

Đáp án chuyên đề:

Giá trị lượng giác của một góc bất kì từ 0 đến 180 độ - Hình học 10

Bài 2.1: a) $A = \frac{\sqrt{2}}{2} + 2 \cdot \frac{1}{2} - \frac{\sqrt{3}}{3} - 5 \cdot \frac{\sqrt{3}}{3} + 4 \cdot \frac{\sqrt{2}}{2} = 1 + \frac{5\sqrt{2}}{2} - 2\sqrt{3}$

b) $B = 4a^2 \cdot \left(\frac{\sqrt{2}}{2}\right)^2 - 3a^2 + 2a^2 = 3a^2$

c) $C = \sin^2 35^\circ + \cos^2 35^\circ - 5 \sin^2 75^\circ + \cos^2 75^\circ = 1 - 5 = -4$

d) $D = 12 \cos^2 76^\circ + 5 \tan 85^\circ \cdot \cot 85^\circ + 12 \sin^2 76^\circ = 12 + 5 = 17$

e) $E = \sin^2 1^\circ + \sin^2 89^\circ + \sin^2 2^\circ + \sin^2 88^\circ + \dots +$
 $+ \sin^2 44^\circ + \sin^2 46^\circ + \sin^2 45^\circ + \sin^2 90^\circ$

$$E = \sin^2 1^\circ + \cos^2 1^\circ + \sin^2 2^\circ + \cos^2 2^\circ + \dots + \sin^2 44^\circ + \cos^2 44^\circ + \frac{1}{2} + 1$$

$$E = \underbrace{1 + 1 + \dots + 1}_{44 \text{ số}} + \frac{1}{2} + 1 = \frac{91}{2}$$

f) $F = \cos^3 1^\circ + \cos^3 179^\circ + \dots + \cos^3 89^\circ + \cos^3 91^\circ + \cos^3 90^\circ + \cos^3 180^\circ$

$$F = \cos^3 90^\circ + \cos^3 180^\circ = 0 - 1 = -1$$

Bài 2.2: Thay vào ta có: $P = 4 \tan 34^\circ \cdot \sin 30^\circ \cdot \cot 146^\circ + \frac{8 \tan^2 27^\circ - 27^\circ}{1 + \tan^2 153^\circ} + 8 \cos^2 27^\circ$

$$P = -4 \cdot \tan 34^\circ \cdot \frac{1}{2} \cdot \cot 34^\circ + 8 \tan^2 27^\circ \cdot \cos^2 27^\circ + 8 \cos^2 27^\circ = -2 + 8 = 6$$

Bài 2.3: a) $VT = \frac{\sin^2 x}{\cos^2 x} - \sin^2 x = \sin^2 x (1 + \tan^2 x) - \sin^2 x = VP$

b) $\sin^6 x + \cos^6 x = \sin^2 x + \cos^2 x^3 - 3 \sin^2 x \cdot \cos^2 x = \sin^2 x + \cos^2 x$
 $= 1 - 3 \sin^2 x \cdot \cos^2 x$

c) $VT = \tan^3 x \cot^2 x + 1 - \tan x \cot^2 x + 1 + \cot^3 x \tan^2 x + 1$
 $= \tan x + \tan^3 x - \cot x - \tan x + \cot x + \cot^3 x = VP$

d) $VP = \tan^6 x \cos^2 x - \tan^6 x \cot^2 x = \tan^4 x \sin^2 x - \tan^4 x$
 $= \tan^4 x \cdot \cos^2 x = \tan^2 x \cdot \sin^2 x = \tan^2 x - \sin^2 x = VT$ (do câu a))

e) $VT = \frac{1}{\tan^2 b} - \frac{1}{\tan^2 a} = \cot^2 b - \cot^2 a = \frac{1}{\sin^2 b} - \frac{1}{\sin^2 a} = VP$

Bài 2.4: a) $A = \tan^2 x + 1 - \tan^2 x - \cos^2 x = \sin^2 x$

b) $B = \frac{\cos^2 x - \sin^2 x}{\frac{1}{\sin^2 x} - 1 - \frac{1}{\cos^2 x} + 1} - \cos^2 x$
 $= \cos^2 x \sin^2 x - \cos^2 x = -\cos^4 x$

$$c) C = \frac{(\sin a + \cos a) \sin^2 a - \sin a \cos a + \cos^2 a}{\sin^2 a - \sin a \cos a + \cos^2 a} = \sin a + \cos a$$

$$d) D^2 = \frac{1 + \sin a}{1 - \sin a} + \frac{1 - \sin a}{1 + \sin a} + 2$$
$$= \frac{1 + \sin a^2 + 1 - \sin a^2}{1 - \sin^2 a} + 2 = \frac{2 + 2\sin^2 a}{\cos^2 a} + 2 = \frac{4}{\cos^2 a}$$

$$\text{Suy ra } D = \frac{2}{|\cos a|}$$

Bài 2.5: a) $(\tan \alpha + \cot \alpha)^2 - (\tan \alpha - \cot \alpha)^2 = 4$

b) $2(\sin^6 \alpha + \cos^6 \alpha) - 3(\sin^4 \alpha + \cos^4 \alpha)$
 $= 2(1 - 3\sin^2 x \cdot \cos^2 x) - 3(1 - 2\sin^2 x \cdot \cos^2 x) = -1$

c) $\cot^2 30^\circ(\sin^8 \alpha - \cos^8 \alpha) + 4 \cos 60^\circ(\cos^6 \alpha - \sin^6 \alpha) - \sin^6(90^\circ - \alpha) \tan^2 \alpha - 1^3$
 $= 3 \sin^2 \alpha - \cos^2 \alpha \sin^4 \alpha + \cos^4 \alpha$
 $- 2 \sin^2 \alpha - \cos^2 \alpha \sin^4 \alpha + \sin^2 \alpha \cos^2 \alpha + \cos^4 \alpha$
 $- \sin^2 \alpha - \cos^2 \alpha^3 = \sin^2 \alpha - \cos^2 \alpha^3 - \sin^2 \alpha - \cos^2 \alpha^3 = 0$

d) $(\sin^4 x + \cos^4 x - 1)(\tan^2 x + \cot^2 x + 2) = -2$

e) $\frac{\sin^4 x + 3\cos^4 x - 1}{\sin^6 x + \cos^6 x + 3\cos^4 x - 1} = \frac{2}{3}$

Bài 2.6: a) $A = 1$ b) $B = 1$

Bài 2.7: a) $\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \frac{4}{5}$, $\tan \alpha = \frac{3}{4}$, $\cot \alpha = \frac{4}{3}$

b) $\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \frac{2}{\sqrt{5}}$, $\tan \alpha = 2$, $\cot \alpha = \frac{1}{2}$

c) $\sin \alpha = \frac{1}{\sqrt{3}}$, $\cos \alpha = -\frac{\sqrt{6}}{3}$, $\tan \alpha = -\frac{1}{\sqrt{2}}$

d) Ta có $\tan \alpha \cot \alpha = 1 > 0$ mà $\tan \alpha + \cot \alpha < 0$ suy ra $\tan \alpha < 0$, $\cot \alpha < 0$

$$\cot \alpha = -\sqrt{\frac{1}{\sin^2 \alpha} - 1} = -2\sqrt{6}$$

$$\Rightarrow \tan \alpha = -\frac{1}{2\sqrt{6}}, \cos \alpha = \cot \alpha \cdot \sin \alpha = -\frac{2\sqrt{6}}{5}$$

Bài 2.8: a) $A = \frac{19}{3}$; b) Từ giả thiết suy ra

$$\cos \alpha = -\frac{2\sqrt{2}}{3}, \tan \alpha = -\frac{1}{2\sqrt{2}}, \cot \alpha = -2\sqrt{2} \Rightarrow B = \frac{26 - 2\sqrt{2}}{9}$$

$$c) C = \frac{2 \tan a + 3}{\tan a + 1} = \frac{7}{3}$$

$$d) \frac{D}{\sin^2 \alpha} = 2 \cot^2 a + 5 \cot a + \frac{1}{\sin^2 \alpha} \Rightarrow \cot^2 a + 1 \cdot D = 3 \cot^2 \alpha + 5 \cot a + 1$$

$$\text{Suy ra } D = \frac{101}{26}$$

$$\text{Bài 2.9: a) } \tan^2 x + \cot^2 x = m^2 - 2$$

$$b) \tan^4 x + \cot^4 x = \tan^2 x + \cot^2 x - 2 = m^2 - 2 - 2 = m^4 - 4m^2 + 2$$

$$\begin{aligned} \Rightarrow \frac{\tan^6 x + \cot^6 x}{\tan^4 x + \cot^4 x} &= \frac{\tan^2 x + \cot^2 x}{m^2 - 2} \cdot \frac{\tan^4 x + \cot^4 x - \tan^2 x \cot^2 x}{m^4 - 4m^2 + 2} \\ &= \frac{m^4 - 4m^2 + 2}{m^4 - 4m^2 + 2} \end{aligned}$$

$$\text{Bài 2.10: } \sin \alpha + \cos \alpha = \frac{7}{5} \Rightarrow \sin^2 \alpha + \cos^2 \alpha = 1 + \frac{24}{25} \Rightarrow \sin \alpha + \cos \alpha = \frac{7}{5} \text{ (do } \cos \alpha > 0)$$

$$\Rightarrow \sin^3 \alpha + \cos^3 \alpha = (\sin \alpha + \cos \alpha)(\sin^2 \alpha - \sin \alpha \cos \alpha + \cos^2 \alpha) = \frac{91}{125}$$

$$\text{Bài 2.11: } \text{ĐS: a) } 11 \quad b) \pm\sqrt{13} \quad c) \pm 33\sqrt{13}$$

$$\text{Bài 2.12: } \text{ĐS: a) } A = \frac{7}{4} \quad b) B = 1; \quad c) C = \frac{7}{4} \text{ hoặc } C = \frac{57}{28}$$

$$\text{Bài 2.13. a) } \overrightarrow{AB} \cdot \overrightarrow{AC} = AB \cdot AC \cdot \cos \angle BAC = a^2 \cos 60^\circ = \frac{a^2}{2}$$

$$b) \overrightarrow{AC} \cdot \overrightarrow{CB} = -\overrightarrow{CA} \cdot \overrightarrow{CB} = -CA \cdot CB \cdot \cos 60^\circ = -\frac{a^2}{2}$$

$$c) \overrightarrow{AB} \cdot \overrightarrow{BC} = -\frac{a^2}{2}$$