ĐẠO HÀM

1: $y=\left(\frac{1}{2}x^5+\frac{2}{3}x^4-x^3-\frac{3}{2}x^2+4x-5\right)$

$y'=\left(\frac{1}{2}x^5+\frac{2}{3}x^4-x^3-\frac{3}{2}x^2+4x-5\right)' \Leftrightarrow y'=\left(\frac{1}{2}x^5\right)'+\left(\frac{2}{3}x^4\right)'-\left(x^3\right)'-\left(\frac{3}{2}x^2\right)'+\left(4x\right)'-5'$

$y'=\frac{5}{2}x^4+\frac{8}{3}x^3-3x^2-3x+4.$

2: $y=\frac{1}{4}-\frac{1}{3}x+x^2-0,5x^4$

$y'=\left(\frac{1}{4}-\frac{1}{3}x+x^2-0,5x^4\right)'$

$\Leftrightarrow y'=\left(\frac{1}{4}\right)'-\left(\frac{1}{3}x\right)'+\left(x^2\right)'-\left(0,5x^4\right)'$

$\Leftrightarrow y'=-\frac{1}{3}+2x-2x^3.$

3: $y=2x^4-\frac{1}{3}x^3+2\sqrt{x}-5$

$y'=\left(2x^4-\frac{1}{3}x^3+2\sqrt{x}-5\right)' \Leftrightarrow y'=\left(2x^4\right)'-\left(\frac{1}{3}x^3\right)'+\left(2\sqrt{x}\right)'-5' \Leftrightarrow y'=8x^3-x^2+\frac{1}{\sqrt{x}}.$

4: $y=\frac{x^4}{4}-\frac{x^3}{3}+\frac{1}{2}x^2-x+a$ (a là hằng số)

$y'=\left(\frac{x^4}{4}-\frac{x^3}{3}+\frac{1}{2}x^2-x+a\right)' \Leftrightarrow y'=x^3-x^2+x-1$.

5: $y=\frac{3}{x^2}-\sqrt{x}+\frac{2}{3}x\sqrt{x}$

$y'=\left(\frac{3}{x^2}-\sqrt{x}+\frac{2}{3}x\sqrt{x}\right)' \Leftrightarrow y'=\left(3.x^{-2}\right)'-\left(\sqrt{x}\right)'+\frac{2}{3}\left(x\sqrt{x}\right)'$

$\Leftrightarrow y'=3.(-2).x^{-3}-\frac{1}{2\sqrt{x}}+\frac{2}{3}\left(x'.\sqrt{x}+\left(\sqrt{x}\right)'.x\right) \Leftrightarrow y'=\frac{-6}{x^3}-\frac{1}{2\sqrt{x}}+\frac{2}{3}\left(\sqrt{x}+\frac{1}{2\sqrt{x}}.x\right)$

$\Leftrightarrow y'=\frac{-6}{x^3}-\frac{1}{2\sqrt{x}}+\frac{2}{3}\left(\sqrt{x}+\frac{\sqrt{x}}{2}\right)=\frac{-6}{x^3}-\frac{1}{2\sqrt{x}}+\sqrt{x}.$

6: $y=2x^4-\frac{1}{3}x^3+2\sqrt{x}-5$

$y'=\left(2x^4-\frac{1}{3}x^3+2\sqrt{x}-5\right)' \Leftrightarrow y'=\left(2x^4\right)'-\left(\frac{1}{3}x^3\right)'+\left(2\sqrt{x}\right)'-5' \Leftrightarrow y'=8x^3-x^2+\frac{1}{\sqrt{x}}.$

7: $y=x^5-4x^3+2x-3\sqrt{x}$

$y'=\left(x^5-4x^3+2x-3\sqrt{x}\right)' \Leftrightarrow y'=\left(x^5\right)'-4\left(x^3\right)'+2.x'-3\left(\sqrt{x}\right)' \Leftrightarrow y'=5x^4-12x+2-\frac{3}{2\sqrt{x}}.$

Bài 2: Tính đạo hàm của các hàm số sau:

a). $y=\left(x^2+3x\right)\left(2-x\right)$. b) $y=\left(2x-3\right)\left(x^5-2x\right)$ c). $y=\left(x^2+1\right)\left(5-3x^2\right)$

d). $y = x(2x-1)(3x+2)$ e). $y = (x^2-2x+3)(2x^2+3)$ f). $y = x^2\sqrt{x}$

g) $y = \dfrac{2x-1}{4x-3}$ h) $y = \dfrac{2x+10}{4x-3}$ k). $y = \dfrac{3}{2x+1}$ l). $y = \dfrac{2x+1}{1-3x}$

m). $y = \dfrac{1+x-x^2}{1-x+x^2}$ n). $y = \dfrac{x^2-3x+3}{x-1}$ o). $y = \dfrac{2x^2-4x+1}{x-3}$

LỜI GIẢI

a). $y = (x^2+3x)(2-x)$.

$$y' = \left((x^2+3x)(2-x)\right)' = (x^2+3x)'.(2-x) + (x^2+3x).(2-x)'$$
$$= (2x+3)(2-x) + (x^2+3x)(-1) = -3x^2-2x+6.$$

b). $y = (2x-3)(x^5-2x)$

$$y' = \left[(2x-3)(x^5-2x)\right]' = (2x-3)'(x^5-2x) + (x^5-2x)'(2x-3)$$
$$= 2(x^5-2x) + (5x^4-2)(2x-3) = 12x^5-15x^4-8x+6.$$

c). $y = (x^2+1)(5-3x^2)$

$$y' = \left[(x^2+1)(5-3x^2)\right]' = (x^2+1)'(5-3x^2) + (5-3x^2)'(x^2+1)$$
$$= 2x(5-3x^2) - 6x(x^2+1) = 10x-6x^3-6x^3-6x = -12x^3+4x.$$

d). $y = x(2x-1)(3x+2) = (2x^2-x)(3x+2)$

$$y' = \left[(2x^2-x)(3x+2)\right]' = (2x^2-x)'(3x+2) + (3x+2)'.(2x^2-x)$$
$$= (4x-1)(3x+2) + 3(2x^2-x) = 18x^2+2x-2.$$

e). $y = (x^2-2x+3)(2x^2+3)$

$$y' = \left[(x^2-2x+3)(2x^2+3)\right]' = (x^2-2x+3)'(2x^2+3) + (2x^2+3)'(x^2-2x+3)$$
$$= (4x-2)(2x^2+3) + (4x)(x^2-2x+3) = 12x^3-4x^2+24x-6.$$

f) $y = x^2\sqrt{x}$

$$y' = \left(x^2\sqrt{x}\right)' = (x^2)'.\sqrt{x} + (\sqrt{x})'.x^2 = 2x.\sqrt{x} + \frac{1}{2\sqrt{x}}.x^2 = 2x\sqrt{x} + \frac{1}{2}x\sqrt{x} = \frac{5x\sqrt{x}}{2}.$$

g) $y = \dfrac{2x-1}{4x-3} \Rightarrow y' = \left(\dfrac{2x-1}{4x-3}\right)'$

$$= \frac{(2x-1)'(4x-3) - (4x-3)'(2x-1)}{(4x-3)^2} = \frac{2(4x-3)-4(2x-1)}{(4x-3)^2} = \frac{-2}{(4x-3)^2}.$$

h) $y = \dfrac{2x+10}{4x-3} \Rightarrow y' = \left(\dfrac{2x+10}{4x-3}\right)'$

$$= \frac{(2x+10)'.(4x-3) - (4x-3)'.(2x+10)}{(4x-3)^2} = \frac{2(4x-3)-4(2x+10)}{(4x-3)^2} = \frac{-46}{(4x-3)^2}$$

k). $y=\dfrac{3}{2x+1}\Rightarrow y'=3.\left(\dfrac{1}{2x+1}\right)'=-3.\dfrac{(2x+1)'}{(2x+1)^2}=\dfrac{-6}{(2x+1)^2}$.

l). $y=\dfrac{2x+1}{1-3x}\Rightarrow y'=\left(\dfrac{2x+1}{1-3x}\right)'$

$$y'=\frac{(2x+1)'(1-3x)-(1-3x)'(2x+1)}{(1-3x)^2}=\frac{2(1-3x)+3(2x+1)}{(1-3x)^2}=\frac{5}{(1-3x)^2}.$$

m). $y=\dfrac{1+x-x^2}{1-x+x^2}\Rightarrow y'=\left(\dfrac{1+x-x^2}{1-x+x^2}\right)'$

$$=\frac{(1+x-x^2)'(1-x+x^2)-(1-x+x^2)'(1+x-x^2)}{(1-x+x^2)^2}$$

$$=\frac{(1-2x)(1-x+x^2)-(-1+2x)(1+x-x^2)}{(1-x+x^2)^2}$$

n). $y=\dfrac{x^2-3x+3}{x-1}\Rightarrow y'=\dfrac{(x^2-3x+3)'(x-1)-(x-1)'(x^2-3x+3)}{(x-1)^2}$

$$=\frac{(2x-3)(x-1)-(x^2+3x+3)}{(x-1)^2}=\frac{x^2-2x}{(x-1)^2}.$$

o). $y=\dfrac{2x^2-4x+1}{x-3}\Rightarrow y'=\dfrac{(2x^2-4x+1)'(x-3)-(x-3)'(2x^2-4x+1)}{(x-3)^2}$

$$=\frac{(4x-4)(x-3)-(2x^2-4x+1)}{(x-3)^2}=\frac{2x^2-12x+11}{(x-3)^2}.$$

Bài 3: Tính đạo hàm của các hàm số sau:

a). $y=(x^7+x)^2$ b). $y=(2x^3-3x^2-6x+1)^2$ c). $y=(1-2x^2)^3$

d). $y=(x-x^2)^{32}$ e). $y=(x^2+x+1)^4$ f). $y=(x^2-x+1)^3.(x^2+x+1)^2$

g) $y=\left(\dfrac{2x+1}{x-1}\right)^3$ h). $y=\dfrac{1}{(x^2-x+1)^5}$ k). $y=\dfrac{(2-x^2)(3-x^3)}{1-x+x^2}$

l). $y=(1+2x)(2+3x^2)(3-4x^3)$

LỜI GIẢI

a). $y=(x^7+x)^2$. Sử dụng công thức $(u^\alpha)'=\alpha.u^{\alpha-1}.u'$ (với $u=x^7+x$)

$y'=2(x^7+x).(x^7+x)'=2(x^7+x)(7x^6+1)$

b). $y=(2x^3-3x^2-6x+1)^2$. Sử dụng công thức $(u^\alpha)'$ với $u=2x^3-3x^2-6x+1$

$y'=2(2x^3-3x^2+6x+1)(2x^3-3x^2+6x+1)'=2(2x^3-3x^2+6x+1)(6x^2-6x+6)$.

c). $y=(1-2x^2)^3$. Sử dụng công thức $(u^\alpha)'$ với $u=1-2x^2$

$y'=3\left(1-2x^2\right)^2\left(1-2x^2\right)'=3\left(1-2x^2\right)^2(-4x)=-12x\left(1-2x^2\right)^2$.

d). $y=\left(x-x^2\right)^{32}$. Sử dụng công thức $\left(u^{\alpha}\right)'$ với $u=x-x^2$

$y'=32\left(x-x^2\right)^{31}.\left(x-x^2\right)'=32\left(x-x^2\right)^{31}.(1-2x)$

e). $y=\left(x^2+x+1\right)^4$. Sử dụng công thức $\left(u^{\alpha}\right)'$ với $u=x^2+x+1$

$y'=4\left(x^2+x+1\right)^3.\left(x^2+x+1\right)'=4\left(x^2+x+1\right)^3.(2x+1)$

f). $y=\left(x^2-x+1\right)^3.\left(x^2+x+1\right)^2$

Đầu tiên sử dụng quy tắc nhân.

$y'=\left[\left(x^2-x+1\right)^3\right]'\left(x^2+x+1\right)^2+\left[\left(x^2+x+1\right)^2\right]'\left(x^2-x+1\right)^3$.

Sau đó sử dụng công thức $\left(u^{\alpha}\right)'$

$y'=3\left(x^2-x+1\right)^2\left(x^2-x+1\right)'\left(x^2+x+1\right)+2\left(x^2+x+1\right)\left(x^2+x+1\right)'\left(x^2-x+1\right)^3$

$y'=3\left(x^2-x+1\right)^2(2x-1)\left(x^2+x+1\right)^2+2\left(x^2+x+1\right)(2x+1)\left(x^2-x+1\right)^3$

$y'=\left(x^2-x+1\right)^2\left(x^2+x+1\right)\left[3(2x-1)\left(x^2+x+1\right)+2(2x+1)\left(x^2-x+1\right)\right]$.

g) $y=\left(\dfrac{2x+1}{x-1}\right)^3$

Bước đầu tiên sử dụng $\left(u^{\alpha}\right)'$, với $u=\dfrac{2x+1}{x-1}$

$y'=3.\left(\dfrac{2x+1}{x-1}\right)^2.\left(\dfrac{2x+1}{x-1}\right)'=3.\left(\dfrac{2x+1}{x-1}\right)^2.\dfrac{-1}{(x-1)^2}=-\dfrac{3(2x+1)^2}{(x-1)^4}$.

h). $y=\dfrac{1}{\left(x^2-x+1\right)^5}$

Đầu tiên sử dụng công thức $\left(\dfrac{1}{u}\right)'$ với $u=\left(x^2-x+1\right)^5$

$$y'=-\frac{\left(\left(x^2-x+1\right)^5\right)'}{\left(\left(x^2-x+1\right)^5\right)^2}=\frac{-5\left(x^2-x+1\right)^4.\left(x^2-x+1\right)'}{\left(x^2-x+1\right)^{10}}=-\frac{5(2x-1)}{\left(x^2-x+1\right)^6}$$

k). $y=\dfrac{\left(2-x^2\right)\left(3-x^3\right)}{1-x+x^2}$

Đầu tiên sử dụng $\left(\dfrac{u}{v}\right)'$

$$y'=\frac{\left[\left(2-x^2\right)\left(3-x^3\right)\right]'.\left(1-x+x^2\right)-\left(1-x+x^2\right)'\left(2-x^2\right)\left(3-x^3\right)}{\left(1-x+x^2\right)^2}$$

Tính $\left[\left(2-x^2\right)\left(3-x^3\right)\right]'=\left(2-x^2\right)'\left(3-x^3\right)+\left(3-x^3\right)'\left(2-x^2\right)$

$=-2x\left(3-x^3\right)-3x^2\left(2-x^2\right)=5x^4-6x^2-6x.$

Vậy $y'=\dfrac{\left(5x^4-6x^2-6x\right)\left(1-x+x^2\right)-\left(-1+2x\right)\left(2-x^2\right)\left(3-x^3\right)}{\left(1-x+x^2\right)^2}$

l). $y=\left(1+2x\right)\left(2+3x^2\right)\left(3-4x^3\right)$

$y'=\left(1+2x\right)'\left(2+3x^2\right)\left(3-4x^3\right)+\left(1+2x\right)\left(2+3x^2\right)'\left(3-4x^3\right)+\left(1+2x\right)\left(2+3x^2\right)\left(3-4x^3\right)'$

$y'=2\left(2+3x^2\right)\left(3-4x^3\right)+\left(1+2x\right)\left(6x\right)\left(3-4x^3\right)+\left(1+2x\right)\left(2+3x^2\right)\left(-12x^2\right)$.

Bài 4: Tính đạo hàm của các hàm số sau:

a). $y=x^2+x\sqrt{x}+1$ b). $y=\sqrt{1+2x-x^2}$ c). $y=\sqrt{x^2+1}-\sqrt{1-x^2}$

d). $y=\sqrt{\dfrac{x^2+1}{x}}$ e). $y=\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)$ f) $y=\sqrt{x-1}+\dfrac{1}{\sqrt{x-1}}$

g). $y=\left(\sqrt{x}-\dfrac{1}{\sqrt{x}}\right)^5$ h). $y=\dfrac{1+x}{\sqrt{1-x}}$ i). $y=\sqrt{x+\sqrt{x+\sqrt{x}}}$

k). $y=\dfrac{4x+1}{\sqrt{x^2+2}}$ l). $y=\sqrt{\dfrac{x^3}{x-1}}$ m). $y=\sqrt{\left(x-2\right)^3}$ n) $y=\left(1+\sqrt{1-2x}\right)^3$

LỜI GIẢI

a). $y=x^2+x\sqrt{x}+1$

$y'=\left(x^2\right)'+\left(x\sqrt{x}\right)'+1'=2x+x'.\sqrt{x}+\left(\sqrt{x}\right)'.x=2x+\sqrt{x}+\dfrac{1}{2\sqrt{x}}.x=2x+\dfrac{3\sqrt{x}}{2}.$

b). $y=\sqrt{1+2x-x^2}$. Sử dụng công thức $\left(\sqrt{u}\right)'$ với $u=1+2x-x^2$

$y'=\dfrac{\left(1+2x-x^2\right)'}{\sqrt{1+2x-x^2}}=\dfrac{1-x}{\sqrt{1+2x-x^2}}.$

c). $y=\sqrt{x^2+1}-\sqrt{1-x^2}$

$y'=\left(\sqrt{x^2+1}\right)'-\left(\sqrt{1-x^2}\right)'=\dfrac{\left(x^2+1\right)'}{2\sqrt{x^2+1}}-\dfrac{\left(1-x^2\right)'}{2\sqrt{1-x^2}}=\dfrac{x}{\sqrt{x^2+1}}+\dfrac{x}{\sqrt{1-x^2}}.$

d). $y=\sqrt{\dfrac{x^2+1}{x}}$. Sử dụng công thức $\left(\sqrt{u}\right)'$ với $u=\dfrac{x^2+1}{x}$

$y'=\dfrac{1}{2\sqrt{\dfrac{x^2+1}{x}}}.\left(\dfrac{x^2+1}{x}\right)'=\dfrac{1}{2\sqrt{\dfrac{x^2+1}{x}}}\left(1-\dfrac{1}{x^2}\right)$

e). $y=\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)$. Đầu tiên sử dụng công thức $\left(u^\alpha\right)'$ với $u=\dfrac{1-\sqrt{x}}{1+\sqrt{x}}$

$y'=2\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right).\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)'$

Tính $\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)' = \frac{\left(1-\sqrt{x}\right)'\left(1+\sqrt{x}\right)-\left(1+\sqrt{x}\right)'\left(1-\sqrt{x}\right)}{\left(1+\sqrt{x}\right)^2}$

$$= \frac{\frac{-1}{2\sqrt{x}}\left(1+\sqrt{x}\right)-\frac{1}{2\sqrt{x}}\left(1-x\right)}{\left(1+\sqrt{x}\right)^2} = \frac{-1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}$$

Vậy $y' = 2\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right).\frac{-1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}$.

f). $y = \sqrt{x-1}+\frac{1}{\sqrt{x-1}}$

$$y' = \left(\sqrt{x-1}\right)' + \left(\frac{1}{\sqrt{x-1}}\right)' = \frac{1}{2\sqrt{x-1}} + \frac{-\left(\sqrt{x-1}\right)'}{\left(\sqrt{x-1}\right)^2} = \frac{1}{2\sqrt{x-1}} + \frac{-1}{2\sqrt{x-1}\left(x-1\right)}.$$

g). $y = \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^5$. Bước đầu tiên sử dụng $\left(u^\alpha\right)'$ với $u = \sqrt{x}-\frac{1}{\sqrt{x}}$

$$y' = 5\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^4.\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)' = 5\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^4.\left(\frac{1}{2\sqrt{x}}+\frac{\left(\sqrt{x}\right)'}{\left(\sqrt{x}\right)^2}\right)$$

$$= 5\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^4\left(\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{x}.x}\right)$$

h). $y = \frac{1+x}{\sqrt{1-x}}$. Sử dụng $\left(\frac{u}{v}\right)'$ được: $y' = \frac{(1+x)'\sqrt{1-x}-\left(\sqrt{1-x}\right)'(1+x)}{\left(\sqrt{1-x}\right)^2} = \frac{\sqrt{1-x}-\frac{(1-x)'}{2\sqrt{1-x}}.(1+x)}{(1-x)}$

$$= \frac{2(1-x)+(1+x)}{2\sqrt{1-x}.(1-x)} = \frac{3-x}{2\sqrt{1-x}(1-x)}.$$

i) $y = \sqrt{x+\sqrt{x+\sqrt{x}}}$. Đầu tiên áp dụng $\sqrt{u}$ với $u = x+\sqrt{x+\sqrt{x}}$

$$y' = \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(x+\sqrt{x+\sqrt{x}}\right)' = \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}.\left(x+\sqrt{x}\right)'\right)$$

$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}.\left[1+\frac{1}{2\sqrt{x+\sqrt{x}}}.\left(1+\frac{1}{2\sqrt{x}}\right)\right].$$

k). $y = \frac{4x+1}{\sqrt{x^2+2}}$ (áp dụng u chia v đạo hàm)

$$y' = \frac{(4x+1)'\sqrt{x^2+2}-\left(\sqrt{x^2+2}\right)'.(4x+1)}{\left(\sqrt{x^2+2}\right)^2} = \frac{4.\sqrt{x^2+2}-\frac{\left(x^2+2\right)'}{2\sqrt{x^2+2}}.(4x+1)}{\left(x^2+2\right)}$$

$$=\frac{4\sqrt{x^2+2}-\frac{x}{\sqrt{x^2+2}}(4x+1)}{x^2+2}=\frac{4(x^2+2)-x(4x+1)}{(x^2+2)\sqrt{x^2+2}}=\frac{-x+8}{(x^2+2)\sqrt{x^2+2}}$$

l). $y=\sqrt{\frac{x^3}{x-1}}$ (Áp dụng căn bặc hai của u đạo hàm).

$$y'=\frac{1}{2\sqrt{\frac{x^3}{x-1}}}.\left(\frac{x^3}{x-1}\right)'$$

Ta có: $\left(\frac{x^3}{x-1}\right)'=\frac{(x^3)'(x-1)-(x-1)'.x^3}{(x-1)^2}=\frac{3x^2(x-1)-x^3}{(x-1)^2}=\frac{2x^3-3x^2}{(x-1)^2}$

Vậy $y'=\frac{1}{2\sqrt{\frac{x^3}{x-1}}}.\frac{2x^3-3x^2}{(x-1)^2}$.

m). $y=\sqrt{(x-2)^3}$. Đầu tiên áp dụng $(\sqrt{u})'$ với $u=(x-2)^3$

$$y'=\frac{1}{2\sqrt{(x-2)^3}}.\left((x-2)^3\right)'=\frac{1}{2\sqrt{(x-2)^3}}.3.(x-2)^2=\frac{3(x-2)}{2\sqrt{x-2}}.$$

n) $y=\left(1+\sqrt{1-2x}\right)^3$. Bước đầu tiên áp dụng $(u^\alpha)'$ với $u=1+\sqrt{1-2x}$

$$y'=3\left(1+\sqrt{1-2x}\right)^2.\left(1+\sqrt{1-2x}\right)'=3\left(1+\sqrt{1-2x}\right)^2.\frac{(1-2x)'}{2\sqrt{1-2x}}=\frac{-6\left(1+\sqrt{1-2x}\right)^2}{2\sqrt{1-2x}}.$$

Bài 5: Tính đạo hàm của các hàm số sau:

a). $y=x\cos x$ b). $y=\left(\frac{\sin x}{1+\cos x}\right)^3$ c). $y=\sin^3(2x+1)$

d). $y=\sin\sqrt{2+x^2}$ e). $y=\sqrt{\sin x+2x}$ f). $y=2\sin^2 4x-3\cos^3 5x$

h). $y=(2+\sin^2 2x)^3$ i). $y=\sin(\cos^2 x.\tan^2 x)$ j). $y=\cos^2\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$

k). $y=\frac{\sin 2x+\cos 2x}{2\sin 2x-\cos 2x}$ l). $y=\frac{1}{\cos^2 x-\sin^2 x}$ m). $y=\sin x.\cos 2x$

n). $y=(\cos^4 x-\sin^4 x)^5$ o). $y=\sin^2(\cos(\tan^4 3x))$ q). $y=(\sin x+\cos x)^3$

r). $y=5\sin x-3\cos x$ s). $y=\sin(x^2-3x+2)$

LỜI GIẢI

a). $y=x\cos x$. Ta áp dụng đạo hàm tích.

$y'=x'\cos x+x.(\cos x)'=\cos x-x\sin x.$

b) $y=\left(\frac{\sin x}{1+\cos x}\right)^3$. Bước đầu tiên ta áp dụng công thức $(u^\alpha)'$ với $u=\frac{\sin x}{1+\cos x}$

$$y'=3\left(\frac{\sin x}{1+\cos x}\right)^2.\left(\frac{\sin}{1+\cos x}\right)'$$

Tính : $\left(\dfrac{\sin x}{1+\cos x}\right)' = \dfrac{(\sin x)'(1+\cos x)-(1+\cos x)'.\sin x}{(1+\cos x)^2} = \dfrac{\cos x(1+\cos x)+\sin^2 x}{(1+\cos x)^2}$

$= \dfrac{\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2} = \dfrac{1}{1+\cos x}$.

Vậy $y' = 3\left(\dfrac{\sin x}{1+\cos x}\right)^2 . \dfrac{1}{1+\cos x} = \dfrac{3\sin^2 x}{(1+\cos x)^3}$.

c). $y=\sin^3(2x+1)$. Bước đầu tiên áp dung công thức $(u^\alpha)'$ với $u=\sin(2x+1)$

Vậy $y'=\left(\sin^3(2x+1)\right)' = 3\sin^2(2x+1).\left(\sin(2x+1)\right)'$.

Tính $\left(\sin(2x+1)\right)'$: Áp dụng $(\sin u)'$, với $u=(2x+1)$

Ta được: $\left(\sin(2x+1)\right)' = \cos(2x+1).(2x+1)' = 2\cos(2x+1)$.

$\Rightarrow y' = 3.\sin^2(2x+1).2\cos(2x+1) = 6\sin^2(2x+1)\cos(2x+1)$.

d). $y=\sin\sqrt{2+x^2}$. Áp dụng công thức $(\sin u)'$ với $u=\sqrt{2+x^2}$

$y' = \cos\sqrt{2+x^2}.\left(\sqrt{2+x^2}\right)' = \cos\sqrt{2+x^2}.\dfrac{(2+x^2)'}{2\sqrt{2+x^2}} = \dfrac{x}{\sqrt{2+x^2}}.\cos\sqrt{2+x^2}$.

e). $y=\sqrt{\sin x+2x}$. Áp dụng $\left(\sqrt{u}\right)'$, với $u=\sin x+2x$

$y' = \dfrac{(\sin x+2x)'}{2\sqrt{\sin x+2x}} = \dfrac{\cos x+2}{2\sqrt{\sin x+2x}}$.

f). $y=2\sin^2 4x-3\cos^3 5x$. Bước đầu tiên áp dụng $(u+v)'$

$y' = \left(2\sin^2 4x\right)' - 3\left(\cos^3 5x\right)'$

Tính $\left(\sin^2 4x\right)'$: Áp dụng $(u^\alpha)'$, với $u=\sin 4x,$ ta được:

$\left(\sin^2 4x\right)' = 2\sin 4x.(\sin 4x)' = 2\sin 4x.\cos 4x(4x)' = 4\sin 8x$.

Tương tự: $\left(\cos^3 5x\right)' = 3\cos^2 5x.(\cos 5x)' = 3\cos^2 5x.(-\sin 5x).(5x)'$

$= -15\cos^2 5x.\sin 5x = \dfrac{-15}{2}\cos 5x.\sin 10x$.

Kết luận: $y' = 8\sin 8x + \dfrac{45}{2}\cos 5x.\sin 10x$

h). $y=\left(2+\sin^2 2x\right)^3$. Áp dụng $(u^\alpha)'$, với $u=2+\sin^2 2x$.

$y' = 3\left(2+\sin^2 2x\right)^2\left(2+\sin^2 2x\right)' = 3\left(2+\sin^2 2x\right)^2\left(\sin^2 2x\right)'$.

Tính $\left(\sin^2 2x\right)'$, áp dụng $(u^\alpha)'$, với $u=\sin 2x$.

$\left(\sin^2 2x\right)' = 2.\sin 2x(\sin 2x)' = 2.\sin 2x.\cos 2x(2x)' = 2\sin 4x$.

$\Rightarrow y' = 6\sin 4x\left(2+\sin^2 2x\right)^2$.

i). $y=\sin\left(\cos^2 x.\tan^2 x\right)$. Áp dụng $(\sin u)'$, với $u=\cos^2 x\tan^2 x$

$y' = \cos\left(\cos^2 x.\tan^2 x\right).\left(\cos^2 x.\tan^2 x\right)'$.

Tính $\left(\cos^2 x.\tan^2 x\right)'$, bước đầu sử dụng $(u.v)'$, sau đó sử dụng $\left(u^\alpha\right)'$.

$$\left(\cos^2 x.\tan^2 x\right)' = \left(\cos^2 x\right)'.\tan^2 x + \left(\tan^2 x\right)'.\cos^2 x$$

$$= 2\cos x\left(\cos x\right)'\tan^2 x + 2\tan x\left(\tan x\right)'\cos^2 x$$

$$= -2\sin x\cos x\tan^2 x + 2\tan x\frac{1}{\cos^2 x}\cos^2 x = -\sin 2x\tan^2 x + 2\tan x.$$

Vậy $y' = \cos\left(\cos^2 x.\tan^2 x\right)\left(-\sin 2x\tan^2 x + 2\tan x\right)$

j). $y = \cos^2\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$. Áp dụng $\left(u^\alpha\right)'$, với $u = \cos\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$

$$y' = 2.\cos\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right).\left[\cos\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)\right]' = -2.\cos\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right).\sin\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right).\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)'$$

$$y' = -\sin\left(2\frac{\sqrt{x}+1}{\sqrt{x}-1}\right).\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)'.$$

Tính $\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)' = \frac{\left(\sqrt{x}+1\right)'.\left(\sqrt{x}-1\right) - \left(\sqrt{x}-1\right)'.\left(\sqrt{x}+1\right)}{\left(\sqrt{x}-1\right)^2} = \frac{-1}{\sqrt{x}\left(\sqrt{x}-1\right)^2}$.

Vậy $y' = \frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)^2}.\sin\left(2.\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$.

k). $y = \frac{\sin 2x + \cos 2x}{2\sin 2x - \cos 2x}$.

$$y' = \frac{\left(\sin 2x + \cos 2x\right)'.\left(2\sin 2x - \cos 2x\right) - \left(2\sin 2x - \cos 2x\right)'.\left(\sin 2x + \cos 2x\right)}{\left(2\sin 2x - \cos 2x\right)^2}$$

$$y' = \frac{\left(2\cos 2x - 2\sin 2x\right)\left(2\sin 2x - \cos 2x\right) - \left(4\cos 2x + 2\sin 2x\right)\left(\sin 2x + \cos 2x\right)}{\left(2\sin 2x - \cos 2x\right)^2}$$

$$y' = \frac{-6\cos^2 2x - 6\sin^2 2x}{\left(2\sin 2x - \cos 2x\right)^2} = \frac{-6}{\left(2\sin 2x - \cos 2x\right)^2}.$$

l). $y = \frac{1}{\cos^2 x - \sin^2 x} = \frac{1}{\cos 2x}$. Áp dụng $\left(\frac{1}{u}\right)'$.

$$y' = \frac{-\left(\cos 2x\right)'}{\left(\cos 2x\right)^2} = \frac{\sin 2x.\left(2x\right)'}{\cos^2 2x} = \frac{2\sin 2x}{\cos^2 2x}.$$

m). $y = \sin x.\cos 2x$. Áp dụng $(u.v)'$

$$y' = \left(\sin x\right)'.\cos 2x + \left(\cos 2x\right)'.\sin x = \cos x.\cos 2x - \sin 2x.\left(2x\right)'.\sin x$$

$$y' = \cos x.\cos 2x - 2\sin 2x.\sin x.$$

n). $y = \left(\cos^4 x - \sin^4 x\right)^5 = \left[\left(\cos^2 x - \sin^2 x\right)\left(\cos^2 x + \sin^2 x\right)\right]^5 = \left(\cos 2x\right)^5$.

Áp dụng $\left(u^{\alpha}\right)'$, với $u=\cos 2x$

$y'=5.\cos^4 2x.\left(\cos 2x\right)'=5.\cos^4 2x.\left(-\sin 2x\right).\left(2x\right)'=-10\cos^4 2x.\sin 2x.$

o). $y=\sin^2\left(\cos\left(\tan^4 3x\right)\right)$

Đầu tiên áp dụng $\left(u^{\alpha}\right)'$, với $u=\sin\left(\cos\left(\tan^4 3x\right)\right)$

$y'=2\sin\left(\cos\left(\tan^4 3x\right)\right).\left[\sin\left(\cos\left(\tan^4 3x\right)\right)\right]'$

Sau đó áp dụng $\left(\sin u\right)'$, với $u=\cos\left(\tan^4 3x\right)$

$y'=2\sin\left(\cos\left(\tan^4 3x\right)\right).\cos\left(\cos\left(\tan^4 3x\right)\right).\left(\cos\left(\tan^4 3x\right)\right)'$

Áp dụng $\left(\cos u\right)'$, với $u=\tan^4 3x.$

$y'=-\sin\left(2\cos\left(\tan^4 3x\right)\right).\left(\sin\left(\tan^4 3x\right)\right).\left(\tan^4 3x\right)'.$

Áp dụng $\left(u^{\alpha}\right)'$, với $u=\tan 3x$

$y'=-\sin\left(2\cos\left(\tan^4 3x\right)\right).\left(\sin\left(\tan^4 3x\right)\right).4\tan^3 3x.\left(\tan 3x\right)'.$

$y'=-\sin\left(2\cos\left(\tan^4 3x\right)\right).\left(\sin\left(\tan^4 3x\right)\right).4\tan^3 3x.\left(1+\tan^2 3x\right).\left(3x\right)'.$

$y'=-\sin\left(2\cos\left(\tan^4 3x\right)\right).\left(\sin\left(\tan^4 3x\right)\right).4\tan^3 3x.\left(1+\tan^3 3x\right).3\ .$

p) $y=\sin^3 2x.\cos^3 2x=\left(\sin 2x.\cos 2x\right)^3=\left(\frac{1}{2}\sin 4x\right)^3=\frac{1}{8}.\sin^3 4x$

Áp dụng $\left(u^{\alpha}\right)'$, $u=\sin 4x.$

$y'=\frac{1}{8}.3\sin^2 4x\left(\sin 4x\right)'=\frac{1}{8}.3\sin^2 4x.\cos 4x.\left(4x\right)'=\frac{3}{2}\sin^2 4x.\cos 4x.$

q) $y=\left(\sin x+\cos x\right)^3$. Áp dụng $\left(u^{\alpha}\right)'$, với $u=\sin x+\cos x$

$y'=3\left(\sin x+\cos x\right)^2.\left(\sin x+\cos x\right)'=3\left(\sin x+\cos x\right)^2\left(\cos x-\sin x\right).$

r). $y=5\sin x-3\cos x$

$y'=\left(5\sin x\right)'-\left(3\cos x\right)'=5\cos x+3\sin x.$

s). $y=\sin\left(x^2-3x+2\right)$

Áp dụng $\left(\sin u\right)'$, với $u=\left(x^2-3x+2\right)$

$y'=\cos\left(x^2-3x+2\right).\left(x^2-3x+2\right)'=\left(2x-3\right).\cos\left(x^2-3x+2\right)$

Bài 6:

a). $y=\sin\sqrt{x}$ b). $y=\cos^2 x$ c). $y=\cos\sqrt{2x+1}$ d). $y=\sin 3x.\cos 5x$

e). $y=\dfrac{\sin x+\cos x}{\sin x-\cos x}$ f). $y=\sqrt{\cos 2x}$ g). $y=\dfrac{\sin x}{x}+\dfrac{x}{\sin x}$ h). $y=\sin\left(\cos x\right)+\cos\left(\sin x\right)$ i). $y=\dfrac{x+\sin x}{x-\sin x}$ k). $y=\left(\dfrac{1+\cos 2x}{1-\cos 2x}\right)^2$

l). $y=\sin^4 x+\cos^4 x$ m). $y=\cos\left(2x-\frac{\pi}{4}\right)^2$ n). $y=\frac{\sin x-x\cos x}{\cos x+x\sin x}$

LỜI GIẢI

a). $y=\sin\sqrt{x}$. Áp dụng $(\sin u)'$, với $u=\sqrt{x}$

$$y'=\left(\sin\sqrt{x}\right)'=\cos\sqrt{x}.\left(\sqrt{x}\right)'=\frac{1}{2\sqrt{x}}.\cos\sqrt{x}.$$

b). $y=\cos^2 x$. Áp dụng công thức $\left(u^{\alpha}\right)'$, với $u=\cos x$

$$y'=\left(\cos^2 x\right)'=2.\cos(\cos x)'=2\cos x.(-\sin x)=-\sin 2x.$$

c). $y=\cos\sqrt{2x+1}$. Áp dụng $(\cos u)'$, với $u=\sqrt{2x+1}$

$$y'=\left(\cos\sqrt{2x+1}\right)'=-\sin\sqrt{2x+1}\left(\sqrt{2x+1}\right)'=-\sin\sqrt{2x+1}.\frac{(2x+1)'}{2\sqrt{2x+1}}$$

$$=-\sin\sqrt{2x+1}.\frac{2}{2\sqrt{2x+1}}=-\frac{1}{\sqrt{2x+1}}.\sin\sqrt{2x+1}.$$

d). $y=\sin 3x.\cos 5x=\frac{1}{2}(\sin(-2x)+\sin 8x)=\frac{1}{2}(-\sin 2x+\sin 8x)$

$$y'=\frac{1}{2}(\sin 8x-\sin 2x)'=\frac{1}{2}(\sin 8x)'-\frac{1}{2}(\sin 2x)'=\frac{1}{2}\cos 8x(8x)'-\frac{1}{2}\cos 2x.(2x)'$$
$$=4\cos 8x-\cos 2x$$

e). $y=\frac{\sin x+\cos x}{\sin x-\cos x}$. Áp dụng $\left(\frac{u}{v}\right)'$

$$y'=\frac{(\sin x+\cos x)'(\sin x-\cos x)-(\sin x-\cos x)'.(\sin x+\cos x)}{(\sin x-\cos x)^2}$$

$$y'=\frac{(\cos x-\sin x)(\sin x-\cos x)-(\cos x+\sin x)(\sin x+\cos x)}{(\sin x-\cos x)^2}$$

$$y'=\frac{-(\sin x-\cos x)^2-(\sin x+\cos x)^2}{(\sin x-\cos x)^2}=\frac{2\sin 2x}{(\sin x-\cos x)^2}.$$

f). $y=\sqrt{\cos 2x}$. Áp dụng $\left(\sqrt{u}\right)'$, với $u=\cos 2x$

$$y'=\frac{(\cos 2x)'}{2\sqrt{\cos 2x}}=\frac{-\sin 2x.(2x)'}{2\sqrt{\cos 2x}}=\frac{-\sin 2x}{\sqrt{\cos 2x}}.$$

g) $y=\frac{\sin x}{x}+\frac{x}{\sin x}\Rightarrow y'=\left(\frac{\sin x}{x}\right)'+\left(\frac{x}{\sin x}\right)'$

$=\frac{(\sin x)'.x-x'.\sin x}{x^2}+\frac{x'.\sin x-(\sin x)'.x}{\sin^2 x}=\frac{x\cos x-\sin x}{x^2}+\frac{\sin x-x\cos x}{\sin^2 x}$. h) h). $y=\sin(\cos x)+\cos(\sin x)$

Bước đầu tiên sử dụng đạo hàm tổng, sau đó sử dụng $(\sin u)',(\cos u)'$.

$$y'=(\sin(\cos x))'+(\cos(\sin x))'=\cos(\cos x).(\cos x)'-\sin(\sin x).(\sin x)'$$
$$=-\sin x.\cos(\cos x)-\cos x.\sin(\sin x)=-(\sin x.\cos(\cos x)+\cos x.\sin(\sin x))$$

$=-\sin(x+\cos x)$

i). $y=\dfrac{x+\sin x}{x-\sin x}$. Sử dụng $\left(\dfrac{u}{v}\right)'$

$$y'=\frac{(x+\sin x)'.(x-\sin x)-(x-\sin x)'.(x+\sin x)}{(x-\sin x)^2}$$

$$=\frac{(1+\cos x)(x-\sin x)-(1-\cos x)(x+\sin x)}{(x-\sin x)^2}=\frac{-2\sin x+2x\cos x}{(x-\sin x)^2}.$$

k). $y=\left(\dfrac{1+\cos 2x}{1-\cos 2x}\right)^2$. Sử dụng $(u^\alpha)'$ với $u=\dfrac{1+\cos 2x}{1-\cos 2x}$

$$y'=2\left(\frac{1+\cos 2x}{1-\cos 2x}\right).\left(\frac{1+\cos 2x}{1-\cos 2x}\right)'$$

$$=2\left(\frac{1+\cos 2x}{1-\cos 2x}\right).\left(\frac{(1+\cos 2x)'(1-\cos 2x)-(1-\cos 2x)'(1+\cos 2x)}{(1-\cos 2x)^2}\right)$$

$$=2\left(\frac{1+\cos 2x}{1-\cos 2x}\right).\left(\frac{-2\sin 2x(1-\cos 2x)-2\sin 2x(1+\cos 2x)}{(1-\cos 2x)^2}\right)$$

$$=2\left(\frac{1+\cos 2x}{1-\cos 2x}\right).\left(\frac{-4\sin 2x}{(1-\cos 2x)^2}\right).$$

l). $y=\sin^4 x+\cos^4 x\ =1-\dfrac{1}{2}\sin^2 2x=\dfrac{3}{4}+\dfrac{1}{4}\cos 4x.$

$$y'=\left(\frac{3}{4}+\frac{1}{4}\cos 4x\right)'=\frac{1}{4}(\cos 4x)'=\frac{1}{4}(-\sin 4x).(4x)'=-\sin 4x.$$

m). $y=\cos\left(2x-\dfrac{\pi}{4}\right)^2$. Áp dụng $(\cos u)'$ với $u=\left(2x-\dfrac{\pi}{4}\right)^2$

$$y'=-\sin\left(2x-\frac{\pi}{4}\right)^2.\left[\left(2x-\frac{\pi}{4}\right)^2\right]'=-\sin\left(2x-\frac{\pi}{4}\right)^2.2\left(2x-\frac{\pi}{4}\right).\left(2x-\frac{\pi}{4}\right)'$$

$$=-4\left(2x-\frac{\pi}{4}\right).\sin\left(2x-\frac{\pi}{4}\right)^2.$$

n). $y=\dfrac{\sin x-x\cos x}{\cos x+x\sin x}$

$$y'=\frac{(\sin x-x\cos x)'(\cos x+x\sin x)-(\cos x-x\sin x)'(\sin x-x\cos x)}{(\cos x+x\sin x)^2}$$

Tính $(\sin x-x\cos x)'=\cos x-(x\cos x)'=\cos x-\left(x'.\cos x+x.(\cos x)'\right)$

$=\cos x-(\cos x-x\sin x)=x\sin x$

Tính $(\cos x+x\sin x)'=-\sin x+\left(x'.\sin x+x.(\sin x)'\right)$

$=-\sin x+(\sin x+x\cos x)=x\cos x$

$$\Rightarrow y'=\frac{x\sin x\left(\cos x+x\sin x\right)-x\cos x\left(\sin x-x\cos x\right)}{\left(\cos x+x\sin x\right)^2}=\frac{x^2}{\left(\cos x+x\sin x\right)^2}.$$

Bài 7:

a). $y=\tan\frac{x+1}{2}$ b). $y=\tan^3 x+\cot 2x$ c). $y=\cot\sqrt{x^2+1}$ d). $y=\tan 3x-\cot 3x$

e). $y=x\cot 2x$ f). $y=\frac{1+\tan^2 3x}{1-\tan^2 3x}$ j). $y=\frac{1}{2}\left(1+\tan^2 x\right)^2$. h) $y=\sqrt{\cot^3 2x}$

LỜI GIẢI

a). $y=\tan\frac{x+1}{2}$. Áp dụng $\left(\tan x\right)'$ với $u=\frac{x+1}{2}$

$$y'=\left(1+\tan^2\left(\frac{x+1}{2}\right)\right).\left(\frac{x+1}{2}\right)'=\frac{1}{2}\left(1+\tan^2\left(\frac{x+1}{2}\right)\right).$$

b). $y=\tan^3 x+\cot 2x$. Đầu tiên áp dụng $\left(u+v\right)'$

$$y'=\left(\tan^3 x\right)'+\left(\cot 2x\right)'$$

$\left(\tan^3 x\right)'$ áp dụng $\left(u^\alpha\right)'$ với $u=\tan x$

Vậy $\left(\tan^3 x\right)'=3\tan^2 x\left(\tan x\right)'=3\tan^2 x\left(1+\tan^2 x\right)$

$\left(\cot 2x\right)'$ áp dụng $\left(\cot u\right)'$ với $u=2x$

Vậy $\left(\cot 2x\right)'=-\left(1+\cot^2 2x\right).\left(2x\right)'=-2\left(1+\cot^2 2x\right)$

$$\Rightarrow y'=3\tan^2 x\left(1+\tan^2 x\right)-2\left(1+\cot^2 2x\right).$$

c). $y=\cot\sqrt{x^2+1}$. Áp dụng $\left(\cot u\right)'$ với $u=\sqrt{x^2+1}$

$$\Rightarrow y'=-\left(1+\cot^2\left(\sqrt{x^2+1}\right)\right).\left(\sqrt{x^2+1}\right)'=-\left(1+\cot^2\left(\sqrt{x^2+1}\right)\right).\frac{\left(x^2+1\right)'}{2\sqrt{x^2+1}}$$

$$=\frac{-x}{\sqrt{x^2+1}}\left(1+\cot^2\sqrt{x^2+1}\right).$$

d). $y=\tan 3x-\cot 3x$

$$y'=\left(\tan 3x\right)'-\left(\cot 3x\right)'=\left(1+\tan^2 3x\right)\left(3x\right)'+\left(1+\cot^2 3x\right)\left(3x\right)'$$

$$=3\left(1+\tan^2 3x\right)+3\left(1+\cot^2 3x\right)=3\left(2+\tan^2 3x+\cot^2 3x\right).$$

e). $y=x\cot 2x$

$$y'=x'.\cot 2x+x\left(\cot 2x\right)'=\cot 2x-x\left(1+\cot^2 2x\right).\left(2x\right)'=\cot 2x-2x\left(1+\cot^2 2x\right).$$

f). $y=\frac{1+\tan^2 3x}{1-\tan^2 3x}$. Bước đầu tiên biến đổi lượng giác, rút gọn biểu thức

$y=\frac{1+\frac{\sin^2 3x}{\cos^2 3x}}{1-\frac{\sin^2 3x}{\cos^2 3x}}=\frac{1}{\cos 6x}$. Sau đó áp dụng $\left(\frac{1}{u}\right)'$ với $u=\cos 6x$

$$y'=\frac{-\left(\cos 6x\right)'}{\cos^2 6x}=\frac{6\sin 6x}{\cos^2 6x}.$$

j). $y=\frac{1}{2}\left(1+\tan^2 x\right)^2$. Áp dụng $\left(u^\alpha\right)'$ với $u=1+\tan^2 x$

$y'=\left(1+\tan^2 x\right)\left(1+\tan^2 x\right)'=\left(1+\tan^2 x\right).2\tan x.\left(\tan x\right)'$

$=\left(1+\tan^2 x\right).2\tan x.\left(1+\tan^2 x\right)=2\tan x.\left(1+\tan^2 x\right)^2$.

h) $y=\sqrt{\cot^3 2x}$. Áp dụng $\left(\sqrt{u}\right)'$ với $u=\cot 2x$

$y'=\frac{1}{2\sqrt{\cot^3 2x}}.3.\cot^2 2x.\left(\cot 2x\right)'=\frac{1}{2\sqrt{\cot^3 2x}}.3.\cot^2 2x.(-1).\left(1+\cot^2 2x\right)$.

Bài 8:

a). $y=\tan^2\left(\sin\left(\cos^3 2x\right)\right)$ b). $y=\tan^3\left(\frac{\pi}{4}-2x\right)^2$ c). $y=\sqrt{\cot\left(x^2+1\right)}$

d). $y=\frac{(x+1)^2}{(x-1)^3}$ e). $y=\tan 2x+\frac{2}{3}\tan^3 2x+\frac{1}{5}\tan^5 2x$

e). $y=4\sin.\cos 5x.\sin 6x$ f). $y=\cot^5\left[\cos^2\left(\frac{x-3}{x+2}\right)^2\right]$

LỜI GIẢI

a). $y=\tan^2\left(\sin\left(\cos^3 2x\right)\right)$. Đầu tiên áp dụng $\left(u^\alpha\right)'$ với $u=\tan\left(\sin\left(\cos^3 2x\right)\right)$

$y'=2.\tan\left(\sin\left(\cos^3 2x\right)\right).\left(\tan\left(\sin\left(\cos^3 2x\right)\right)\right)'$

Áp dụng $\left(\tan u\right)'$ với $u=\sin\left(\cos^2 2x\right)$

$y'=2\tan\left(\sin\left(\cos^3 2x\right)\right).\left(1+\tan^2\left(\sin\left(\cos^3 2x\right)\right)\right).\left(\sin\left(\cos^3 2x\right)\right)'$

Tính $\sin\left(\cos^3 2x\right)'$ áp dụng $\left(\sin u\right)'$ với $u=\cos^3 2x$

$\left(\sin\left(\cos^3 2x\right)\right)'=\cos\left(\cos^3 2x\right).\left(\cos^3 2x\right)'=\cos\left(\cos^3 2x\right).3\cos^2 2x.\left(\cos 2x\right)'$

$=\cos\left(\cos^3 2x\right).(-6).\sin 2x.\cos^2 2x$.

Vậy $y'=2\tan\left(\sin\left(\cos^3 2x\right)\right)\left[1+\tan^2\left(\sin\left(\cos^3 2x\right)\right)\right].\cos\left(\cos^3 2x\right).(-6).\sin 2x.\cos^2 2x$.

b). $y=\tan^3\left(\frac{\pi}{4}-2x\right)^2$. Bước đầu tiên áp dụng $\left(u^\alpha\right)'$ với $u=\tan\left(\frac{\pi}{4}-2x\right)^2$

$y'=3\tan^2\left(\frac{\pi}{4}-2x\right)^2.\left[\tan\left(\frac{\pi}{4}-2x\right)^2\right]'$

Tính $\left[\tan\left(\frac{\pi}{4}-2x\right)^2\right]'$ áp dụng $\left(\tan u\right)'$ với $u=\frac{\pi}{4}-2x$ được

$2\left(\frac{\pi}{4}-2x\right)\left(\frac{\pi}{4}-2x\right)'=-4\left(\frac{\pi}{4}-2x\right)$

$\Rightarrow y'=-12.\left(\frac{\pi}{4}-2x\right).\tan^2\left(\frac{\pi}{4}-2x\right)^2.\left(1+\tan^2\left(\frac{\pi}{4}-2x\right)^2\right)$

c). $y=\sqrt{\cot\left(x^2+1\right)}$. Áp dụng $\left(\sqrt{u}\right)'$ với $u=\cot\left(x^2+1\right)$

$y'=\dfrac{1}{2\sqrt{\cot\left(x^2+1\right)}}.\left(\cot\left(x^2+1\right)\right)'$ áp dụng $\left(\cot u\right)'$ với $u=x^2+1$

$$\Rightarrow y'=\frac{1}{2\sqrt{\cot\left(x^2+1\right)}}.(-1).\left(1+\cot^2\left(x^2+1\right)\right).\left(x^2+1\right)'=\frac{-x\left(1+\cot^2\left(x^2+1\right)\right)}{2\sqrt{\cot\left(x^2+1\right)}}.$$

d). $y=\dfrac{(x+1)^2}{(x-1)^3}$. Áp dụng $\left(\dfrac{u}{v}\right)'$ được:

$$y'=\frac{\left[(x+1)^2\right]'.(x-1)^3-\left[(x-1)^3\right]'.(x+1)^2}{(x-1)^6}=\frac{2(x+1)(x-1)^3-3(x-1)^2.(x+1)^2}{(x-1)^6}$$

$$=\frac{(x+1)(x-1)^2(-x-5)}{(x-1)^6}=\frac{(x+1)(-x-5)}{(x-1)^4}.$$

e). $y=\tan 2x+\dfrac{2}{3}\tan^3 2x+\dfrac{1}{5}\tan^5 2x$

$$y'=\left(\tan 2x\right)'+\frac{2}{3}\left(\tan^3 2x\right)'+\frac{1}{5}\left(\tan^5 2x\right)'$$

Áp dụng $\left(u^\alpha\right)'$ với $u=\tan 2x$ và $\left(\tan u\right)'$ với $u=2x$ được:

$$y'=\left(\tan 2x\right)'+2\tan^2 2x.\left(\tan 2x\right)'+\tan^4 2x\left(\tan 2x\right)'$$

$$=\left(1+2\tan^2 2x+\tan^4 2x\right)\left(1+\tan^2 2x\right).\left(2x\right)'$$

$$=\left(1+\tan^2 2x\right)^2.\left(1+\tan^2 2x\right).2=2\left(1+\tan^2 2x\right)^3.$$

f). $y=4\sin.\cos 5x.\sin 6x$

Áp dụng công thức biến đổi tích thành tổng của lượng giác , rút gọn y:

$$y=2\left(\sin 6x-\sin 4x\right).\sin 6x=2\sin^2 6x-2\sin 6x.\sin 4x=1-\cos 12x-\left(\cos 2x+\cos 10x\right)$$

$$=1-\cos 12x-\cos 2x-\cos 10x=1+12\sin 2x+2\sin 2x+10\sin 10x.$$

g). $y=\cot^5\left[\cos^2\left(\dfrac{x-3}{x+2}\right)^2\right]$

Bước đầu tiên áp dụng $\left(u^\alpha\right)'$ với $u=\cot\left[\cos^2\left(\dfrac{x-3}{x+2}\right)^2\right]$

$$y'=5.\cot^4\left[\cos^2\left(\frac{x-3}{x+2}\right)^2\right].\cot\left[\cos^2\left(\frac{x-3}{x+2}\right)^2\right]'$$

Áp dụng $\left(\cot u\right)'$ với $u=\cos^2\left(\dfrac{x-3}{x+2}\right)^2$

Vậy $\cot\left[\cos^2\left(\dfrac{x-3}{x+2}\right)^2\right]'=-\left\{1+\cot^2\left[\cos^2\left(\dfrac{x-3}{x+2}\right)^2\right]\right\}.\left[\cos^2\left(\dfrac{x-3}{x+2}\right)^2\right]'$

Tính $\left[\cos^2\left(\frac{x-3}{x+2}\right)^2\right]'$ áp dụng $(u^\alpha)'$ với $u=\cos\left(\frac{x-3}{x+2}\right)^2$

$\Rightarrow 2.\cos\left(\frac{x-3}{x+2}\right)^2.\left[\cos\left(\frac{x-3}{x+2}\right)^2\right]'$

Tính $\left[\cos\left(\frac{x-3}{x+2}\right)^2\right]'$ áp dụng $(\cos u)'$ với $u=\left(\frac{x-3}{x+2}\right)$ được:

$-\sin\left(\frac{x-3}{x+2}\right)^2.\left[\left(\frac{x-3}{x+2}\right)^2\right]'$

Tính $\left[\left(\frac{x-3}{x+2}\right)^2\right]'$ áp dụng $(u^\alpha)'$ với $u=\frac{x-3}{x+2}$ được:

$2\left(\frac{x-3}{x+2}\right).\left(\frac{x-3}{x+2}\right)'=2.\left(\frac{x-3}{x-2}\right).\frac{5}{(x-2)^2}=\frac{10(x-3)}{(x-2)^3}$

Vậy $y'=?$

Câu : Tính:

1). Cho $f(x)=\frac{1}{x}+\frac{2}{x^2}+\frac{3}{x^3}$. Tính $f'(-1)$. 2). Cho $f(x)=\frac{1}{x}+\frac{1}{\sqrt{x}}+x^2$. Tính $f'(1)$

3). Cho $f(x)=x^5+x^3-2x-3$. Tính $f'(1)+f'(-1)+4f(0)$

4). Cho $f(x)=\frac{x}{\sqrt{4-x^2}}$. Tính $f'(0)$

LỜI GIẢI

1). Cho $f(x)=\frac{1}{x}+\frac{2}{x^2}+\frac{3}{x^3}$. Tính $f'(-1)$.

Bước đầu tiên tính đạo hàm sử dụng công thức $\left(\frac{1}{x^\alpha}\right)'=\frac{-\alpha}{x^{\alpha+1}}$

$f'(x)=\left(\frac{1}{x}+\frac{2}{x^2}+\frac{3}{x^3}\right)'=-\frac{1}{x^2}-\frac{4}{x^3}-\frac{9}{x^4}\Rightarrow f'(1)=-1-4-9=-14$

2). Cho $f(x)=\frac{1}{x}+\frac{1}{\sqrt{x}}+x^2$. Tính $f'(1)$

Ta có $f'(x)=\left(\frac{1}{x}+\frac{1}{\sqrt{x}}+x^2\right)'=-\frac{1}{x^2}-\frac{(\sqrt{x})'}{x}+2x=-\frac{1}{x^2}-\frac{1}{2x\sqrt{x}}+2x$

Vậy $f'(1)=-1-\frac{1}{2}+2=\frac{1}{2}$

3). Cho $f(x)=x^5+x^3-2x-3$. Tính $f'(1)+f'(-1)+4f(0)$

Ta có $f'(x)=(x^5+x^3-2x-3)'=5x^4+3x^2-2$

$f'(1)+f'(-1)+4f(0)=(5+3-2)+(5+3-2)+4.(-2)=4$

4). Cho $f(x)=\frac{x}{\sqrt{4-x^2}}$. Tính $f'(0)$

$$f'(x)=\left(\frac{x}{\sqrt{4-x^2}}\right)'=\frac{x'\sqrt{4-x^2}-x\left(\sqrt{4-x^2}\right)'}{\left(\sqrt{4-x^2}\right)^2}=\frac{\sqrt{4-x^2}+\frac{x^2}{\sqrt{4-x^2}}}{\left(4-x^2\right)}=\frac{4}{\left(4-x^2\right)\sqrt{4-x^2}}$$

Vậy $f'(0)=\frac{1}{4}$.

Cho $f(x)=\frac{x^3}{3}+\frac{x^2}{2}-2x$. Với những giá trị nào của x thì:

a. $f'(x)=0$ b. $f'(x)=-2$ c. $f'(x)=10$

LỜI GIẢI

Ta có $f'(x)=\left(\frac{x^3}{3}+\frac{x^2}{2}-2x\right)=x^2+x-2$

a). $f'(x)=0\Leftrightarrow x^2+x-2=0\Leftrightarrow x=1\vee x=-2$

b). $f'(x)=-2\Leftrightarrow x^2+x-2=-2\Leftrightarrow x^2+x=0\Leftrightarrow x=0\vee x=-1$

c). $f'(x)=10\Leftrightarrow x^2+x-2=10\Leftrightarrow x^2+x-12=0\Leftrightarrow x=3\vee x=-4$

Câu : Giải

a). Cho $f(x)=2x^3+x-\sqrt{2}, g(x)=3x^2+x+\sqrt{2}$. Giải bất phương trình $f'(x)>g'(x)$.

b).Cho $f(x)=2x^3-x^2+\sqrt{3}, g(x)=x^3+\frac{x^2}{2}-\sqrt{3}$. Giải bất phương trình $f'(x)>g'(x)$.

Cho $f(x)=3x+\frac{60}{x}-\frac{64}{x^3}+5$. Giải phương trình $f'(x)=0$

LỜI GIẢI

a). Ta có $f'(x)=\left(2x^3+x-\sqrt{2}\right)'=6x^2+1$, $g'(x)=\left(3x^2+x+\sqrt{2}\right)'=6x+1$

$f'(x)>g'(x)\Leftrightarrow 6x^2+1>6x+1\Leftrightarrow 6x^2-6x>0\Leftrightarrow x\in(-\infty;0)\cup(1;+\infty)$

b). $f'(x)=\left(2x^3-x^2+\sqrt{3}\right)'=6x^2-2x$, $g'(x)=\left(x^3+\frac{x^2}{2}-\sqrt{3}\right)'=3x^2+x$

$f'(x)>g'(x)\Leftrightarrow 6x^2-2x>3x^2+x\Leftrightarrow 3x^2-3x>0\Leftrightarrow x\in(-\infty;0)\cup(1;+\infty)$

c). Ta có $f'(x)=\left(3x+\frac{60}{x}-\frac{64}{x^3}+5\right)'=3-\frac{60}{x^2}+\frac{192}{x^4}$

$f'(x)=0\Leftrightarrow 3-\frac{60}{x^2}+\frac{192}{x^4}=0\ (1)$. Đặt $t=\frac{1}{x^2},(t>0)$

$(1)\Leftrightarrow 192t^2-60t+3=0\Leftrightarrow t=\frac{1}{4}\vee t=\frac{1}{16}$

Với $t=\frac{1}{4}\Leftrightarrow\frac{1}{x^2}=\frac{1}{4}\Leftrightarrow x^2=4\Leftrightarrow x=\pm2$

Với $t=\frac{1}{16}\Leftrightarrow\frac{1}{x^2}=\frac{1}{16}\Leftrightarrow x^2=16\Leftrightarrow x=\pm4$

Vậy $f'(x)=0$ có 4 nghiệm $x=\pm2$, $x=\pm4$

Bài 10: Tính đạo hàm của các hàm số sau:

a). $y=3\left(\sin^4x+\cos^4x\right)-2\left(\sin^{6\pi}x+\cos^6x\right)$

b). $y=\cos^4x\left(2\cos^2x-3\right)+\sin^4x\left(2\sin^2x-3\right)$

LỜI GIẢI

a). $y=3\left(\sin^4x+\cos^4x\right)-2\left(\sin^{6\pi}x+\cos^6x\right)$

Ta có $y=3\left(1-\frac{1}{2}\sin^2 2x\right)-2\left(1-\frac{3}{4}\sin^2 2x\right)=1$

Vậy $y'=0$.

b). $y=\cos^4x\left(2\cos^2x-3\right)+\sin^4x\left(2\sin^2x-3\right)$

$y=2\cos^6x-3\cos^4x+2\sin^6x-3\sin^4x=2\left(\sin^6x+\cos^6x\right)-3\left(\sin^4x+\cos^4x\right)$

$=2\left(1-\frac{3}{4}\sin^2 2x\right)-3\left(1-\frac{1}{2}\sin^2 2x\right)=1$

$\Rightarrow y'=0.$

Bài 11: Chứng minh

a). Cho $y=\tan x$ chứng minh $y'-y^2-1=0$ $(*)$

b). $y=\cot 2x$ chứng minh : $y'+2y^2+2=0$ $(*)$

c). Cho $y=x\sin x$ chứng minh: $x.y-2\left(y'-\sin x\right)+x\left(2\cos x-y\right)=0$

d). Cho hàm số $y=\sqrt{x+\sqrt{1+x^2}}$. Chứng minh: $2\sqrt{1+x^2}.y'=y$ $(*)$

LỜI GIẢI

a). Cho $y=\tan x$ chứng minh $y'-y^2-1=0$ $(*)$

Ta có: $y'=(\tan x)'=1+\tan^2x$

$(*)\Leftrightarrow 1+\tan^2x-\tan^2x-1=0$ (đúng) (đpcm).

b). $y=\cot 2x$ chứng minh : $y'+2y^2+2=0$ $(*)$

Ta có: $y'=(\cot 2x)'=-2\left(1+\cot^2 2x\right)$

$(*)\Leftrightarrow -2\left(1+\cot^2 2x\right)+2\cot^2 2x+2=0$ (đpcm).

c). $y=x\sin x$ chứng minh: $x.y-2\left(y'-\sin x\right)+x\left(2\cos x-y\right)=0$ $(*)$

Ta có: $y'=(x\sin x)'=x'.\sin x+x.(\sin x)'=\sin x+x\cos x.$

$(*)\Leftrightarrow x^2.\sin x-2\left(\sin x+x\cos x-\sin x\right)+x\left(2\cos x-x\sin x\right)=0$

$\Leftrightarrow x^2\sin x-2x\cos x+2x\cos x-x^2\sin x=0\Leftrightarrow 0=0$ (đpcm).

d). Cho hàm số $y=\sqrt{x+\sqrt{1+x^2}}$. Chứng minh: $2\sqrt{1+x^2}.y'=y$ $(*)$

Ta có: $y'=\left(\sqrt{x+\sqrt{1+x^2}}\right)'=\frac{1}{2\sqrt{x+\sqrt{1+x^2}}}.\left(x+\sqrt{1+x^2}\right)'=\frac{1}{2\sqrt{x+\sqrt{1+x^2}}}.\left(1+\frac{x}{\sqrt{1+x^2}}\right)$

$$=\frac{1}{2\sqrt{x+\sqrt{1+x^2}}}.\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}=\frac{\sqrt{\sqrt{1+x^2}+x}}{2\sqrt{1+x^2}}.$$

$(*)\Leftrightarrow 2\sqrt{1+x^2}.\frac{\sqrt{x+\sqrt{1+x^2}}}{2\sqrt{1+x^2}}=\sqrt{x+\sqrt{1+x^2}}\Leftrightarrow\sqrt{x+\sqrt{1+x^2}}=\sqrt{x+\sqrt{1+x^2}}$ (đpcm).