Đáp án chuyên đề:
Giá trị lượng giác của một góc bất kì từ 0 đến 180 độ - Hình học 10

Bài 2.1: a) $A=\frac{\sqrt{2}}{2}+2.\frac{1}{2}-\frac{\sqrt{3}}{3}-5.\frac{\sqrt{3}}{3}+4.\frac{\sqrt{2}}{2}=1+\frac{5\sqrt{2}}{2}-2\sqrt{3}$

b) $B=4a^2.\left(\frac{\sqrt{2}}{2}\right)^2-3a^2+\ 2a\ ^2=3a^2$

c) $C=\ \sin^2 35^0+\cos^2 35^0\ -5\ \sin^2 75^0+\cos^2 75^0\ =1-5=-4$

d) $D=12\cos^2 76^0+5\tan 85^0.\cot 85^0+12\sin^2 76^0=12+5=17$

e) $E=\ \sin^2 1^0+\sin^2 89^0\ +\ \sin^2 2^0+\sin^2 88^0\ +...+$

$+\ \sin^2 44^0+\sin^2 46^0\ +\sin^2 45^0+\sin^2 90^0$

$E=\ \sin^2 1^0+\cos^2 1^0\ +\ \sin^2 2^0+\cos^2 2^0\ +...+\ \sin^2 44^0+\cos^2 44^0\ +\frac{1}{2}+1$

$E=\underbrace{1+1+...+1}_{44\ số}+\frac{1}{2}+1=\frac{91}{2}$

f) $F=\ \cos^3 1^0+\cos^3 179^0\ +...+\ \cos^3 89^0+\cos^3 91^0\ +\cos^3 90^0+\cos^3 180^0$

$F=\cos^3 90^0+\cos^3 180^0=0-1=-1$

Bài 2.2: Thay vào ta có: $P=4\tan 34^0.\sin 30^0.\cot 146^0+\frac{8\tan^2\ -27^0}{1+\tan^2 153^0}+8\cos^2 27^0$

$P=-4.\tan 34^0.\frac{1}{2}.\cot 34^0+8\tan^2 27^0.\cos^2 27^0+8\cos^2 27^0=-2+8=6$

Bài 2.3: a) $VT=\frac{\sin^2 x}{\cos^2 x}-\sin^2 x=\sin^2 x\ \ 1+\tan^2 x\ \ -\sin^2 x=VP$

b) $\sin^6 x+\cos^6 x=\ \sin^2 x+\cos^2 x\ ^3-3\sin^2 x.\cos^2 x\ \ \sin^2 x+\cos^2 x$

$=1-3\sin^2 x.\cos^2 x$

c) $VT=\tan^3 x\ \ \cot^2 x+1\ \ -\tan x\ \ \cot^2 x+1\ \ +\cot^3 x\ \ \tan^2 x+1$

$=\tan x+\tan^3 x-\cot x-\tan x+\cot x+\cot^3 x=VP$

d) $VP=\tan^6 x\cos^2 x-\tan^6 x\cot^2 x=\tan^4 x\sin^2 x-\tan^4 x$

$=\tan^4 x.\cos^2 x=\tan^2 x.\sin^2 x=\tan^2 x-\sin^2 x=VT$ (do câu a))

e) $VT=\frac{1}{\tan^2 b}-\frac{1}{\tan^2 a}=\cot^2 b-\cot^2 a=\frac{1}{\sin^2 b}-\frac{1}{\sin^2 a}=VP$

Bài 2.4: a) $A=\tan^2 x+1-\tan^2 x-\cos^2 x=\sin^2 x$

b) $B==\frac{\cos^2 x-\sin^2 x}{\frac{1}{\sin^2 x}-1-\frac{1}{\cos^2 x}+1}-\cos^2 x$

$=\cos^2 x\sin^2 x-\cos^2 x=-\cos^4 x$

c) $C = \dfrac{(\sin a + \cos a)\ \sin^2 a - \sin a\cos a + \cos^2 a}{\sin^2 a - \sin a\cos a + \cos^2 a} = \sin a + \cos a$

d) $D^2 = \dfrac{1+\sin a}{1-\sin a} + \dfrac{1-\sin a}{1+\sin a} + 2$

$= \dfrac{1+\sin a\ ^2 + \ 1-\sin a\ ^2}{1-\sin^2 a} + 2 = \dfrac{2+2\sin^2 a}{\cos^2 a} + 2 = \dfrac{4}{\cos^2 a}$

Suy ra $D = \dfrac{2}{|\cos a|}$

Bài 2.5: a) $(\tan\alpha + \cot\alpha)^2 - (\tan\alpha - \cot\alpha)^2 = 4$

b) $2(\sin^6\alpha + \cos^6\alpha) - 3(\sin^4\alpha + \cos^4\alpha)$

$= 2\ \ 1 - 3\sin^2 x.\cos^2 x\ \ - 3\ \ 1 - 2\sin^2 x.\cos^2 x\ \ = -1$

c) $\cot^2 30^0(\sin^8\alpha - \cos^8\alpha) + 4\cos 60^0(\cos^6\alpha - \sin^6\alpha) - \sin^6(90^0 - \alpha)\ \ \tan^2\alpha - 1\ ^3$

$= 3\ \ \sin^2\alpha - \cos^2\alpha\ \ \ \sin^4\alpha + \cos^4\alpha$

$-2\ \ \sin^2\alpha - \cos^2\alpha\ \ \ \sin^4\alpha + \sin^2\alpha\cos^2\alpha + \cos^4\alpha$

$-\ \ \sin^2\alpha - \cos^2\alpha\ \ ^3 = \ \ \sin^2\alpha - \cos^2\alpha\ \ ^3 - \ \ \sin^2\alpha - \cos^2\alpha\ \ ^3 = 0$

d) $(\sin^4 x + \cos^4 x - 1)(\tan^2 x + \cot^2 x + 2) = -2$

e) $\dfrac{\sin^4 x + 3\cos^4 x - 1}{\sin^6 x + \cos^6 x + 3\cos^4 x - 1} = \dfrac{2}{3}$

Bài 2.6: a) $A = 1$ b) $B = 1$

Bài 2.7: a) $\cos\alpha = \sqrt{1 - \sin^2\alpha} = \dfrac{4}{5}$, $\tan\alpha = \dfrac{3}{4}$, $\cot\alpha = \dfrac{4}{3}$

b) $\sin\alpha = \sqrt{1 - \cos^2\alpha} = \dfrac{2}{\sqrt{5}}$, $\tan\alpha = 2$, $\cot\alpha = \dfrac{1}{2}$

c) $\sin\alpha = \dfrac{1}{\sqrt{3}}$, $\cos\alpha = -\dfrac{\sqrt{6}}{3}$, $\tan\alpha = -\dfrac{1}{\sqrt{2}}$

d) Ta có $\tan\alpha\cot\alpha = 1 > 0$ mà $\tan\alpha + \cot\alpha < 0$ suy ra $\tan\alpha < 0,\ \cot\alpha < 0$

$\cot\alpha = -\sqrt{\dfrac{1}{\sin^2\alpha} - 1} = -2\sqrt{6}$

$\Rightarrow \tan\alpha = -\dfrac{1}{2\sqrt{6}}$, $\cos\alpha = \cot\alpha.\sin\alpha = -\dfrac{2\sqrt{6}}{5}$

Bài 2.8: a) $A = \dfrac{19}{3}$; b) Từ giả thiết suy ra

$\cos\alpha = -\dfrac{2\sqrt{2}}{3}$, $\tan\alpha = -\dfrac{1}{2\sqrt{2}}$, $\cot\alpha = -2\sqrt{2} \Rightarrow B = \dfrac{26 - 2\sqrt{2}}{9}$

c) $C = \dfrac{2\tan a + 3}{\tan a + 1} = \dfrac{7}{3}$

d) $\dfrac{D}{\sin^2 \alpha} = 2\cot^2 a + 5\cot a + \dfrac{1}{\sin^2 \alpha} \Rightarrow \cot^2 a + 1 \;\; D = 3\cot^2 \alpha + 5\cot a + 1$

Suy ra $D = \dfrac{101}{26}$

Bài 2.9: a) $\tan^2 x + \cot^2 x = m^2 - 2$

b) $\tan^4 x + \cot^4 x = \left(\tan^2 x + \cot^2 x\right)^2 - 2 = \left(m^2 - 2\right)^2 - 2 = m^4 - 4m^2 + 2$

$$\Rightarrow \frac{\tan^6 x + \cot^6 x}{\tan^4 x + \cot^4 x} = \frac{\left(\tan^2 x + \cot^2 x\right)\left(\tan^4 x + \cot^4 x - \tan^2 x \cot^2 x\right)}{m^4 - 4m^2 + 2} = \frac{\left(m^2 - 2\right)\left(m^4 - 4m^2 + 1\right)}{m^4 - 4m^2 + 2}$$

Bài 2.10: $\left(\sin\alpha + \cos\alpha\right)^2 = 1 + \dfrac{24}{25} \Rightarrow \sin\alpha + \cos\alpha = \dfrac{7}{5}$ (do $\cos\alpha > 0$)

$$\Rightarrow \sin^3 \alpha + \cos^3 \alpha = \left(\sin\alpha + \cos\alpha\right)\left(\sin^2 \alpha - \sin\alpha\cos\alpha + \cos^2 \alpha\right) = \frac{91}{125}$$

Bài 2.11: ĐS: a) 11 b) $\pm\sqrt{13}$ c) $\pm 33\sqrt{13}$

Bài 2.12: ĐS: a) $A = \dfrac{7}{4}$ b) B = 1 ; c) $C = \dfrac{7}{4}$ hoặc $C = \dfrac{57}{28}$

Bài 2.13. a) $\overrightarrow{AB}.\overrightarrow{AC} = AB.AC.\cos\left(\overrightarrow{AB};\overrightarrow{AC}\right) = a^2\cos 60^0 = \dfrac{a^2}{2}$

b) $\overrightarrow{AC}.\overrightarrow{CB} = -\overrightarrow{CA}.\overrightarrow{CB} = -CA.CB.\cos 60^0 = -\dfrac{a^2}{2}$

c) $\overrightarrow{AB}.\overrightarrow{BC} = -\dfrac{a^2}{2}$