1) sin2a = 2sinacosa 2) $\cos 2a = \cos^2 a - \sin^2 a$ $= 2\cos^2 a - 1$ $= 1 - 2\sin^2 a$ 3) $\tan 2a = \dfrac{2\tan a}{1-\tan^2 a}$	1) $\cos^2\alpha = \dfrac{1+\cos 2\alpha}{2} = \dfrac{1}{2}(1+\cos 2\alpha)$ 2) $\sin^2\alpha = \dfrac{1-\cos 2\alpha}{2} = \dfrac{1}{2}(1-\cos 2\alpha)$ 3) $\tan^2\alpha = \dfrac{1-\cos 2\alpha}{1+\cos 2\alpha}$

8. Công thức biến đổi tích thành tổng:

1) $\cos a\cos b = \frac{1}{2}[\cos(a-b)+\cos(a+b)]$ 2) $\sin a\sin b = \frac{1}{2}[\cos(a-b)-\cos(a+b)]$

3) $\sin a\cos b = \frac{1}{2}[\sin(a+b)+\sin(a-b)]$ 4) $\cos a\sin b = \frac{1}{2}[\sin(a+b)-\sin(a-b)]$

9. Công thức biến đổi tổng thành tích:

1) $\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2}$ 2) $\cos a - \cos b = -2\sin\frac{a+b}{2}\sin\frac{a-b}{2}$

3) $\sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a-b}{2}$ 4) $\sin a - \sin b = 2\cos\frac{a+b}{2}\sin\frac{a-b}{2}$

II: BÀI TẬP MẪU

Bài 1: Cho $\sin\alpha = \frac{3}{5}$, với $\frac{\pi}{2} < \alpha < \pi$. Tính $\cos\alpha$, $\tan\alpha$, $\cot\alpha$

Giải: Ta có: $\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = 1 - \sin^2\alpha = 1 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} \Rightarrow \cos\alpha = \pm\frac{4}{5}$

Vì $\frac{\pi}{2} < \alpha < \pi \Rightarrow \cos\alpha < 0$. Suy ra: $\cos\alpha = -\frac{4}{5}$

Khi đó: * $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\frac{3}{5}}{-\frac{4}{5}} = -\dfrac{3}{4}$ * $\cot\alpha = \dfrac{1}{\tan\alpha} = \dfrac{1}{-\frac{3}{5}} = -\dfrac{5}{3}$

Bài 2: Cho $\tan\alpha = -\frac{4}{5}$, với $\frac{3\pi}{2} < \alpha < 2\pi$. Tính $\cos\alpha$, $\sin\alpha$, $\cot\alpha$

Giải: Ta có: $\cot\alpha = \dfrac{1}{\tan\alpha} = \dfrac{1}{-\dfrac{4}{5}} = -\dfrac{5}{4}$

* $\cos^2\alpha = \dfrac{1}{1+\tan^2\alpha} = \dfrac{1}{1+\left(-\dfrac{4}{5}\right)^2} = \dfrac{25}{41} \Rightarrow \cos\alpha = \pm\sqrt{\dfrac{25}{41}} = \pm\dfrac{5\sqrt{41}}{41}$

Vì $\dfrac{3\pi}{2} < \alpha < 2\pi \Rightarrow \cos\alpha > 0$. Suy ra: $\cos\alpha = \dfrac{5\sqrt{41}}{41}$

* $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} \Rightarrow \sin\alpha = \tan\alpha.\cos\alpha = \left(-\dfrac{4}{5}\right).\dfrac{5\sqrt{41}}{41} = -\dfrac{4\sqrt{41}}{41}$

Bài 3: a) Tính $\cos\left(\alpha+\dfrac{\pi}{3}\right)$, biết $\sin\alpha = \dfrac{1}{\sqrt{3}}$ và $0 < \alpha < \dfrac{\pi}{2}$

b) Tính sin(a – b), biết $\sin a = \dfrac{4}{5}$ với $0^0 < a < 90^0$ và $\sin b = \dfrac{2}{3}$ với $90^0 < b < 180^0$

c) Tính $\tan\left(a+\dfrac{\pi}{4}\right)$, biết $\cos a = -\dfrac{5}{13}$ và $a \in \left(\pi; \dfrac{3\pi}{2}\right)$

Giải: a) Ta có: $A = \cos\left(\alpha+\dfrac{\pi}{3}\right) = \cos\alpha.\cos\dfrac{\pi}{3} - \sin\alpha.\sin\dfrac{\pi}{3} = \dfrac{1}{2}\cos\alpha - \dfrac{\sqrt{3}}{2}\sin\alpha$

* Tính $\cos\alpha$: $\cos^2\alpha = 1 - \sin^2\alpha = 1 - \left(\dfrac{1}{\sqrt{3}}\right)^2 = \dfrac{2}{3} \Rightarrow \cos\alpha = \pm\sqrt{\dfrac{2}{3}} = \pm\dfrac{\sqrt{6}}{3}$

Vì $0 < \alpha < \dfrac{\pi}{2} \Rightarrow \cos\alpha > 0$ nên $\cos\alpha = \dfrac{\sqrt{6}}{3}$

Vậy: $A = \dfrac{1}{2}\cos\alpha - \dfrac{\sqrt{3}}{2}\sin\alpha = \dfrac{1}{2}.\dfrac{\sqrt{6}}{3} - \dfrac{\sqrt{3}}{2}.\dfrac{1}{\sqrt{3}} = \dfrac{-3+\sqrt{6}}{6}$

b) Ta có: B = sin(a – b) = sina.sinb – cosa.cosb

* Tính cosa: $\cos^2 a = 1 - \sin^2 a = 1 - \left(\dfrac{4}{5}\right)^2 = \dfrac{9}{25} \Rightarrow \cos a = \pm\dfrac{3}{5}$

Vì $0^0 < a < 90^0 \Rightarrow \cos a > 0$ nên $\cos a = \dfrac{3}{5}$

* Tính cosb: $\cos^2 b = 1 - \sin^2 b = 1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9} \Rightarrow \cos b = \pm\sqrt{\frac{5}{9}} = \pm\frac{\sqrt{5}}{3}$

Vì $0^0 < a < 90^0 \Rightarrow \cos b < 0$ nên $\cos b = -\frac{\sqrt{5}}{3}$

Vậy: $B = \sin a.\sin b - \cos a.\cos b = \frac{4}{5}.\frac{2}{3} - \frac{3}{5}.\left(-\frac{\sqrt{5}}{3}\right) = \frac{8+3\sqrt{5}}{15}$

c) Ta có: $C = \tan\left(a + \frac{\pi}{4}\right) = \frac{\tan a + \tan\frac{\pi}{4}}{1 - \tan a.\tan\frac{\pi}{4}} = \frac{\tan a + 1}{1 - \tan a}$

* Tính tana: $1 + \tan^2 a = \frac{1}{\cos^2 a} \Rightarrow \tan^2 a = \frac{1}{\cos^2 a} - 1 = \frac{1}{\left(-\frac{5}{13}\right)^2} - 1 = \frac{144}{25} \Rightarrow \tan a = \pm\frac{12}{5}$

Vì $a \in \left(\pi; \frac{3\pi}{2}\right) \Rightarrow \tan a > 0$. Suy ra: $\tan a = \frac{12}{5}$. Vậy: $C = \frac{\tan a + 1}{1 - \tan a} = \frac{\frac{12}{5} + 1}{1 - \frac{12}{5}} = -\frac{17}{7}$

Bài 4: Tính sin2a, cos2a và tan2a, biết:

a) $\sin a = -0,6$ và $\pi < a < \frac{3\pi}{2}$ b) $\sin a + \cos a = \frac{1}{2}$ và $\frac{3\pi}{4} < a < \pi$

Giải: a) Ta có: $\sin 2a = 2\sin a.\cos a$

* Tính cosa: $\cos^2 a = 1 - \sin^2 a = 1 - (-0,6)^2 = \frac{16}{25} \Rightarrow \cos a = \pm\frac{4}{5}$

Vì $\pi < a < \frac{3\pi}{2} \Rightarrow \cos a < 0$ nên $\cos a = -\frac{4}{5}$. Vậy: $\sin 2a = 2.(-0,6).\left(-\frac{4}{5}\right) = \frac{24}{25}$

* $\cos 2a = \cos^2 a - \sin^2 a = \left(-\frac{4}{5}\right)^2 - (-0,6)^2 = \frac{7}{25}$ * $\tan 2a = \frac{\sin 2a}{\cos 2a} = \frac{\frac{24}{25}}{\frac{7}{25}} = \frac{24}{7}$

b) Ta có: $\sin a + \cos a = \frac{1}{2} \Rightarrow (\sin a + \cos a)^2 = \frac{1}{4} \Leftrightarrow \sin^2 a + \cos^2 a + 2\sin a\cos a = \frac{1}{4}$

$\Leftrightarrow \sin 2a = \frac{1}{4} - 1 = -\frac{3}{4}$

* Tính cos2a: Tính cos2a: $\cos^2 2a = 1 - \sin^2 2a = 1 - \left(-\frac{3}{4}\right)^2 = \frac{7}{16} \Rightarrow \cos 2a = \pm\sqrt{\frac{7}{16}} = \pm\frac{\sqrt{7}}{4}$

Vì $\frac{3\pi}{4} < a < \pi \Rightarrow \frac{3\pi}{2} < 2a < 2\pi \Rightarrow \cos 2a > 0 \Rightarrow \cos 2a = \frac{\sqrt{7}}{4}$ * $\tan 2a = \frac{120}{119}$

Bài 5: Tính giá trị của biểu thức: a) $A = \sin\frac{\pi}{8}\cos\frac{3\pi}{8}$ b) $B = \sin\frac{13\pi}{24}\sin\frac{5\pi}{24}$

Giải: a) Ta có: $A = \sin\frac{\pi}{8}\cos\frac{3\pi}{8} = \frac{1}{2}\left[\sin\left(\frac{\pi}{8} - \frac{3\pi}{8}\right) + \sin\left(\frac{\pi}{8} + \frac{3\pi}{8}\right)\right]$

$$= \frac{1}{2}\left[\sin\left(-\frac{\pi}{4}\right) + \sin\frac{\pi}{2}\right] = \frac{1}{2}\left(-\frac{\sqrt{2}}{2} + 1\right) = \frac{2 - \sqrt{2}}{4}$$

b) $B = \sin\frac{13\pi}{24}\sin\frac{5\pi}{24} = \frac{1}{2}\left[\cos\left(\frac{13\pi}{24} - \frac{5\pi}{24}\right) - \cos\left(\frac{13\pi}{24} + \frac{5\pi}{24}\right)\right]$

$$= \frac{1}{2}\left[\cos\frac{\pi}{3} - \cos\frac{3\pi}{4}\right] = \frac{1}{2}\left(\frac{1}{2} + \frac{\sqrt{2}}{2}\right) = \frac{1 + \sqrt{2}}{4}$$

Bài 6: Biến đổi thành tích các biểu thức sau:

a) $A = 1 - \sin x$ b) $B = 1 - 2\sin x$ c) $C = \sin 3x + \sin 2x$

d) $D = 1 + \cos x + \cos 2x + \cos 3x$ e) $E = \cos^2 x + \cos^2 2x + \cos^2 3x - 1$

f) $F = \sin 11x.\sin 5x + \sin^2 3x - \cos^2 x$

Giải: a) $A = 1 - \sin x = \sin\frac{\pi}{2} - \sin x = 2\cos\left(\frac{\frac{\pi}{2} + x}{2}\right)\sin\left(\frac{\frac{\pi}{2} - x}{2}\right) = 2\cos\left(\frac{\pi}{4} + \frac{x}{2}\right)\sin\left(\frac{\pi}{4} - \frac{x}{2}\right)$

b) $1 - 2\sin x = 2\left(\frac{1}{2} - \sin x\right) = 2\left(\sin\frac{\pi}{6} - \sin x\right)$

$$= 4\cos\left(\frac{\frac{\pi}{6} + x}{2}\right)\sin\left(\frac{\frac{\pi}{6} - x}{2}\right) = 2\cos\left(\frac{\pi}{12} + \frac{x}{2}\right)\sin\left(\frac{\pi}{12} - \frac{x}{2}\right)$$

c) $C = \sin 3x + \sin 2x = 2\sin\left(\frac{3x+2x}{2}\right)\cos\left(\frac{3x-2x}{2}\right) = 2\sin\frac{5x}{2}\cos\frac{x}{2}$

d) $D = 1 + \cos x + \cos 2x + \cos 3x = (1 + \cos x) + (\cos 3x + \cos 2x) = 2\cos^2\frac{x}{2} + 2\cos\frac{5x}{2}\cos\frac{x}{2}$

$= 2\cos\frac{x}{2}(\cos\frac{5x}{2} + \cos\frac{x}{2}) = 4\cos\frac{x}{2}\cos\frac{3x}{2}\cos x$

e) $E = \cos^2 x + \cos^2 2x + \cos^2 3x - 1 = \frac{1}{2}(1+\cos 2x) + \frac{1}{2}(1+\cos 4x) + \frac{1}{2}(1+\cos 6x) - 1$

$= \frac{1}{2} + \frac{1}{2}\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x + \frac{1}{2} + \frac{1}{2}\cos 6x - 1 = \frac{1}{2} + \frac{1}{2}\cos 2x + \frac{1}{2}\cos 4x + \frac{1}{2}\cos 6x$

$= \frac{1}{2}[(1 + \cos 2x) + (\cos 6x + \cos 4x)] = \frac{1}{2}(2\cos^2 x + 2\cos 5x\cos x) = \cos x(\cos 5x + \cos x)$

$= 2\cos x\cos 3x\cos 2x$

f) $F = \sin 11x.\sin 5x + \sin^2 3x - \cos^2 x = \frac{1}{2}(\cos 6x - \cos 16x) + \frac{1}{2}(1 - \cos 6x) - \frac{1}{2}(1 + \cos 2x)$

$= \frac{1}{2}\cos 6x - \frac{1}{2}\cos 16x + \frac{1}{2} - \frac{1}{2}\cos 6x - \frac{1}{2} - \frac{1}{2}\cos 2x = -\frac{1}{2}(\cos 16x + \cos 2x)$

$= -\sin 9x.\sin 7x$

Bài 7: Biến đổi thành tổng các biểu thức sau:

a) $A = 2\sin x\sin 2x\sin 3x$ b) $B = 8\cos x\sin 2x\sin 3x$

c) $C = \sin(a + 30^0).\cos(a - 30^0)$ d) $D = \sin\left(x+\frac{\pi}{6}\right).\sin\left(x-\frac{\pi}{6}\right).\cos 2x$

Giải: a) $A = 2\sin x\sin 2x\sin 3x = (2\sin 2x\sin x).\sin 3x = (\cos x - \cos 3x).\sin 3x$

$= \sin 3x\cos x - \sin 3x\cos 3x = \frac{1}{2}(\sin 4x + \sin 2x) - \frac{1}{2}(\sin 6x + \sin 0)$

$= \frac{1}{2}\sin 4x + \frac{1}{2}\sin 2x - \frac{1}{2}\sin 6x$

b) $B = 8\cos x\sin 2x\sin 3x = (8\sin 3x\sin 2x)\cos x = 4(\cos x - \cos 5x).\cos x = 4\cos^2 x - 4\cos 5x.\cos x$

$= 2(1 + \cos 2x) - 2(\cos 4x + \cos 6x) = 2 + 2\cos 2x - 2\cos 4x - 2\cos 6x$