LỜI GIẢI

a). $\lim\limits_{x\to-\infty} x\sqrt{\dfrac{2x^3+x}{x^5-x^2+3}} = \lim\limits_{x\to-\infty} x\sqrt{\dfrac{x^3\left(2+\dfrac{1}{x^2}\right)}{x^5\left(1-\dfrac{1}{x^3}+\dfrac{3}{x^5}\right)}} = \lim\limits_{x\to-\infty} x\dfrac{\sqrt{2+\dfrac{1}{x^2}}}{\sqrt{x^2}\sqrt{1-\dfrac{1}{x^3}+\dfrac{3}{x^5}}}$

$= \lim\limits_{x\to-\infty} \dfrac{\sqrt{2+\dfrac{1}{x^2}}}{\sqrt{1-\dfrac{1}{x^3}+\dfrac{3}{x^5}}} = \sqrt{2}$

b). $\lim\limits_{x\to+\infty}\left[(x+2)\sqrt{\dfrac{x-1}{x^3+x}}\right] = \lim\limits_{x\to+\infty}\left[x\left(1+\dfrac{2}{x}\right)\sqrt{\dfrac{x\left(1-\dfrac{1}{x}\right)}{x^3\left(1+\dfrac{1}{x^2}\right)}}\right] = \lim\limits_{x\to+\infty}\left[x\left(1+\dfrac{2}{x}\right)\dfrac{\sqrt{1-\dfrac{1}{x}}}{x\sqrt{1+\dfrac{1}{x^2}}}\right]$

$= \lim\limits_{x\to+\infty}\left[\left(1+\dfrac{2}{x}\right)\dfrac{\sqrt{1-\dfrac{1}{x}}}{\sqrt{1+\dfrac{1}{x^2}}}\right] = 1.$

c). $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+2x+3}+4x+1}{\sqrt{4x^2+1}-x+2} = \lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2\left(1+\dfrac{2}{x}+\dfrac{3}{x^2}\right)}+4x+1}{\sqrt{x^2\left(4+\dfrac{1}{x^2}\right)}-x+2} = \lim\limits_{x\to-\infty}\dfrac{|x|\sqrt{\left(1+\dfrac{2}{x}+\dfrac{3}{x^2}\right)}+4x+1}{|x|\sqrt{4+\dfrac{1}{x^2}}-x+2}$

$= \lim\limits_{x\to-\infty}\dfrac{-x\sqrt{1+\dfrac{2}{x}+\dfrac{3}{x^2}}+4x+1}{-x\sqrt{4+\dfrac{1}{x^2}}-x+2} = \lim\limits_{x\to-\infty}\dfrac{-x+4x+1}{-4x-x+2} = \lim\limits_{x\to-\infty}\dfrac{-x\left(1-4-\dfrac{1}{x}\right)}{-x\left(4+1-\dfrac{2}{x}\right)} = \lim\limits_{x\to-\infty}\dfrac{-3-\dfrac{1}{x}}{5-\dfrac{2}{x}} = \dfrac{-3}{5}$

d). $\lim\limits_{x\to+\infty}\left(\sqrt{x^2-4x}-x\right) = \lim\limits_{x\to+\infty}\dfrac{x^2-4x-x}{\sqrt{x^2-4x}+x} = \lim\limits_{x\to+\infty}\dfrac{-4x}{\sqrt{x^2\left(1-\dfrac{1}{x}\right)}+x} = \lim\limits_{x\to+\infty}\dfrac{-4x}{x+x} = \lim\limits_{x\to+\infty}\dfrac{-4x}{2x} = -2$

e). $\lim\limits_{x\to-\infty}\left(\sqrt[3]{8x^3+1}-2x+1\right) = \lim\limits_{x\to-\infty}\dfrac{8x^3+1-8x^3}{\left(\sqrt[3]{8x^3+1}\right)^2+\sqrt[3]{8x^3+1}.2x+4x^2}+1$

$$= \lim_{x\to-\infty} \frac{1}{\left(\sqrt[3]{x^3\left(8+\frac{1}{x^3}\right)}\right)^2 + \sqrt[3]{x^3\left(8+\frac{1}{x^3}\right)}.2x + 4x^2} + 1$$

$$= \lim_{x\to-\infty} \frac{1}{4\left(\sqrt[3]{x^3}\right)^2 + 2\sqrt[3]{x^3}.2x + 4x^2} + 1 = \lim_{x\to-\infty} \frac{1}{4x^2+4x^2+4x^2} + 1 = \lim_{x\to-\infty} \frac{1}{12x^2} + 1 = 1$$

Câu 5: Tìm các giới hạn sau:

a). $\lim\limits_{x\to+\infty}\left(\sqrt{x^2-4x}-x\right)$ b). $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+1}-x\right)$ c). $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-3x+2}+x\right)$

d). $\lim\limits_{x\to-\infty}\left(\sqrt{4x^2-4x+1}+2x+3\right)$ e). $\lim\limits_{x\to+\infty}\left(\sqrt{x^2-4x+1}-2x+3\right)$

LỜI GIẢI

a). $\lim\limits_{x\to+\infty}\left(\sqrt{x^2-4x}-x\right) = \lim\limits_{x\to+\infty}\frac{x^2-4x-x^2}{\sqrt{x^2-4x}+x} = \lim\limits_{x\to+\infty}\frac{-4x}{\sqrt{x^2\left(1-\frac{4}{x}\right)}+x} = \lim\limits_{x\to+\infty}\frac{-4x}{|x|+x} = \lim\limits_{x\to+\infty}\frac{-4x}{2x} = -2.$

b). $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+1}-x\right) = \lim\limits_{x\to+\infty}\frac{x^2+1-x^2}{\sqrt{x^2+1}+x} = \lim\limits_{x\to+\infty}\frac{1}{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}+x} = \lim\limits_{x\to+\infty}\frac{1}{|x|+x} = \lim\limits_{x\to+\infty}\frac{1}{2x} = 0.$

c). $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-3x+2}+x\right) = \lim\limits_{x\to-\infty}\frac{x^2-3x+2-x^2}{\sqrt{x^2-3x+2}+x} = \lim\limits_{x\to-\infty}\frac{-3x+2}{\sqrt{x^2\left(1-\frac{3}{x}+\frac{2}{x^2}\right)}-x}$

$= \lim\limits_{x\to-\infty}\frac{-x\left(3+\frac{2}{x}\right)}{|x|-x} = \lim\limits_{x\to-\infty}\frac{-3x}{-2x} = \frac{3}{2}.$

d). $\lim\limits_{x\to-\infty}\left(\sqrt{4x^2-4x+1}+2x+3\right) = \lim\limits_{x\to-\infty}\left(\frac{4x^2-4x+1-4x^2}{\sqrt{4x^2-4x+1}-2x}\right)+3 = \lim\limits_{x\to-\infty}\frac{-4x+1}{\sqrt{x^2\left(4-\frac{4}{x}+\frac{1}{x}\right)}-2x}+3$

$= \lim\limits_{x\to-\infty}\frac{-x\left(4+\frac{1}{x}\right)}{2|x|-2x}+3 = \lim\limits_{x\to-\infty}\frac{-4x}{-2x}+3 = 2+3 = 5.$

e). $\lim\limits_{x\to+\infty}\left(\sqrt{x^2-4x+1}-2x+3\right)=\lim\limits_{x\to+\infty}\dfrac{x^2-4x+1-4x^2}{\sqrt{x^2-4x+1}+2x}+3$ $=\lim\limits_{x\to+\infty}\dfrac{-3x^2-4x+1}{\sqrt{x^2\left(1-\dfrac{4}{x}+\dfrac{1}{x^2}\right)}+2x}+3$

$=\lim\limits_{x\to+\infty}\dfrac{x^2\left(-3-\dfrac{4}{x}+\dfrac{1}{x^2}\right)}{|x|+2x}+3=\lim\limits_{x\to+\infty}\dfrac{-8x^2}{x+2x}+3$ $=\lim\limits_{x\to+\infty}\dfrac{-8x^2}{3x}+3=\lim\limits_{x\to+\infty}(-3x)+3=-\infty$.

Câu 6: Tìm các giới hạn sau:

a). $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-4x+3}-\sqrt{x^2-3x+2}\right)$ b). $\lim\limits_{x\to-\infty}\left(\sqrt{4x^2-9x-21}-\sqrt{4x^2-7x+13}\right)$

c). $\lim\limits_{x\to-\infty}\left(\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2}\right)$

LỜI GIẢI

a). $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-4x+3}-\sqrt{x^2-3x+2}\right)=\lim\limits_{x\to-\infty}\dfrac{x^2-4x+3-x^2+3x-2}{\sqrt{x^2-4x+3}+\sqrt{x^2-3x+2}}$

$=\lim\limits_{x\to-\infty}\dfrac{-x+1}{\sqrt{x^2\left(1-\dfrac{4}{x}+\dfrac{3}{x^2}\right)}+\sqrt{x^2\left(1-\dfrac{4}{x}+\dfrac{3}{x^2}\right)}}=\lim\limits_{x\to-\infty}\dfrac{-x+1}{\sqrt{x^2}\left(\sqrt{1-\dfrac{1}{x}+\dfrac{3}{x^2}}+\sqrt{1-\dfrac{4}{x}+\dfrac{3}{x^2}}\right)}$

$=\lim\limits_{x\to-\infty}\dfrac{-x\left(1-\dfrac{1}{x}\right)}{|x|\left(\sqrt{1-\dfrac{1}{x}+\dfrac{3}{x^2}}+\sqrt{1-\dfrac{4}{x}+\dfrac{3}{x^2}}\right)}=\lim\limits_{x\to-\infty}\dfrac{-x\left(1-\dfrac{1}{x}\right)}{-x\left(\sqrt{1-\dfrac{1}{x}+\dfrac{3}{x^2}}+\sqrt{1-\dfrac{4}{x}+\dfrac{3}{x^2}}\right)}=\dfrac{1}{2}$

b). $\lim\limits_{x\to-\infty}\left(\sqrt{4x^2-9x-21}-\sqrt{4x^2-7x+13}\right)=\lim\limits_{x\to-\infty}\dfrac{4x^2-9x-21-4x^2+7x-13}{\sqrt{4x^2-9x-21}+\sqrt{4x^2-7x+13}}$

$=\lim\limits_{x\to-\infty}\dfrac{-2x-34}{\sqrt{x^2\left(4-\dfrac{9}{x}-\dfrac{21}{x^2}\right)}+\sqrt{x^2\left(4-\dfrac{7}{x}+\dfrac{13}{x^2}\right)}}=\lim\limits_{x\to-\infty}\dfrac{-x\left(2+\dfrac{34}{x}\right)}{\sqrt{x^2}\left(\sqrt{4-\dfrac{9}{x}-\dfrac{21}{x^2}}+\sqrt{4-\dfrac{7}{x}+\dfrac{13}{x^2}}\right)}$

$=\lim\limits_{x\to-\infty}\dfrac{-x\left(2+\dfrac{34}{x}\right)}{|x|\left(\sqrt{4-\dfrac{9}{x}-\dfrac{21}{x^2}}+\sqrt{4-\dfrac{7}{x}+\dfrac{13}{x^2}}\right)}=\lim\limits_{x\to-\infty}\dfrac{-x\left(2+\dfrac{34}{x}\right)}{-x\left(\sqrt{4-\dfrac{9}{x}-\dfrac{21}{x^2}}+\sqrt{4-\dfrac{7}{x}+\dfrac{13}{x^2}}\right)}=\dfrac{1}{2}$

c). $L = \lim\limits_{x\to-\infty}\left(\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2}\right)$

$= \lim\limits_{x\to-\infty}\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2} = \lim\limits_{x\to-\infty}\dfrac{x^3(3x+2)-x^2(3x^2-4)}{(3x^2-4)(3x+2)} = \lim\limits_{x\to-\infty}\dfrac{2x^3+4x^2}{(3x^2-4)(3x+2)}$

$= \lim\limits_{x\to-\infty}\dfrac{x^3\left(\dfrac{2x^3+4x^2}{x^3}\right)}{x^2\left(\dfrac{3x^2-4}{x^2}\right)x\left(\dfrac{3x+2}{x}\right)} = \lim\limits_{x\to-\infty}\dfrac{2+\dfrac{4}{x}}{\left(3-\dfrac{4}{x^2}\right)\left(3+\dfrac{2}{x}\right)}.$

Do $\lim\limits_{x\to-\infty}\dfrac{4}{x} = \lim\limits_{x\to-\infty}\dfrac{4}{x^2} = \lim\limits_{x\to-\infty}\dfrac{2}{x^2} = 0$ nên $L = \dfrac{2}{3.3} = \dfrac{2}{9}$.