Câu 2: Giải các phương trình sau:

1). $\dfrac{\sqrt{3}-\sqrt{3}\cos 2x}{2\sin x}=\cos x$.

2). $\tan\dfrac{\pi}{7}.\sin x+2\cos^2\dfrac{x}{2}=2$

3). $\sqrt{2}\left(\cos^4 x-\sin^4 x\right)=\sin x+\cos x$

4). $\sqrt{3}\cos 2x+\sin 2x+2\sin\left(2x-\dfrac{\pi}{6}\right)=2\sqrt{2}$

5). $\sqrt{3}\sin 7x-\cos 7x=2\sin\left(5x-\dfrac{\pi}{6}\right)$

6). $\sin\left(\dfrac{\pi}{2}+2x\right)+\sqrt{3}\sin\left(\pi-2x\right)=2$

7). $\cos x+\sqrt{3}\sin x+2\cos\left(2x+\dfrac{\pi}{3}\right)=0$

8). $2\cos 2x=\left(1+\sqrt{3}\right)\left(\cos x-\sin x\right)$

9). $\left(\sqrt{3}-1\right)\sin x-\left(\sqrt{3}+1\right)\cos x=1-\sqrt{3}$.

10). $3\sin 3x-\sqrt{3}\cos 9x=1+4\sin^3 3x$

LỜI GIẢI

1). $\dfrac{\sqrt{3}-\sqrt{3}\cos 2x}{2\sin x}=\cos x\quad(1)$. Điều kiện $\sin x\neq 0\Leftrightarrow x\neq k\pi$

$(1)\Leftrightarrow\sqrt{3}-\sqrt{3}\cos 2x=2\sin x\cos x\qquad\Leftrightarrow\sqrt{3}\cos 2x+\sin 2x=\sqrt{3}$

$\Leftrightarrow\dfrac{\sqrt{3}}{2}\cos 2x+\dfrac{1}{2}\sin 2x=\dfrac{\sqrt{3}}{2}\qquad\Leftrightarrow\cos 2x.\cos\dfrac{\pi}{6}+\sin 2x.\sin\dfrac{\pi}{6}=\dfrac{\sqrt{3}}{2}$

$$\Leftrightarrow\cos\left(2x-\dfrac{\pi}{6}\right)=\cos\dfrac{\pi}{6}\Leftrightarrow\left[\begin{array}{l}2x-\dfrac{\pi}{6}=\dfrac{\pi}{6}+k2\pi\\2x-\dfrac{\pi}{6}=-\dfrac{\pi}{6}+k2\pi\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=\dfrac{\pi}{6}+k\pi\\x=k\pi\end{array}\right.,\left(k\in\mathbb{Z}\right)$$

So với điều kiện thì nghiệm $x=k\pi$ loại.

Vậy nghiệm phương trình: $x=\dfrac{\pi}{6}+k\pi,\left(k\in\mathbb{Z}\right)$

2). $\tan\frac{\pi}{7}.\sin x+2\cos^2\frac{x}{2}=2 \Leftrightarrow \tan\frac{\pi}{7}.\sin x+1+\cos x=2$ $\Leftrightarrow \frac{\sin\frac{\pi}{7}}{\cos\frac{\pi}{7}}\sin x+\cos x=1$

$\Leftrightarrow \sin\frac{\pi}{7}\sin x+\cos\frac{\pi}{7}\cos x=\cos\frac{\pi}{7}$ $\Leftrightarrow \cos\left(x-\frac{\pi}{7}\right)=\cos\frac{\pi}{7} \Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{7}=\frac{\pi}{7}+k2\pi \\ x-\frac{\pi}{7}=-\frac{\pi}{7}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{2\pi}{7}+k2\pi \\ x=k2\pi \end{array}\right.$

Nghiệm phương trình: $x=\frac{2\pi}{7}+k2\pi, x=k2\pi, (k\in\mathbb{Z})$

3). $\sqrt{2}\left(\cos^4 x-\sin^4 x\right)=\sin x+\cos x$

$\Leftrightarrow \sqrt{2}\left(\cos^2 x-\sin^2 x\right)\left(\cos^2 x+\sin^2 x\right)=\sin x+\cos x$

$\Leftrightarrow \sqrt{2}\cos 2x=\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$

$\Leftrightarrow \cos 2x=\cos\left(x-\frac{\pi}{4}\right) \Leftrightarrow \left[\begin{array}{l} 2x=x-\frac{\pi}{4}+k2\pi \\ 2x=-x+\frac{\pi}{4}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\frac{\pi}{4}+k2\pi \\ x=\frac{\pi}{12}+\frac{k2\pi}{3} \end{array}\right., (k\in\mathbb{Z})$

Nghiệm của phương trình: $x=-\frac{\pi}{4}+k2\pi, x=\frac{\pi}{12}+\frac{k2\pi}{3}, (k\in\mathbb{Z})$

4). $\sqrt{3}\cos 2x+\sin 2x+2\sin\left(2x-\frac{\pi}{6}\right)=2\sqrt{2}$

$\Leftrightarrow \frac{\sqrt{3}}{2}\cos 2x+\frac{1}{2}\sin 2x+\sin\left(2x-\frac{\pi}{6}\right)=\sqrt{2}$

$\Leftrightarrow \cos 2x.\cos\frac{\pi}{6}+\sin 2x.\sin\frac{\pi}{6}+\sin\left(2x-\frac{\pi}{6}\right)=\sqrt{2}$

$\Leftrightarrow \cos\left(2x-\frac{\pi}{6}\right)+\sin\left(2x-\frac{\pi}{6}\right)=\sqrt{2}$

$\Leftrightarrow \frac{1}{\sqrt{2}}\cos\left(2x-\frac{\pi}{6}\right)+\frac{1}{\sqrt{2}}\sin\left(2x-\frac{\pi}{6}\right)=1$

$\Leftrightarrow \cos\left(2x-\frac{\pi}{6}\right).\cos\frac{\pi}{4}+\sin\left(2x-\frac{\pi}{6}\right)\sin\frac{\pi}{4}=1$

$\Leftrightarrow \cos\left(2x-\frac{\pi}{6}-\frac{\pi}{4}\right)=1 \Leftrightarrow 2x-\frac{\pi}{6}-\frac{\pi}{4}=k2\pi \Leftrightarrow x=\frac{5\pi}{24}+k\pi$.

Nghiệm của phương trình: $x=\frac{5\pi}{24}+k\pi$.

5). $\sqrt{3}\sin 7x-\cos 7x=2\sin\left(5x-\frac{\pi}{6}\right)$

$\Leftrightarrow \frac{\sqrt{3}}{2}\sin 7x-\frac{1}{2}\cos 7x=\sin\left(5x-\frac{\pi}{6}\right)$

$\Leftrightarrow \sin 7x\cos\frac{\pi}{6}-\cos 7x\sin\frac{\pi}{6}=\sin\left(5x-\frac{\pi}{6}\right) \Leftrightarrow \sin\left(7x-\frac{\pi}{6}\right)=\sin\left(5x-\frac{\pi}{6}\right)$

$\Leftrightarrow \left[\begin{array}{l} 7x-\frac{\pi}{6}=5x-\frac{\pi}{6}+k2\pi \\ 7x-\frac{\pi}{6}=\pi-\left(5x-\frac{\pi}{6}\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=k\pi \\ x=\frac{\pi}{9}+\frac{k\pi}{6} \end{array}\right.$

6). $\sin\left(\frac{\pi}{2}+2x\right)+\sqrt{3}\sin\left(\pi-2x\right)=2 \Leftrightarrow \cos 2x+\sqrt{3}\sin 2x=2 \Leftrightarrow \frac{1}{2}\cos 2x+\frac{\sqrt{3}}{2}\sin 2x=1$

$\Leftrightarrow \cos 2x.\cos\frac{\pi}{3}+\sin 2x.\sin\frac{\pi}{3}=1 \Leftrightarrow \cos\left(2x-\frac{\pi}{3}\right)=1$

$\Leftrightarrow 2x-\frac{\pi}{3}=k2\pi \Leftrightarrow x=\frac{\pi}{6}+k\pi,\left(k\in\mathbb{Z}\right)$

7). $\Leftrightarrow \cos x+\sqrt{3}\sin x+2\cos\left(2x+\frac{\pi}{3}\right)=0$

$\Leftrightarrow \frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x=-\cos\left(2x+\frac{\pi}{3}\right) \Leftrightarrow \cos x\cos\frac{\pi}{3}+\sin x\sin\frac{\pi}{3}=\cos\left(2x+\frac{\pi}{3}+\pi\right)$

$\Leftrightarrow \cos\left(x-\frac{\pi}{3}\right)=\cos\left(2x+\frac{4\pi}{3}\right) \Leftrightarrow \left[\begin{array}{l} 2x+\frac{4\pi}{3}=x-\frac{\pi}{3}+k2\pi \\ 2x+\frac{4\pi}{3}=-\left(x-\frac{\pi}{3}\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\frac{5\pi}{3}+k2\pi \\ x=-\pi+\frac{k2\pi}{3} \end{array}\right.$

8). $2\cos 2x=\left(1+\sqrt{3}\right)\left(\cos x-\sin x\right)$

$\Leftrightarrow 2\left(\cos^2 x-\sin^2 x\right)=\left(1+\sqrt{3}\right)\left(\cos x-\sin x\right)$

$$\Leftrightarrow 2(\cos x-\sin x)(\cos x+\sin x)=\left(1+\sqrt{3}\right)(\cos x-\sin x)$$

$$\Leftrightarrow (\cos x-\sin x)\left[2(\cos x+\sin x)-\left(1+\sqrt{3}\right)\right]=0$$

$$\Leftrightarrow \left[\begin{array}{l}\cos x-\sin x=0\\ 2(\cos x+\sin x)-\left(1+\sqrt{3}\right)=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)=0\\ \sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=\frac{1+\sqrt{3}}{2}\end{array}\right. \Leftrightarrow \left[\begin{array}{ll}\cos\left(x+\frac{\pi}{4}\right)=0 & (1)\\ \cos\left(x-\frac{\pi}{4}\right)=\frac{1+\sqrt{3}}{2\sqrt{2}} & (2)\end{array}\right.$$

Giải (1): $\Leftrightarrow x+\frac{\pi}{4}=\frac{\pi}{2}+k\pi \Leftrightarrow x=\frac{\pi}{4}+k\pi$.

Giải (2): $\cos\left(x-\frac{\pi}{4}\right)=\cos\frac{\pi}{12} \Leftrightarrow \left[\begin{array}{l}x-\frac{\pi}{4}=\frac{\pi}{12}+k2\pi\\ x-\frac{\pi}{4}=-\frac{\pi}{12}+k2\pi\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{3}+k2\pi\\ x=\frac{\pi}{6}+k2\pi\end{array}\right.$

Vậy nghiệm của phương trình: $x=\frac{\pi}{4}+k\pi$, $x=\frac{\pi}{3}+k2\pi$, $x=\frac{\pi}{6}+k2\pi,(k\in\mathbb{Z})$

9). $\left(\sqrt{3}-1\right)\sin x-\left(\sqrt{3}+1\right)\cos x=1-\sqrt{3}$.

Ta có $a=\sqrt{3}-1, b=\sqrt{3}+1, c=1-\sqrt{3} \Rightarrow \sqrt{a^2+b^2}=2\sqrt{2}$

$$\Leftrightarrow \frac{\sqrt{3}-1}{2\sqrt{2}}\sin x-\frac{\sqrt{3}+1}{2\sqrt{2}}\cos x=\frac{1-\sqrt{3}}{2\sqrt{2}} \Leftrightarrow \sin x.\cos\frac{5\pi}{12}-\cos x.\sin\frac{5\pi}{12}=\frac{1-\sqrt{3}}{2\sqrt{2}}$$

$$\Leftrightarrow \sin\left(x-\frac{5\pi}{12}\right)=\sin\left(-\frac{\pi}{12}\right) \Leftrightarrow \left[\begin{array}{l}x-\frac{5\pi}{12}=-\frac{\pi}{12}+k2\pi\\ x-\frac{5\pi}{12}=\pi+\frac{\pi}{12}+k2\pi\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{3}+k2\pi\\ x=\frac{3\pi}{2}+k2\pi\end{array}\right.$$

Vậy nghiệm của phương trình: $x=\frac{\pi}{3}+k2\pi, x=\frac{3\pi}{2}+k2\pi,(k\in\mathbb{Z})$

10). $3\sin 3x-\sqrt{3}\cos 9x=1+4\sin^3 3x$

$\Leftrightarrow 3\sin 3x-4\sin^3 3x-\sqrt{3}\cos 9x=1 \quad \Leftrightarrow \sin 9x-\sqrt{3}\cos 9x=1$

$\Leftrightarrow \frac{1}{2}\sin 9x-\frac{\sqrt{3}}{2}\cos 9x=\frac{1}{2} \quad \Leftrightarrow \sin 9x.\cos\frac{\pi}{3}-\cos 9x.\sin\frac{\pi}{3}=\frac{1}{2}$

$$\Leftrightarrow \sin\left(9x-\frac{\pi}{3}\right)=\sin\frac{\pi}{6} \Leftrightarrow \left[\begin{array}{l} 9x-\frac{\pi}{3}=\frac{\pi}{6}+k2\pi \\ 9x-\frac{\pi}{3}=\pi-\frac{\pi}{6}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{18}+\frac{k2\pi}{9} \\ x=\frac{7\pi}{54}+\frac{k2\pi}{9} \end{array}\right.$$

Vậy nghiệm của phương trình: $x=\frac{\pi}{18}+\frac{k2\pi}{9}, x=\frac{7\pi}{54}+\frac{k2\pi}{9}, (k\in\mathbb{Z})$

Câu 3: Giải các phương trình sau:

1). $2\cos\left(2x+\frac{\pi}{6}\right)+4\sin x\cos x-1=0$

2). $4\sin\left(x+\frac{\pi}{4}\right)+2\cos\left(x-\frac{\pi}{4}\right)-3\sqrt{2}=0$

3). $8\sin x\sin 2x+6\sin\left(x+\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}-2x\right)=5+7\cos x$

4). $2\sqrt{3}\sin\left(x-\frac{\pi}{8}\right)\cos\left(x-\frac{\pi}{8}\right)+2\cos^2\left(x-\frac{\pi}{8}\right)=\sqrt{3}+1$

5). $\frac{1+\cos x+\cos 2x+\cos 3x}{2\cos^2 x+\cos x-1}=\frac{2}{3}\left(3-\sqrt{3}\sin x\right)$

6). $8\sin x=\frac{\sqrt{3}}{\cos x}+\frac{1}{\sin x}$

7). $2\cos^3 x+2\sin^3 x+2\sin^2 x\cos x+2\cos^2 x\sin x-\sqrt{2}=0$

8). $5(\cos x+\sin x)+\sin 3x-\cos 3x=2\sqrt{2}(2+\sin 2x)$

LỜI GIẢI

1). $2\cos\left(2x+\frac{\pi}{6}\right)+4\sin x\cos x-1=0$

$$\Leftrightarrow 2\left(\cos 2x\cos\frac{\pi}{6}-\sin 2x\sin\frac{\pi}{6}\right)+2\sin 2x-1=0 \Leftrightarrow \sqrt{3}\cos 2x+\sin 2x=1$$

$$\Leftrightarrow \frac{\sqrt{3}}{2}\cos 2x+\frac{1}{2}\sin 2x=\frac{1}{2} \Leftrightarrow \cos 2x\cos\frac{\pi}{6}+\sin 2x\sin\frac{\pi}{6}=\frac{1}{2}$$