

GIỚI HẠN KHI x TIẾN TỚI VÔ CỰC

Câu 1: Tìm các giới hạn sau:

a). $\lim_{x \rightarrow -\infty} \frac{3x^2 - x + 7}{2x^3 - 1}$ b). $\lim_{x \rightarrow +\infty} \frac{(4x^2 + 1)(7x - 1)}{(2x^3 - 1)(x + 3)}$ c). $\lim_{x \rightarrow +\infty} \frac{x\sqrt{x} + 3}{x^2 - x + 2}$

d). $\lim_{x \rightarrow +\infty} \left[(x+1) \sqrt{\frac{x}{2x^4 + x^2 + 1}} \right]$ e). $\lim_{x \rightarrow -\infty} x \sqrt{\frac{2x^3 + x}{x^5 - x^2 + 3}}$.

LỜI GIẢI

a). $\lim_{x \rightarrow -\infty} \frac{3x^2 - x + 7}{2x^3 - 1} = \lim_{x \rightarrow -\infty} \frac{x^2 \left(3 - \frac{1}{x} + \frac{7}{x^2} \right)}{x^3 \left(2 - \frac{1}{x^3} \right)} = \lim_{x \rightarrow -\infty} \frac{3 - \frac{1}{x} + \frac{7}{x^2}}{x \left(2 - \frac{1}{x^3} \right)} = \lim_{x \rightarrow -\infty} \frac{3}{2x} = 0$

b). $\lim_{x \rightarrow +\infty} \frac{(4x^2 + 1)(7x - 1)}{(2x^3 - 1)(x + 3)} = \lim_{x \rightarrow +\infty} \frac{x^2 \left(4 + \frac{1}{x^2} \right) x \left(7 - \frac{1}{x} \right)}{x^3 \left(2 - \frac{1}{x^3} \right) x \left(1 + \frac{3}{x} \right)} = \lim_{x \rightarrow +\infty} \frac{\left(4 + \frac{1}{x^2} \right) \left(7 - \frac{1}{x} \right)}{x \left(2 - \frac{1}{x^3} \right) \left(1 + \frac{3}{x} \right)} = \lim_{x \rightarrow +\infty} \frac{28}{2x} = 0$

c). $\lim_{x \rightarrow +\infty} \frac{x\sqrt{x} + 3}{x^2 - x + 2} = \lim_{x \rightarrow +\infty} \frac{x\sqrt{x} \left(1 + \frac{3}{x\sqrt{x}} \right)}{x^2 \left(1 - \frac{1}{x} + \frac{2}{x^2} \right)} = \lim_{x \rightarrow +\infty} \frac{1 + \frac{3}{x\sqrt{x}}}{\sqrt{x} \left(1 - \frac{1}{x} + \frac{2}{x^2} \right)} = \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{x}} = 0$.

d). $\lim_{x \rightarrow +\infty} \left[(x+1) \sqrt{\frac{x}{2x^4 + x^2 + 1}} \right] = \lim_{x \rightarrow +\infty} \left[(x+1) \sqrt{\frac{x}{x^4 \left(2 + \frac{1}{x^2} + \frac{1}{x^4} \right)}} \right]$

$= \lim_{x \rightarrow +\infty} \left[(x+1) \sqrt{\frac{1}{2x^3}} \right] = \lim_{x \rightarrow +\infty} \left[(x+1) \frac{1}{x\sqrt{x}\sqrt{2}} \right] = \lim_{x \rightarrow +\infty} \left[x \left(1 + \frac{1}{x} \right) \frac{1}{x\sqrt{x}\sqrt{2}} \right] = 0$

e). $\lim_{x \rightarrow -\infty} x \sqrt{\frac{2x^3 + x}{x^5 - x^2 + 3}} = \lim_{x \rightarrow -\infty} x \sqrt{\frac{x^3 \left(x + \frac{1}{x^2} \right)}{x^5 \left(1 - \frac{1}{x^3} + \frac{3}{x^5} \right)}} = \lim_{x \rightarrow -\infty} x \frac{\sqrt{2}}{\sqrt{x^2}} = \lim_{x \rightarrow -\infty} x \frac{\sqrt{2}}{|x|} = \lim_{x \rightarrow -\infty} x \frac{\sqrt{2}}{-x} = -\sqrt{2}$

Câu 2: Tìm các giới hạn sau:

a). $\lim_{x \rightarrow -\infty} (\sqrt{2x^2 + 1} + x)$ b). $\lim_{x \rightarrow +\infty} \frac{3x^2 - x + 3}{x - 4}$ c). $\lim_{x \rightarrow +\infty} \frac{x^4 - x^3 + 11}{2x - 7}$
 d). $\lim_{x \rightarrow +\infty} \frac{\sqrt{2x^4 + x^2 - 1}}{1 - 2x}$ e). $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 - x}}{1 - 2x}$ f). $\lim_{x \rightarrow -\infty} \frac{2x^4 + 7x^3 - 15}{x^4 + 1}$.

LỜI GIẢI

a). $\lim_{x \rightarrow -\infty} (\sqrt{2x^2 + 1} + x) = \lim_{x \rightarrow -\infty} \left(\sqrt{x^2 \left(2 + \frac{1}{x^2} \right)} + x \right) = \lim_{x \rightarrow -\infty} (\sqrt{2x^2} + x)$
 $= \lim_{x \rightarrow -\infty} (|x|\sqrt{2} + x) = \lim_{x \rightarrow -\infty} (-\sqrt{2}x + x) = \lim_{x \rightarrow -\infty} x(-\sqrt{2} + 1) = +\infty$

b). $\lim_{x \rightarrow +\infty} \frac{3x^2 - x + 3}{x - 4} = \lim_{x \rightarrow +\infty} \frac{x^2 \left(3 - \frac{1}{x} + \frac{3}{x^2} \right)}{x \left(1 - \frac{4}{x} \right)} = \lim_{x \rightarrow +\infty} 3x = +\infty$

c). $\lim_{x \rightarrow +\infty} \frac{x^4 - x^3 + 11}{2x - 7} = \lim_{x \rightarrow +\infty} \frac{x^4 \left(1 - \frac{1}{x} + \frac{11}{x^3} \right)}{x \left(2 - \frac{7}{x} \right)} = \lim_{x \rightarrow +\infty} \frac{1}{2} x^3 = +\infty$

d). $\lim_{x \rightarrow +\infty} \frac{\sqrt{2x^4 + x^2 - 1}}{1 - 2x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 \left(2 + \frac{1}{x^2} - \frac{1}{x^4} \right)}}{x \left(\frac{1}{x} - 2 \right)} = \lim_{x \rightarrow +\infty} \frac{x \sqrt{2 + \frac{1}{x^2} - \frac{1}{x^4}}}{\frac{1}{x} - 2} = \lim_{x \rightarrow +\infty} \frac{\sqrt{2}x}{-2} = -\infty$

e). $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 - x}}{1 - 2x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 \left(1 - \frac{1}{x^3} \right)}}{x \left(\frac{1}{x} - 2 \right)} = \lim_{x \rightarrow +\infty} \frac{x \sqrt{1 - \frac{1}{x^3}}}{\frac{1}{x} - 2} = +\infty$

f). $\lim_{x \rightarrow -\infty} \frac{2x^4 + 7x^3 - 15}{x^4 + 1} = \lim_{x \rightarrow -\infty} \frac{x^4 \left(2 + \frac{7}{x} - \frac{15}{x^3} \right)}{x^4 \left(1 + \frac{1}{x^4} \right)} = 2$

Câu 3: Tìm các giới hạn sau:

a). $\lim_{x \rightarrow -\infty} \frac{(x-1)^2(5x+2)^2}{(3x+1)^4}$ b). $\lim_{x \rightarrow -\infty} \frac{2|x|+3}{\sqrt{x^2+x+5}}$ c). $\lim_{x \rightarrow -\infty} \frac{2|x|+3}{\sqrt{x^2+x+5}}$

d). $\lim_{x \rightarrow +\infty} \sqrt{\frac{2x^5 + x^3 - 1}{(2x^2 - 1)(x^3 + x)}}$ e). $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + x} + 2x}{2x + 3}$ f). $\lim_{x \rightarrow -\infty} \frac{|x| - \sqrt{x^2 + x}}{x + 10}$

LỜI GIẢI

a). $\lim_{x \rightarrow -\infty} \frac{(x-1)^2(5x+2)^2}{(3x+1)^4} = \lim_{x \rightarrow -\infty} \frac{x^2 \left(1 - \frac{1}{x}\right)^2 x^2 \left(5 + \frac{2}{x}\right)^2}{x^4 \left(3 + \frac{1}{x}\right)^4} = \lim_{x \rightarrow -\infty} \frac{\left(1 - \frac{1}{x}\right)^2 \left(5 + \frac{2}{x}\right)^2}{\left(3 + \frac{1}{x}\right)^4} = \frac{25}{81}$

b). $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^6 + 2}}{3x^3 - 1} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^6 \left(1 + \frac{2}{x^6}\right)}}{x^3 \left(3 - \frac{1}{x^3}\right)} = \lim_{x \rightarrow +\infty} \frac{x^3 \sqrt{1 + \frac{2}{x^6}}}{x^3 \left(3 - \frac{1}{x^3}\right)} = \lim_{x \rightarrow +\infty} \frac{\sqrt{1 + \frac{2}{x^6}}}{3 - \frac{1}{x^3}} = \frac{1}{3}$

c). $\lim_{x \rightarrow -\infty} \frac{2|x| + 3}{\sqrt{x^2 + x} + 5} = \lim_{x \rightarrow -\infty} \frac{-2x + 3}{\sqrt{x^2 \left(1 + \frac{1}{x} + \frac{5}{x^2}\right)}} = \lim_{x \rightarrow -\infty} \frac{x \left(-2 + \frac{3}{x}\right)}{x \sqrt{1 + \frac{1}{x} + \frac{5}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{-2 + \frac{3}{x}}{1 + \frac{1}{x} + \frac{5}{x^2}} = -2$

d). $\lim_{x \rightarrow +\infty} \sqrt{\frac{2x^5 + x^3 - 1}{(2x^2 - 1)(x^3 + x)}} = \lim_{x \rightarrow +\infty} \frac{x^5 \left(2 + \frac{1}{x^2} - \frac{1}{x^5}\right)}{x^2 \left(2 - \frac{1}{x^2}\right) x^3 \left(1 + \frac{1}{x^2}\right)} = \lim_{x \rightarrow +\infty} \frac{2 + \frac{1}{x^2} - \frac{1}{x^5}}{\left(2 - \frac{1}{x^2}\right) \left(1 + \frac{1}{x^2}\right)} = 1$

e). $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + x} + 2x}{2x + 3} = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 \left(1 + \frac{1}{x}\right)} + 2x}{x \left(2 + \frac{3}{x}\right)} = \lim_{x \rightarrow -\infty} \frac{x \left(\sqrt{1 + \frac{1}{x}} + 2\right)}{x \left(2 + \frac{3}{x}\right)} = \lim_{x \rightarrow -\infty} \frac{\sqrt{1 + \frac{1}{x}} + 2}{2 + \frac{3}{x}} = 1$

f). $\lim_{x \rightarrow -\infty} \frac{|x| - \sqrt{x^2 + x}}{x + 10} = \lim_{x \rightarrow -\infty} \frac{-x - \sqrt{x^2 \left(1 + \frac{1}{x}\right)}}{x \left(1 + \frac{10}{x}\right)} = \lim_{x \rightarrow -\infty} \frac{-x \left(1 + \sqrt{1 + \frac{1}{x}}\right)}{x \left(1 + \frac{10}{x}\right)} = \lim_{x \rightarrow -\infty} \frac{1 + \sqrt{1 + \frac{1}{x}}}{1 + \frac{10}{x}} = 1$

Câu 4: Tìm các giới hạn sau:

a). $\lim_{x \rightarrow -\infty} x \sqrt{\frac{2x^3 + x}{x^5 - x^2 + 3}}$ b). $\lim_{x \rightarrow +\infty} \left[(x+2) \sqrt{\frac{x-1}{x^3 + x}} \right]$ c). $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 2x + 3} + 4x + 1}{\sqrt{4x^2 + 1 - x} + 2}$

d). $\lim_{x \rightarrow +\infty} (\sqrt{x^2 - 4x} - x)$ e). $\lim_{x \rightarrow -\infty} (\sqrt[3]{8x^3 + 1} - 2x + 1)$