

## HƯỚNG DẪN GIẢI

**Câu 1.** Chọn đáp án A

$$\text{Ta có: } A = \frac{\tan \alpha}{1 + \tan^2 \alpha} = \frac{\tan \alpha}{\frac{1}{\cos^2 \alpha}} = \frac{\sin \alpha}{\cos \alpha} \cdot \cos^2 \alpha = \sin \alpha \cos \alpha$$

$$\text{Do } \sin \alpha = \frac{3}{5} \Rightarrow \cos^2 \alpha = 1 - \sin^2 \alpha = \frac{16}{25} \xrightarrow{\frac{\pi}{2} < \alpha < \pi} \cos \alpha = \frac{-4}{5}$$

$$\text{Do đó } A = \frac{-12}{25}$$

**Câu 2.** Chọn đáp án B

$$\begin{aligned} \text{Ta có: } A &= \sin^4 \alpha + \cos^4 \alpha = (\sin^2 \alpha + \cos^2 \alpha)^2 - 2\sin^2 \alpha \cos^2 \alpha = 1 - \frac{1}{2}(\sin 2\alpha)^2 \\ &= 1 - \frac{1}{2} \cdot \frac{4}{9} = 1 - \frac{2}{9} = \frac{7}{9}. \end{aligned}$$

**Câu 3.** Chọn đáp án C

$$\text{Ta có: } A = \cos\left(\alpha + \frac{\pi}{3}\right) = \cos \alpha \cdot \cos \frac{\pi}{3} - \sin \alpha \cdot \sin \frac{\pi}{3} = \frac{\cos \alpha - \sqrt{3} \sin \alpha}{2}$$

$$\text{Mặt khác } \sin \alpha = \frac{1}{\sqrt{3}} \Rightarrow \cos^2 \alpha = \frac{2}{3} \xrightarrow{0 < \alpha < \frac{\pi}{2}} \cos \alpha = \sqrt{\frac{2}{3}}$$

$$\text{Do đó } A = \frac{\sqrt{\frac{2}{3}} - 1}{2} = \frac{\sqrt{6} - 3}{6}.$$

**Câu 4.** Chọn đáp án D

$$\sin(\alpha + \pi) = -\frac{1}{3} \Rightarrow -\sin \alpha = -\frac{1}{3} \Rightarrow \cos^2 \alpha = 1 - \sin^2 \alpha = \frac{8}{9}$$

$$\begin{aligned} \text{Do } \frac{\pi}{2} < \alpha < \pi \text{ nên } \cos \alpha < 0 \Rightarrow \cos \alpha = \frac{-2\sqrt{2}}{3} \text{ do đó } A &= \tan\left(\frac{7\pi}{2} - \alpha\right) = \tan\left(\frac{\pi}{2} - \alpha\right) = \cot \alpha \\ &= \frac{\cos \alpha}{\sin \alpha} = -2\sqrt{2}. \end{aligned}$$

**Câu 5.** Chọn đáp án A

$$\text{Ta có } \cos^6 \alpha + \sin^6 \alpha = (\cos^2 \alpha + \sin^2 \alpha)^3 - 3\cos^2 \alpha \sin^2 \alpha (\cos^2 \alpha + \sin^2 \alpha) = 1 - 3\cos^2 \alpha \sin^2 \alpha$$

$$\text{Do đó } A = \sqrt{\frac{5}{4} - \frac{3}{4} \sin^2 2\alpha} = \sqrt{\frac{5}{4} - \frac{3}{4} \cdot \frac{1 - \cos 4\alpha}{2}} = \sqrt{\frac{5}{4} - \frac{3}{4} \cdot \frac{1}{3}} = 1.$$

**Câu 6.** Chọn đáp án B

$$\text{Ta có: } A = \frac{\sin \alpha}{\sin^3 \alpha + 3\cos^3 \alpha} = \frac{\frac{\sin \alpha}{\cos^3 \alpha}}{\frac{\sin^3 \alpha}{\cos^3 \alpha} + 3} = \frac{\tan \alpha \cdot \frac{1}{\cos^2 \alpha}}{\tan^3 \alpha + 3} = \frac{\tan \alpha (1 + \tan^2 \alpha)}{\tan^3 \alpha + 3} = \frac{10}{11}$$

**Câu 7.** Chọn đáp án A

$$\text{Ta có: } y = 2\sin^2 x - 2\cos^2 x - 3\sin x \cos x = -2(\cos x - \sin^2 x) - \frac{3}{2}\sin 2x$$

$$\Rightarrow y = -2\cos 2x - \frac{3}{2}\sin 2x - 1 \Rightarrow -\sqrt{4 + \frac{9}{4}} - 1 \leq y \leq \sqrt{4 + \frac{9}{4}} - 1$$

$$\text{Hay } \frac{-7}{2} \leq y \leq \frac{3}{2}.$$

**Câu 8.** Chọn đáp án B

$$\text{Hàm số đã cho xác định khi } \cos x - \cos 3x = 2\sin 2x \sin x \neq 0 \Leftrightarrow \sin 2x \neq 0 \Leftrightarrow x \neq \frac{k\pi}{2}.$$

**Câu 9.** Chọn đáp án A

$$\text{Ta có } y = \frac{1 + 4\cos^2 x}{3} \leq \frac{1 + 4}{3} \text{ dấu bằng xảy ra } \Leftrightarrow \cos^2 x = 1 \Leftrightarrow 1 - \cos^2 x = \sin^2 x = 0 \Leftrightarrow x = k\pi.$$

**Câu 10.** Chọn đáp án B

$$\text{Ta có: PT } \Leftrightarrow \sin^2 x - 4\sin x \cos x + 4\cos^2 x = 5(\sin^2 x + \cos^2 x)$$

$$\Leftrightarrow 4\sin^2 x + 4\sin x \cos x + \cos^2 x = 0 \Leftrightarrow (2\sin x + \cos x)^2 = 0 \Leftrightarrow 2\sin x + \cos x = 0$$

$$\Leftrightarrow 2\sin x = -\cos x \Leftrightarrow \tan x = -\frac{1}{2}$$

**Câu 11.** Chọn đáp án A

$$\text{Giả sử } \frac{\cos x + 2\sin x + 3}{2\cos x - \sin x + 4} = m \Leftrightarrow \cos x + 2\sin x + 3 = 2m\cos x - m\sin x + 4m$$

$$\Leftrightarrow (m + 2)\sin x + (1 - 2m)\cos x = 4m - 3 \quad (1)$$

$$\text{PT (1) có nghiệm } \Leftrightarrow (m + 2)^2 + (1 - 2m)^2 \geq (4m - 3)^2 \Leftrightarrow 11m^2 - 24m + 4 \leq 0$$

$$\Leftrightarrow \frac{2}{11} \leq m \leq 2 \text{ suy ra GTLN của hàm số là 2.}$$