \cdot \sin a} = \frac{\cos 6a}{\cos 5a \cdot \sin a} + \frac{1 - \cos 6a}{\cos 5a \cdot \sin a}$$

$$= \frac{\cos 6a + 1 - \cos 6a}{\cos 5a \cdot \sin a} = \frac{1}{\sin a \cdot \cos 5a} = \text{VT (đpcm)}$$

b) VT = $\frac{1 - \cos x + \cos 2x}{\sin 2x - \sin x} = \frac{2 \cos^2 x - \cos x}{2 \sin x \cos x - \sin x} = \frac{\cos x(2 \cos x - 1)}{\sin x(2 \cos x - 1)} = \frac{\cos x}{\sin x} = \cot x = \text{VP (đpcm)}$

c) VT = $\frac{\sin^3 a + \cos^3 a}{\sin a + \cos a} = \frac{(\sin a + \cos a)(\sin^2 a - \sin a \cdot \cos a + \cos^2 a)}{\sin a + \cos a} = 1 - \sin a \cdot \cos a = \text{VP (đpcm)}$

d) VT = $\sin^4 x + \cos^4 x - \sin^6 x - \cos^6 x = \sin^4 x + \cos^4 x - (\sin^6 x + \cos^6 x)$

$$= \sin^4 x + \cos^4 x - [(\sin^2 x)^3 + (\cos^2 x)^3]$$

$$= \sin^4 x + \cos^4 x - (\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cdot \cos^2 x + \cos^4 x)$$

$$= \sin^4 x + \cos^4 x - \sin^4 x + \sin^2 x \cdot \cos^2 x - \cos^4 x = \sin^2 x \cdot \cos^2 x = \text{VP (đpcm)}$$

e) VT = $\sin x(1 + 2\cos 2x + 2\cos 4x + 2\cos 6x)$

$$= \sin x + 2\cos 2x \cdot \sin x + 2\cos 4x \cdot \sin x + 2\cos 6x \cdot \sin x$$

$$= \sin x + (\sin 3x - \sin x) + (\sin 5x - \sin 3x) + (\sin 7x - \sin 5x) = \sin 7x = \text{VP (đpcm)}$$

$$\begin{aligned} \text{f) VT} &= \frac{1 - \sin^2 \alpha \cdot \cos^2 \alpha}{\cos^2 \alpha} - \cos^2 \alpha = \frac{1 - \sin^2 \alpha \cdot \cos^2 \alpha - \cos^4 \alpha}{\cos^2 \alpha} = \frac{1 - \cos^2 \alpha (\sin^2 \alpha + \cos^2 \alpha)}{\cos^2 \alpha} \\ &= \frac{1 - \cos^2 \alpha}{\cos^2 \alpha} = \frac{\sin^2 \alpha}{\cos^2 \alpha} = \tan^2 \alpha = \text{VP (đpcm)} \end{aligned}$$

III: BÀI TẬP TỰ LUYỆN

Bài 1: Tính các giá trị lượng giác của cung α , biết:

a) $\sin \alpha = 0,6$ khi $0 < \alpha < \frac{\pi}{2}$

b) $\cos \alpha = -\frac{7}{10}$ khi $\frac{\pi}{2} < \alpha < \pi$

c) $\tan \alpha = 2$ khi $\pi < \alpha < \frac{3\pi}{2}$

d) $\cot \alpha = -3$ khi $\frac{3\pi}{2} < \alpha < 2\pi$

Bài 2: Tính các giá trị lượng giác của cung a , biết:

a) $\cos a = -\frac{1}{4}$, $\pi < a < \frac{3\pi}{2}$

b) $\sin a = \frac{2}{3}$, $\frac{\pi}{2} < a < \pi$

c) $\tan a = \frac{7}{3}$, $0 < a < \frac{\pi}{2}$

d) $\cot a = -\frac{14}{9}$, $\frac{3\pi}{2} < a < 2\pi$

Bài 3: a) Cho $\sin a = -\frac{12}{13}$ và $\frac{3\pi}{4} < \frac{a}{2} < \pi$. Tính $\cos\left(\frac{\pi}{3} - a\right)$

b) Cho $\sin \alpha = \frac{8}{17}$, $0 < \alpha < \frac{\pi}{2}$ và $\sin \beta = \frac{8}{17}$, $\pi < \beta < \frac{3\pi}{2}$. Tính $\sin(\alpha + \beta)$

c) Cho $\cos \alpha = -\frac{1}{3}$, $\alpha \in \left(\frac{\pi}{2}; \pi\right)$. Tính $\tan\left(\alpha - \frac{\pi}{4}\right)$

Bài 4: a) Cho $\cos a = \frac{3}{5}$ và $\frac{\pi}{2} < a < \pi$. Tính $\sin\left(\frac{\pi}{6} + a\right)$

b) Tính $\cos(a + b)$, biết $\sin a = \frac{4}{5}$ với $0^\circ < a < 90^\circ$ và $\sin b = \frac{2}{3}$ với $90^\circ < b < 180^\circ$

c) Cho $\cos x = \frac{4}{5}$ và $\frac{3\pi}{2} < x < 2\pi$. Tính $\tan\left(x + \frac{\pi}{4}\right)$

Bài 5: a) Cho $\cos \alpha = \frac{3}{4}$ và $0 < \alpha < \frac{\pi}{2}$. Tính $\cos 2\alpha$, $\sin 2\alpha$ và $\tan 2\alpha$

b) Cho $\sin \beta = -\frac{3}{5}$ và $\pi < \beta < \frac{3\pi}{2}$. Tính $\cos 2\beta$, $\sin 2\beta$ và $\tan 2\beta$

Bài 6: a) Cho $\sin a - \cos a = \frac{1}{5}$ và $\frac{\pi}{4} < a < \frac{\pi}{2}$. Tính $\sin 2a$, $\cos 2a$, $\tan 2a$

b) Cho $\sin b + \cos b = \frac{2}{7}$ và $0 < b < \frac{\pi}{4}$. Tính $\sin 2b$, $\cos 2b$, $\tan 2b$

Bài 7: Tính giá trị của các biểu thức: a) $A = \cos 75^\circ \cdot \cos 15^\circ$ b) $B = \sin \frac{11\pi}{12} \cdot \cos \frac{5\pi}{12}$

c) $C = \frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$ d) $D = 96\sqrt{3} \sin \frac{\pi}{48} \cdot \cos \frac{\pi}{48} \cdot \cos \frac{\pi}{24} \cdot \cos \frac{\pi}{12} \cdot \cos \frac{\pi}{6}$

Bài 8: Biến đổi thành tích các biểu thức sau:

a) $A = 1 + \sin x$ b) $B = 1 + 2\cos x$ c) $C = \cos 5x - \cos 2x$

d) $D = 1 + \cos 2x + \cos 4x + \cos 6x$ e) $E = 1 + \sin x - \cos 2x$

f) $F = 1 - 2\cos x + \cos 2x$ g) $M = \sin x \cdot \cos 3x + \sin 4x \cdot \cos 2x$

h) $N = \sin^2 x - \sin^2 2x + \sin^2 3x$ i) $P = \cos a + \cos b + \sin(a + b)$

Bài 9: Biến đổi thành tổng các biểu thức sau:

a) $A = 2\cos 2x \cdot \cos 3x \cdot \sin 4x$ b) $B = 8\cos 3x \cdot \sin 5x \cdot \sin 7x$

c) $C = 4\sin(x + 30^\circ) \cdot \cos(x - 30^\circ) \cdot \cos 2x$ d) $D = 4\cos(a - b) \cdot \cos(b - c) \cdot \cos(c - a)$

Bài 10: Rút gọn các biểu thức sau:

a) $A = \frac{\cos 2x \cos 4x + \sin 2x \sin 4x}{\sin 4x}$ b) $B = \frac{\sin x + \sin 3x + \sin 5x}{\cos x + \cos 3x + \cos 5x}$

c) $C = \frac{1 + \sin 2\alpha + \cos 2\alpha}{1 + \sin 2\alpha - \cos 2\alpha}$ d) $D = \frac{\sin 8\alpha + \sin 2\alpha}{2\sin 5\alpha}$

e) $E = \cot \alpha \left(\frac{1 + \sin^2 \alpha}{\cos \alpha} - \cos \alpha \right)$ f) $F = \frac{\sin 2a + \sin a}{1 + \cos 2a + \cos a}$

Bài 11: Chứng minh các đồng nhất thức:

a) $\frac{\cos 4a \cdot \tan 2a - \sin 4a}{\cos 4a \cdot \cot 2a + \sin 4a} = -\tan^2 2a$

b) $\frac{1 + \cos x + \cos 2x + \cos 3x}{2 \cos^2 x + \cos x - 1} = 2 \cos x$

c) $\frac{\sin^3 a - \cos^3 a}{\sin a - \cos a} = 1 + \sin a \cos a$

d) $\cos^2 x \cdot \sin x - \sin^3 x \cdot \cos x = \frac{1}{4} \sin 4x$

e) $\cos x(2 \cos 2x + 2 \cos 4x + 2 \cos 6x - 1) = -\cos 7x$

f) $\frac{1 - \cos x}{1 + \cos x} \cdot \tan^2 \frac{x}{2} - \cos^2 x = \sin^2 x$

g) $\cos x - \frac{1}{2} \cos 3x - \frac{1}{2} \cos 5x = 8 \sin^2 x \cdot \cos^3 x$

h) $3 - 4 \cos 2a + \cos 4a = 8 \sin^4 a$

i) $\frac{\cos 2x - \sin 4x - \cos 6x}{\cos 2x + \sin 4x - \cos 6x} = \tan(x - 15^\circ) \cdot \cot(x - 15^\circ)$

j) $\cos 2a - \cos 3a - \cos 4a + \cos 5a = -4 \sin \frac{a}{2} \cdot \sin a \cdot \cos \frac{7a}{2}$

