PHƯƠNG TRÌNH ĐỐI XỨNG ĐỐI VỚI sin VÀ cos

4)PHƯƠNG TRÌNH DẠNG : $a\left(\sin x \pm \cos x\right)+b\sin x\cos x+c=0\left(1\right)$

Cách giải.Đặt :

$t=\sin x+\cos x \Rightarrow \sin x\cos x=\dfrac{t^2-1}{2}\left(-\sqrt{2}\le t\le\sqrt{2}\right)$ $t=\sin x-\cos x \Rightarrow \sin x\cos x=\dfrac{1-t^2}{2}\left(-\sqrt{2}\le t\le\sqrt{2}\right)$

$t=\cos x-\sin x \Rightarrow \sin x\cos x=\dfrac{1-t^2}{2}\left(-\sqrt{2}\le t\le\sqrt{2}\right)$

Thay vào (1) rồi giải phuong trình bậc 2 theo t.

BÀI TẬP

Bài 1: Giải các phương trình sau:

1). $2\sin 2x-3\sqrt{3}\left(\sin x+\cos x\right)+5=0$ 2). $2(\sin x+\cos x)+6\sin x\cos -2=0.$

3). $2\sqrt{2}\left(\sin x-\cos x\right)-2\sin 2x=1$ 4). $\sin x+\cos x-4\sin x\cos x-1=0.$

5). $\sin x\cos x-\sqrt{2}(\sin x+\cos x)+1=0.$ 6). $\sin x\cos x=6(\sin x-\cos x)-1.$

7). $\sin x-\cos x=2\sqrt{6}\sin x\cos x.$ 8). $2\sqrt{2}(\sin x-\cos x)=3-\sin 2x.$

9). $2\sin 2x+3\sqrt{3}(\sin x+\cos x)+5=0.$ 10). $(1-\sqrt{2})(1+\sin x-\cos x)=\sin 2x$.

11). $(1+\sqrt{2})(\sin x+\cos x)-2\sin x\cos x-1-\sqrt{2}=0.$

LỜI GIẢI

1). $2\sin 2x-3\sqrt{3}\left(\sin x+\cos x\right)+5=0$

Đặt $t=\sin x+\cos x\Rightarrow t^2=\left(\sin x+\cos x\right)^2\Rightarrow \sin 2x=t^2-1$, điều kiện $|t|\le\sqrt{2}$

$\left(1\right)\Leftrightarrow 2\left(t^2-1\right)-3\sqrt{3}t+5=0\Leftrightarrow 2t^2-3\sqrt{3}t+3=0\Leftrightarrow t=\sqrt{3}\vee t=\dfrac{\sqrt{3}}{2}$

So với điều kiện nhận $t=\dfrac{\sqrt{3}}{2}$.

$\Leftrightarrow\sqrt{2}\cos\left(x-\dfrac{\pi}{4}\right)=\dfrac{\sqrt{3}}{2}\Leftrightarrow\cos\left(x-\dfrac{\pi}{4}\right)=\dfrac{\sqrt{6}}{4}$

$$\Leftrightarrow\left[\begin{array}{l}x-\dfrac{\pi}{4}=\arccos\dfrac{\sqrt{6}}{4}+k2\pi\\ x-\dfrac{\pi}{4}=-\arccos\dfrac{\sqrt{6}}{4}+k2\pi\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=\dfrac{\pi}{4}+\arccos\dfrac{\sqrt{6}}{4}+k2\pi\\ x=\dfrac{\pi}{4}-\arccos\dfrac{\sqrt{6}}{4}+k2\pi\end{array}\right.,\left(k\in\mathbb{Z}\right)$$

Nghiệm phương trình: $x=\dfrac{\pi}{4}+\arccos\dfrac{\sqrt{6}}{4}+k2\pi, x=\dfrac{\pi}{4}-\arccos\dfrac{\sqrt{6}}{4}+k2\pi,\left(k\in\mathbb{Z}\right)$

2). $2(\sin x+\cos x)+6\sin x\cos -2=0.$

Đặt $t=\sin x+\cos x\Rightarrow t^2=\left(\sin x+\cos x\right)^2\Rightarrow \sin x\cos x=\dfrac{t^2-1}{2}$, điều kiện $|t|\le\sqrt{2}$

Ta được : $2t+3(t^2-1)-2=0\Leftrightarrow 3t^2+2t-5=0\Leftrightarrow t=1\vee t=-\dfrac{5}{3}$ (loại).

Với $t=1\Leftrightarrow \sin x.\cos x=1$ $\Leftrightarrow\sqrt{2}\sin\left(x+\dfrac{\pi}{4}\right)=1$ $\Leftrightarrow\sin\left(x+\dfrac{\pi}{4}\right)=\dfrac{1}{\sqrt{2}}$

$$\Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=\sin\frac{\pi}{4} \Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{4}=\frac{\pi}{4}+k2\pi \\ x+\frac{\pi}{4}=\pi-\frac{\pi}{4}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=k2\pi \\ x=\frac{\pi}{2}+k2\pi \end{array}\right.,(k\in\mathbb{Z})$$

Vậy nghiệm của phương trình: $x=k2\pi, x=\frac{\pi}{2}+k2\pi,(k\in\mathbb{Z})$

3). $2\sqrt{2}(\sin x-\cos x)-2\sin 2x=1 \ \ (1)$

Đặt $t=\sin x-\cos x \Rightarrow t^2=(\sin x-\cos x)^2 \Rightarrow \sin 2x=1-t^2$, điều kiện $|t|\le\sqrt{2}$

$$(1)\Leftrightarrow 2\sqrt{2}t-2(1-t^2)=1 \Leftrightarrow 2t^2+2\sqrt{2}t-3=0 \Leftrightarrow t=\frac{\sqrt{2}}{2} \vee t=-\frac{3\sqrt{2}}{2}$$

So với điều kiện nhận $t=\frac{\sqrt{2}}{2}$. Suy ra $\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$

$$\Leftrightarrow \sin\left(x-\frac{\pi}{4}\right)=\frac{1}{2} \Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{4}=\frac{\pi}{6}+k2\pi \\ x-\frac{\pi}{4}=\frac{5\pi}{6}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{5\pi}{12}+k2\pi \\ x=\frac{13\pi}{12}+k2\pi \end{array}\right.,(k\in\mathbb{Z})$$

Vậy nghiệm phương trình $x=\frac{5\pi}{12}+k2\pi, x=\frac{13\pi}{12}+k2\pi,(k\in\mathbb{Z})$.

4). $\sin x+\cos x-4\sin x\cos x-1=0.\ (1)$

Đặt $t=\sin x+\cos x$ (ĐK: $|t|\le\sqrt{2}$) $\Rightarrow t^2=(\sin x+\cos x)^2 \Rightarrow \sin x.\cos x=\frac{t^2-1}{2}$

$$(1)\Leftrightarrow t-2(t^2-1)-1=0 \Leftrightarrow -2t^2+t+1=0 \Leftrightarrow t=1 \vee t=-\frac{1}{2}$$

Với $t=1 \Leftrightarrow \sin x+\cos x=1 \Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=1 \Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

$$\Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=\sin\frac{\pi}{4} \Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{4}=\frac{\pi}{4}+k2\pi \\ x+\frac{\pi}{4}=\pi-\frac{\pi}{4}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=k2\pi \\ x=\frac{\pi}{2}+k2\pi \end{array}\right.(k\in\mathbb{Z})$$

Với $t=-\frac{1}{2} \Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=-\frac{1}{2} \Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=-\frac{1}{2\sqrt{2}}$

$$\Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{4}=\arcsin\left(-\frac{1}{2\sqrt{2}}\right)+k2\pi \\ x+\frac{\pi}{4}=\pi-\arcsin\left(-\frac{1}{2\sqrt{2}}\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\frac{\pi}{4}+\arcsin\left(-\frac{1}{2\sqrt{2}}\right)+k2\pi \\ x=\frac{3\pi}{4}-\arcsin\left(-\frac{1}{2\sqrt{2}}\right)+k2\pi \end{array}\right.,(k\in\mathbb{Z})$$

Nghiệm của phương trình: $x=k2\pi, x=\frac{\pi}{2}+k2\pi$, $x=-\frac{\pi}{4}+\arcsin\left(-\frac{1}{2\sqrt{2}}\right)+k2\pi$,

$x=\frac{3\pi}{4}-\arcsin\left(-\frac{1}{2\sqrt{2}}\right)+k2\pi,(k\in\mathbb{Z})$.

5). $\sin x\cos x-\sqrt{2}(\sin x+\cos x)+1=0.$

Đặt $t=\sin x+\cos x$ (ĐK: $|t|\le\sqrt{2}$) $\Rightarrow t^2=(\sin x+\cos x)^2 \Rightarrow \sin x.\cos x=\frac{t^2-1}{2}$

Ta được : $\frac{t^2-1}{2}-\sqrt{2}t+1=0 \Leftrightarrow t^2-2\sqrt{2}t+1=0 \Leftrightarrow \left[\begin{array}{l} t=1+\sqrt{2} \\ t=-1+\sqrt{2} \end{array}\right.$

So với điều kiện chọn $t=-1+\sqrt{2}$. Có nghĩa $\sin x+\cos x=-1+\sqrt{2}$

$\Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=-1+\sqrt{2} \Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=\frac{2-\sqrt{2}}{2}$

$\Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{4}=\arcsin\left(\frac{2-\sqrt{2}}{2}\right)+k2\pi \\ x+\frac{\pi}{4}=\pi-\arcsin\left(\frac{2-\sqrt{2}}{2}\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\frac{\pi}{4}+\arcsin\left(\frac{2-\sqrt{2}}{2}\right)+k2\pi \\ x=\frac{3\pi}{4}-\arcsin\left(\frac{2-\sqrt{2}}{2}\right)+k2\pi \end{array}\right. (k\in\mathbb{Z})$.

Nghiệm của phương trình: $x=-\frac{\pi}{4}+\arcsin\left(\frac{2-\sqrt{2}}{2}\right)+k2\pi, x=\frac{3\pi}{4}-\arcsin\left(\frac{2-\sqrt{2}}{2}\right)+k2\pi, (k\in\mathbb{Z})$

6). $\sin x\cos x=6(\sin x-\cos x)-1.$ (1)

Đặt $t=\sin x-\cos x$ (ĐK : $|t|\le\sqrt{2}$) $\Rightarrow t^2=(\sin x-\cos x)^2 \Rightarrow \sin x\cos x=\frac{1-t^2}{2}$

(1): $\frac{1-t^2}{2}=6t-1 \Leftrightarrow t^2+12t-3=0 \Leftrightarrow t=-6+\sqrt{39} \vee t=-6-\sqrt{39}$ (loại).

Với $t=-6+\sqrt{39} \Leftrightarrow \sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=-6+\sqrt{39} \Leftrightarrow \sin\left(x-\frac{\pi}{4}\right)=\frac{-6+\sqrt{39}}{\sqrt{2}}$

$\Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{4}=\arcsin\left(\frac{-6+\sqrt{39}}{\sqrt{2}}\right)+k2\pi \\ x-\frac{\pi}{4}=\pi-\arcsin\left(\frac{-6-\sqrt{39}}{\sqrt{2}}\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{4}+\arcsin\left(\frac{-6+\sqrt{39}}{\sqrt{2}}\right)+k2\pi \\ x=\frac{5\pi}{4}-\arcsin\left(\frac{-6-\sqrt{39}}{\sqrt{2}}\right)+k2\pi \end{array}\right.$

Nghiệm của phương trình:

$x=\frac{\pi}{4}+\arcsin\left(\frac{-6+\sqrt{39}}{\sqrt{2}}\right)+k2\pi, x=\frac{5\pi}{4}-\arcsin\left(\frac{-6-\sqrt{39}}{\sqrt{2}}\right)+k2\pi, (k\in\mathbb{Z})$

7). $\sin x-\cos x=2\sqrt{6}\sin x\cos x.$ (1)

Đặt $t=\sin x-\cos x$ (Đk: $|t|\le\sqrt{2}$) $\Leftrightarrow t^2=(\sin x-\cos x)^2 \Leftrightarrow \sin x\cos x=\frac{1-t^2}{2}$

$(1)\Leftrightarrow t=2\sqrt{6}\frac{1-t^2}{2} \Leftrightarrow \sqrt{6}t^2+t-\sqrt{6}=0 \Leftrightarrow t=\frac{\sqrt{6}}{3} \vee t=-\frac{\sqrt{6}}{2}$

Với $t=\frac{\sqrt{6}}{3} \Leftrightarrow \sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=\frac{\sqrt{6}}{3} \Leftrightarrow \sin\left(x-\frac{\pi}{4}\right)=\frac{\sqrt{3}}{3}$

$\Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{4}=\arcsin\left(\frac{\sqrt{3}}{3}\right)+k2\pi \\ x-\frac{\pi}{4}=\pi-\arcsin\left(\frac{\sqrt{3}}{3}\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{4}+\arcsin\left(\frac{\sqrt{3}}{3}\right)+k2\pi \\ x=\frac{5\pi}{4}-\arcsin\left(\frac{\sqrt{3}}{3}\right)+k2\pi \end{array}\right.$

Với $t=-\frac{\sqrt{6}}{2} \Leftrightarrow \sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=-\frac{\sqrt{6}}{2} \Leftrightarrow \sin\left(x-\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{2}$

$$\Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{4}=-\frac{\pi}{3}+k2\pi \\ x-\frac{\pi}{4}=\pi+\frac{\pi}{3}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\frac{\pi}{12}+k2\pi \\ x=\frac{19\pi}{12}+k2\pi \end{array}\right. (k\in\mathbb{Z}).$$

Nghiệm của phương trình: $x=-\frac{\pi}{12}+k2\pi, x=\frac{19\pi}{12}+k2\pi$,

$(k\in\mathbb{Z}) x=\frac{\pi}{4}+\arcsin\left(\frac{\sqrt{3}}{3}\right)+k2\pi, x=\frac{5\pi}{4}-\arcsin\left(\frac{\sqrt{3}}{3}\right)+k2\pi, (k\in\mathbb{Z})$

8). $2\sqrt{2}(\sin x-\cos x)=3-\sin 2x.$

Đặt $t=\sin x-\cos x$ (Đk : $|t|\le\sqrt{2}$) $\Leftrightarrow$ $t^2=(\sin-\cos x)^2$ $\Leftrightarrow$ $\sin 2x=1-t^2$

Ta được : $2\sqrt{2}t=3-(1-t^2) \Leftrightarrow t^2-2\sqrt{2}t+2=0 \Leftrightarrow t=\sqrt{2} \Leftrightarrow \sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=\sqrt{2}$

$\Leftrightarrow \sin\left(x-\frac{\pi}{4}\right)=1 \Leftrightarrow x-\frac{\pi}{4}=\frac{\pi}{2}+k2\pi \Leftrightarrow x=\frac{3\pi}{4}+k2\pi \ (k\in\mathbb{Z})$

Nghiệm của phương trình: $x=\frac{3\pi}{4}+k2\pi \ (k\in\mathbb{Z})$

9). $2\sin 2x+3\sqrt{3}(\sin x+\cos x)+5=0.$

Đặt $t=\sin x+\cos x$ (Đk : $|t|\le\sqrt{2}$) $\Leftrightarrow$ $t^2=(\sin x+\cos x)^2$ $\Leftrightarrow$ $\sin 2x=t^2-1$

Ta được : $2(t^2-1)+3\sqrt{3}t+5=0 \Leftrightarrow 2t^2+3\sqrt{3}t+3=0 \Leftrightarrow t=-\sqrt{3} \vee t=-\frac{\sqrt{3}}{2}$

So với điều kiện $t=-\frac{\sqrt{3}}{2}$ nhận.

Với $t=-\frac{\sqrt{3}}{2} \Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{2} \Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=-\frac{\sqrt{6}}{4}$

$$\Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{4}=\arcsin\left(-\frac{\sqrt{6}}{4}\right)+k2\pi \\ x+\frac{\pi}{4}=\pi-\arcsin\left(-\frac{\sqrt{6}}{4}\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\frac{\pi}{4}+\arcsin\left(-\frac{\sqrt{6}}{4}\right)+k2\pi \\ x=\frac{3\pi}{4}-\arcsin\left(-\frac{\sqrt{6}}{4}\right)+k2\pi \end{array}\right. (k\in\mathbb{Z})$$

Nghiệm của phương trình:

$x=-\frac{\pi}{4}+\arcsin\left(-\frac{\sqrt{6}}{4}\right)+k2\pi, x=\frac{3\pi}{4}-\arcsin\left(-\frac{\sqrt{6}}{4}\right)+k2\pi, (k\in\mathbb{Z})$

10). $(1-\sqrt{2})(1+\sin x-\cos x)=\sin 2x$.

Đặt $t=\sin x-\cos x$ (Đk : $|t|\le\sqrt{2}$) $\Leftrightarrow$ $\sin 2x=1-t^2$

Ta được : $(1-\sqrt{2})(1+t)=1-t^2 \Leftrightarrow t^2+(1-\sqrt{2})t-\sqrt{2}=0 \Leftrightarrow t=\sqrt{2} \vee t=-1$

Với $t=\sqrt{2} \Leftrightarrow \sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=\sqrt{2} \Leftrightarrow \sin\left(x-\frac{\pi}{4}\right)=1$

$\Leftrightarrow x-\frac{\pi}{4}=\frac{\pi}{2}+k2\pi \Leftrightarrow x=\frac{3\pi}{4}+k2\pi \ (k\in\mathbb{Z})$

Với $t=-1 \Leftrightarrow \sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=-1 \Leftrightarrow \sin\left(x-\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}$

$$\Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{4}=-\frac{\pi}{4}+k2\pi \\ x-\frac{\pi}{4}=\pi+\frac{\pi}{4}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=k2\pi \\ x=\frac{3\pi}{2}+k2\pi \end{array}\right.(k\in\mathbb{Z})$$

Nghiệm của phương trình: $x=\frac{3\pi}{4}+k2\pi,\ x=k2\pi, x=\frac{3\pi}{2}+k2\pi,(k\in\mathbb{Z})$

11). $(1+\sqrt{2})(\sin x+\cos x)-2\sin x\cos x-1-\sqrt{2}=0.$

Đặt $t=\sin x+\cos x$ (Đk : $|t|\le\sqrt{2}$ $\Rightarrow \sin x\cos x=\frac{t^2-1}{2}$. Ta được : $(1+\sqrt{2})t-(t^2-1)-1-\sqrt{2}=0 \Leftrightarrow$

$-t^2+(1+\sqrt{2})t-\sqrt{2}=0 \Leftrightarrow t=\sqrt{2}\vee t=1$

Với $t=\sqrt{2} \Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=\sqrt{2} \Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=1$

$\Leftrightarrow x+\frac{\pi}{4}=\frac{\pi}{2}+k2\pi \Leftrightarrow x=\frac{\pi}{4}+k2\pi\ (k\in\mathbb{Z})$

Với $t=1 \Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=1 \Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

$$\Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{4}=\frac{\pi}{4}+k2\pi \\ x+\frac{\pi}{4}=\pi-\frac{\pi}{4}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=k2\pi \\ x=\frac{\pi}{2}+k2\pi \end{array}\right.(k\in\mathbb{Z})$$

Nghiệm của phương trình: $x=\frac{\pi}{4}+k2\pi,\ x=k2\pi, x=\frac{\pi}{2}+k2\pi,(k\in\mathbb{Z})$

Bài 2: Giải các phương trình sau:

1). $\frac{1}{\sin x}+\frac{1}{\cos x}=2\sqrt{2}.$ 2). $\sin x-2\sin 2x=\frac{1}{2}-\cos x.$

3). $\sin 2x+\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=1.$ 4). $2\sin 2x-3\sqrt{6}|\sin x+\cos x|+8=0.$

5). $\cot x-1=\frac{\cos 2x}{1+\tan x}+\sin^2 x-\frac{1}{2}\sin 2x$ (1) [ĐH A03]

6). $\frac{1}{\cos x}-\frac{1}{\sin x}=2\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)$ (1) [Dự bị 2 ĐH B04]

7) $\sin 2x-2\sqrt{2}(\sin x+\cos x)-5=0$ [Dự bị 2 ĐH D04]

LỜI GIẢI

1). $\frac{1}{\sin x}+\frac{1}{\cos x}=2\sqrt{2}.$ Điều kiện : $\begin{cases}\sin x\ne 0\\ \cos x\ne 0\end{cases} \Leftrightarrow \begin{cases}x\ne k\pi\\ x\ne\frac{\pi}{2}+k\pi\end{cases}(k\in\mathbb{Z})$

Phương trình $\Leftrightarrow \sin x+\cos x=2\sqrt{2}\sin x\cos x$

Đặt $t=\sin x+\cos x$ (Đk : $|t|\le\sqrt{2}$) $\Rightarrow \sin x\cos x=\frac{t^2-1}{2}$

Ta được : $t=\sqrt{2}(t^2-1) \Leftrightarrow \sqrt{2}t^2-t-\sqrt{2}=0 \Leftrightarrow t=\sqrt{2}\vee t=-\frac{\sqrt{2}}{2}$

Với $t=\sqrt{2} \Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=\sqrt{2} \Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=1$

$\Leftrightarrow x+\frac{\pi}{4}=\frac{\pi}{2}+k2\pi \Leftrightarrow x=\frac{\pi}{4}+k2\pi \ (k\in\mathbb{Z})$

Với $t=-\frac{\sqrt{2}}{2} \Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2} \Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=-\frac{1}{2}$

$$\Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{4}=-\frac{\pi}{6}+k2\pi \\ x+\frac{\pi}{4}=\pi+\frac{\pi}{6}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\frac{5\pi}{12}+k2\pi \\ x=\frac{11\pi}{12}+k2\pi \end{array}\right. (k\in\mathbb{Z})$$

So với điều kiện phương trình có các nghiệm: $x=\frac{\pi}{4}+k2\pi;\ x=-\frac{5\pi}{12}+k2\pi;\ x=\frac{11\pi}{12}+k2\pi,(k\in\mathbb{Z})$

2). $\sin x-2\sin 2x=\frac{1}{2}-\cos x.$

$\Leftrightarrow 2(\sin x+\cos x)-4\sin 2x=1$.

Đặt $t=\sin x+\cos x$ (Đk : $|t|\le\sqrt{2}$) $\Rightarrow \sin 2x=t^2-1$

Ta được : $2t-4(t^2-1)=1 \Leftrightarrow 4t^2-2t-3=0 \Leftrightarrow t=\frac{1+\sqrt{13}}{4}\vee t=\frac{1-\sqrt{13}}{4}$

Với $t=\frac{1+\sqrt{13}}{4} \Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=\frac{1+\sqrt{13}}{4} \Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{2}+\sqrt{26}}{8}$

$$\Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{4}=\arcsin\left(\frac{\sqrt{2}+\sqrt{26}}{8}\right)+k2\pi \\ x+\frac{\pi}{4}=\pi-\arcsin\left(\frac{\sqrt{2}-\sqrt{26}}{8}\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\frac{\pi}{4}+\arcsin\left(\frac{\sqrt{2}+\sqrt{26}}{8}\right)+k2\pi \\ x=\frac{3\pi}{4}-\arcsin\left(\frac{\sqrt{2}-\sqrt{26}}{8}\right)+k2\pi \end{array}\right.$$

Với $t=\frac{1-\sqrt{13}}{4} \Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=\frac{1-\sqrt{13}}{4} \Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{2}-\sqrt{26}}{8}$

$$\Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{4}=\arcsin\left(\frac{\sqrt{2}-\sqrt{26}}{8}\right)+k2\pi \\ x+\frac{\pi}{4}=\pi-\arcsin\left(\frac{\sqrt{2}-\sqrt{26}}{8}\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\frac{\pi}{4}+\arcsin\left(\frac{\sqrt{2}-\sqrt{26}}{8}\right)+k2\pi \\ x=\frac{3\pi}{4}-\arcsin\left(\frac{\sqrt{2}-\sqrt{26}}{8}\right)+k2\pi \end{array}\right.$$

Nghiệm của phương trình: $x=-\frac{\pi}{4}+\arcsin\left(\frac{\sqrt{2}+\sqrt{26}}{8}\right)+k2\pi\ x=\frac{3\pi}{4}-\arcsin\left(\frac{\sqrt{2}-\sqrt{26}}{8}\right)+k2\pi$,

$x=-\frac{\pi}{4}+\arcsin\left(\frac{\sqrt{2}-\sqrt{26}}{8}\right)+k2\pi$, $x=\frac{3\pi}{4}-\arcsin\left(\frac{\sqrt{2}-\sqrt{26}}{8}\right)+k2\pi$, $(k\in\mathbb{Z})$

3). $\sin 2x+\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=1.$

$\Leftrightarrow \sin 2x+(\sin x-\cos x)=1$

Đặt $t=\sin x-\cos x$ (Đk : $|t|\le\sqrt{2}$) $\Rightarrow \sin 2x=1-t^2$

Ta được : $1-t^2+t=1 \Leftrightarrow -t^2+t=0 \Leftrightarrow t=1\vee t=0$

Với $t=1 \Leftrightarrow \sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=1 \Leftrightarrow \sin\left(x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

$$\Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{4}=\frac{\pi}{4}+k2\pi \\ x-\frac{\pi}{4}=\pi-\frac{\pi}{4}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{2}+k2\pi \\ x=\pi+k2\pi \end{array}\right.(k\in\mathbb{Z})$$

Với $t=0 \Leftrightarrow \sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=0 \Leftrightarrow x-\frac{\pi}{4}=k\pi \Leftrightarrow x=\frac{\pi}{4}+k\pi$.

Nghiệm của phương trình: $x=\frac{\pi}{2}+k2\pi, x=\pi+k2\pi$, $x=\frac{\pi}{4}+k\pi,(k\in\mathbb{Z})$

4). $2\sin 2x-3\sqrt{6}\left|\sin x+\cos x\right|+8=0.$

Đặt $t=\left|\sin x+\cos x\right|$ (Đk : $0\le t\le\sqrt{2}$) $\Rightarrow t^2=(\sin x+\cos x)^2 \Rightarrow \sin 2x=t^2-1$

Ta được : $2(t^2-1)-3\sqrt{6}t+8=0 \Leftrightarrow 2t^2-3\sqrt{6}+8=0 \Leftrightarrow t=\frac{\sqrt{6}}{2} \vee t=\sqrt{6}$ (loại).

Với $t=\frac{\sqrt{6}}{2} \Leftrightarrow \left|\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)\right|=\frac{\sqrt{6}}{2} \Leftrightarrow \left|\sin\left(x+\frac{\pi}{4}\right)\right|=\frac{\sqrt{3}}{2} \Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=\pm\frac{\sqrt{3}}{2}$

Với $\sin\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2} \Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{4}=\frac{\pi}{3}+k2\pi \\ x+\frac{\pi}{4}=\pi-\frac{\pi}{3}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{12}+k2\pi \\ x=\frac{5\pi}{12}+k2\pi \end{array}\right.,(k\in\mathbb{Z})$

Với $\sin\left(x+\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{2} \Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{4}=-\frac{\pi}{3}+k2\pi \\ x+\frac{\pi}{4}=\pi+\frac{\pi}{3}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\frac{7\pi}{12}+k2\pi \\ x=\frac{13\pi}{12}+k2\pi \end{array}\right.,(k\in\mathbb{Z})$

Nghiệm của phương trình: $x=\frac{\pi}{12}+k2\pi, x=\frac{5\pi}{12}+k2\pi$, $x=-\frac{7\pi}{12}+k2\pi$,$x=\frac{13\pi}{12}+k2\pi,(k\in\mathbb{Z})$

5). $\cot x-1=\frac{\cos 2x}{1+\tan x}+\sin^2 x-\frac{1}{2}\sin 2x$ (1) [ĐH A03]

LỜI GIẢI

Điều kiện : $\begin{cases} \sin 2x\ne 0 \\ \tan x\ne -1 \end{cases}$

$(1) \Leftrightarrow \frac{\cos x}{\sin x}-1=\frac{\cos^2 x-\sin^2 x}{1+\frac{\sin x}{\cos x}}+\sin x(\sin x-\cos x)$

$\Leftrightarrow \frac{\cos x-\sin x}{\sin x}=\frac{\cos x(\cos^2 x-\sin^2 x)}{\sin x+\cos x}+\sin x(\sin x-\cos x)$

$\Leftrightarrow \frac{\cos x-\sin x}{\sin x}=\cos x(\cos x-\sin x)+\sin x(\sin x-\cos x)$

$\Leftrightarrow (\cos x-\sin x)\left(\sin^2 x-\sin x\cos x+1\right)=0$

$$\Leftrightarrow \left[\begin{array}{l} \cos x-\sin x=0 \\ \sin^2 x-\sin x\cos x+1=0 \end{array}\right.$$

* $\cos x-\sin x=0 \Leftrightarrow \sqrt{2}\cos\left(x+\frac{\pi}{4}\right)=0 \Leftrightarrow \cos\left(x+\frac{\pi}{4}\right)=0 \Leftrightarrow x+\frac{\pi}{4}=\frac{\pi}{2}+k\pi \Leftrightarrow x=\frac{\pi}{4}+k\pi; k\in\mathbb{Z}$

* $\sin^2 x-\sin x\cos x+1=0 \Leftrightarrow \frac{1-\cos 2x}{2}-\frac{\sin 2x}{2}+1=0$

$\Leftrightarrow \sin 2x+\cos 2x-3=0$ (vô nghiệm)

6). $\frac{1}{\cos x}-\frac{1}{\sin x}=2\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)$ (1) [Dự bị 2 ĐH B04]

LỜI GIẢI

Điều kiện : $\sin 2x\neq 0 \Leftrightarrow 2x\neq k\pi \Leftrightarrow x\neq \frac{k\pi}{2},(k\in\mathbb{Z})$

(1) $\Leftrightarrow \frac{\sin x-\cos x}{\sin x\cos x}=2\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)$

$\Leftrightarrow -\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)=2\sqrt{2}\sin x\cos x\cos\left(x+\frac{\pi}{4}\right)$

$\Leftrightarrow \cos\left(x+\frac{\pi}{4}\right)(1+\sin 2x)=0 \Leftrightarrow \left[\begin{array}{l}\cos\left(x+\frac{\pi}{4}\right)=0\\ \sin 2x=-1\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x+\frac{\pi}{4}=\frac{\pi}{2}+k\pi\\ 2x=-\frac{\pi}{2}+k2\pi\end{array}\right.,(k\in\mathbb{Z})$

$\Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{4}+k\pi\\ x=-\frac{\pi}{4}+k\pi\end{array}\right.,(k\in\mathbb{Z}) \Leftrightarrow x=\frac{\pi}{4}+\frac{k\pi}{2},(k\in\mathbb{Z})$

Nghiệm của phương trình: $x=\frac{\pi}{4}+\frac{k\pi}{2},(k\in\mathbb{Z})$

7) $\sin 2x-2\sqrt{2}(\sin x+\cos x)-5=0$ (1) [Dự bị 2 ĐH D04]

LỜI GIẢI

Đặt $t=\sin x+\cos x$ với $-\sqrt{2}\leq t\leq \sqrt{2}$ $\Rightarrow \sin 2x=t^2-1$.

(1) $\Leftrightarrow t^2-2\sqrt{2}t-6=0 \Leftrightarrow t=-\sqrt{2} \vee t=3\sqrt{2}$ (loại).

Với $t=-\sqrt{2} \Leftrightarrow \sin x+\cos x=-\sqrt{2} \Leftrightarrow \sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=-\sqrt{2}$

$\Leftrightarrow \cos\left(x-\frac{\pi}{4}\right)=-1 \Leftrightarrow x-\frac{\pi}{4}=\pi+k2\pi \Leftrightarrow x=\frac{5\pi}{4}+k2\pi,(k\in\mathbb{Z})$