TÌM GIỚI HẠN VÔ ĐỊNH DẠNG $\frac{0}{0}$

DẠNG 1: $L = \lim\limits_{x \to x_0} \frac{P(x)}{Q(x)}$ với P(x), Q(x) là các đa thức và $P(x_0) = Q(x_0) = 0$

Câu 1: Tìm các giới hạn sau:

a). $\lim\limits_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}$ b). $\lim\limits_{x \to 5} \frac{x^2 - 5x}{x^2 - 25}$ c). $\lim\limits_{x \to 2} \frac{x^3 - 8}{x^2 - 3x + 2}$

d). $\lim\limits_{x \to 1} \frac{x^3 - 3x + 2}{x^4 - 4x + 3}$ e). $\lim\limits_{x \to 1} \frac{x^2 + 2x - 3}{2x^2 - x - 1}$ f). $\lim\limits_{x \to -3} \frac{x + 3}{x^2 + 2x - 3}$

LỜI GIẢI

a). $\lim\limits_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4} = \lim\limits_{x \to 2} \frac{(x+3)(x-2)}{(x-2)(x+2)} = \lim\limits_{x \to 2} \frac{x+3}{x+2} = \frac{5}{4}$

b). $\lim\limits_{x \to 5} \frac{x^2 - 5x}{x^2 - 25} = \lim\limits_{x \to 5} \frac{x(x-5)}{(x-5)(x+5)} = \lim\limits_{x \to 5} \frac{x}{x+5} = \frac{1}{2}$

c). $\lim\limits_{x \to 2} \frac{x^3 - 8}{x^2 - 3x + 2} = \lim\limits_{x \to 2} \frac{(x-2)(x^2+2x+4)}{(x-1)(x-2)} = \lim\limits_{x \to 2} \frac{x^2+2x+4}{x-1} = 12$

d). $L = \lim\limits_{x \to 1} \frac{x^3 - 3x + 2}{x^4 - 4x + 3}$

Phân tích $x^3 - 3x + 2$ thành nhân tử bằng Hoocner:

	1	0	-3	2
1	1	1	-2	0

$\Rightarrow x^3 - 3x + 2 = (x-1)(x^2 + x - 2)$

Phân tích $x^4 - 4x + 3$ thành nhân tử bằng Hoocner:

	1	0	0	-4	3
1	1	1	1	-3	0

$\Rightarrow x^4 - 4x + 3 = (x-1)(x^3 + x^2 + x - 3)$

Vậy $L = \lim\limits_{x \to 1} \frac{(x-1)(x^2+x-2)}{(x-1)(x^3+x^2+x-3)} = \lim\limits_{x \to 1} \frac{x^2+x-2}{x^3+x^2+x-3}$ (khi $x \to 1$ thì ta thấy cả tử và mẫu đều dần về 0, có nghĩa vẫn còn vô định $\frac{0}{0}$, nên ta phải phân tích thành nhân tử tiếp).

Phân tích $x^2 + x - 2$ thành nhân tử bằng Hoocner:

	1	1	-2
1	1	2	0

$\Rightarrow x^2 + x - 2 = (x-1)(x+2)$

Phân tích $x^3 + x^2 + x - 3$ thành nhân tử bằng Hoocner:

	1	1	1	-3
1	1	2	3	0

$\Rightarrow x^3 + x^2 + x - 3 = (x-1)(x^2+2x+3)$

$L = \lim\limits_{x\to 1} = \dfrac{(x-1)(x+2)}{(x-1)(x^2+2x+3)} = \lim\limits_{x\to 1}\dfrac{x+2}{x^2+2x+3} = \dfrac{1}{2}$

e). $\lim\limits_{x\to 1}\dfrac{x^2+2x-3}{2x^2-x-1} = \lim\limits_{x\to 1}\dfrac{(x-1)(x+3)}{(x-1)(2x+1)} = \lim\limits_{x\to 1}\dfrac{x+3}{2x+1} = \dfrac{4}{3}$

f). $\lim\limits_{x\to -3}\dfrac{x+3}{x^2+2x-3} = \lim\limits_{x\to -3}\dfrac{x+3}{(x-1)(x+3)} = \lim\limits_{x\to -3}\dfrac{1}{x-1} = -\dfrac{1}{4}$

Câu 2: Tìm các giới hạn sau :

a). $\lim\limits_{x\to 0}\dfrac{(1+x)^3-1}{x}$ b). $\lim\limits_{x\to 0}\dfrac{(x+3)^3-27}{x}$ c). $\lim\limits_{x\to 3}\dfrac{2x^3-5x^2-2x-3}{4x^3-12x^2+4x-12}$

d). $\lim\limits_{x\to -\sqrt{2}}\dfrac{x^3+2\sqrt{2}}{x^2-2}$ e). $\lim\limits_{x\to 1}\dfrac{x^2+4x-5}{x-1}$ f). $\lim\limits_{x\to 3}\dfrac{x^4-27x}{2x^2-3x-9}$

LỜI GIẢI

a). $\lim\limits_{x\to 0}\dfrac{(1+x)^3-1}{x} = \lim\limits_{x\to 0}\dfrac{x^3+3x^2+3x}{x} = \lim\limits_{x\to 0}(x^2+3x+3) = 3$

b). $\lim\limits_{x\to 0}\dfrac{(x+3)^3-27}{x} = \lim\limits_{x\to 0}\dfrac{x^3+9x^2+27x}{x} = \lim\limits_{x\to 0}(x^2+9x+27) = 27$

c). $L = \lim\limits_{x\to 3}\dfrac{2x^3-5x^2-2x-3}{4x^3-12x^2+4x-12}$

Phân tích $2x^3-5x^2-2x-3$ thành nhân tử bằng Hoocner:

	2	-5	-2	-3
3	2	1	1	0

$\Rightarrow 2x^3-5x^2-2x-3 = (x-3)(2x^2+x+1)$

Phân tích $4x^3-12x^2+4x-12$ thành nhân tử bằng Hoocner:

	4	-12	4	-12
3	4	0	4	0

$\Rightarrow 4x^3-12x^2+4x-12 = (x-3)(4x^2+4)$

$L = \lim\limits_{x\to 3}\dfrac{(x-3)(2x^2+x+1)}{(x-3)(4x^2+4)} = \lim\limits_{x\to 3}\dfrac{2x^2+x+1}{4x^2+4} = \dfrac{2.9+3+1}{4.3^2+4} = \dfrac{11}{20}$.

d). $\lim\limits_{x\to -\sqrt{2}}\dfrac{x^3+2\sqrt{2}}{x^2-2} = \lim\limits_{x\to -\sqrt{2}}\dfrac{x^3+\left(\sqrt{2}\right)^3}{x^2-2} = \lim\limits_{x\to -\sqrt{2}}\dfrac{\left(x+\sqrt{2}\right)\left(x^2-\sqrt{2}x+2\right)}{\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right)}$

$= \lim\limits_{x\to -\sqrt{2}}\dfrac{x^2-\sqrt{2}x+2}{x-\sqrt{2}} = -\dfrac{3\sqrt{2}}{2}$

e). $\lim\limits_{x\to 1}\dfrac{x^2+4x-5}{x-1} = \lim\limits_{x\to 1}\dfrac{(x-1)(x+5)}{x-1} = \lim\limits_{x\to 1}(x+5) = 6$

f). $\lim\limits_{x\to 3}\dfrac{x^4-27x}{2x^2-3x-9} = \lim\limits_{x\to 3}\dfrac{x(x^3-27)}{(x-3)(2x+3)} = \lim\limits_{x\to 3}\dfrac{x(x-3)(x^2+3x+9)}{(x-3)(2x+3)} = \lim\limits_{x\to 3}\dfrac{x(x^2+3x+9)}{2x+3} = 9$

Câu 3: Tìm các giới hạn sau:

a). $\lim\limits_{x\to 2}\dfrac{x^3+x^2-5x-2}{x^2-3x+2}$ b). $\lim\limits_{x\to -2}\dfrac{x^4-16}{x^2+6x+8}$ c). $\lim\limits_{x\to 2}\dfrac{x^5-2x^4+x-2}{x^2-4}$

d). $\lim\limits_{x\to 1}\dfrac{x^4-x^3-x+1}{x^3-5x^2+7x-3}$ e). $\lim\limits_{x\to 1}\dfrac{x+2\sqrt{x}-3}{x-5\sqrt{x}+4}$ f). $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}$

LỜI GIẢI

a). $\lim\limits_{x\to 2}\dfrac{x^3+x^2-5x-2}{x^2-3x+2}$

Phân tích x^3+x^2-5x-2 thành nhân tử bằng Hoocner:

	1	1	-5	-2
2	1	3	1	0

$\Rightarrow x^3+x^2-5x-2=(x-2)(x^2+3x+1)$

Vậy $\lim\limits_{x\to 2}\dfrac{(x-2)(x^2+3x+1)}{(x-1)(x+2)}=\lim\limits_{x\to 2}\dfrac{x^2+3x+1}{x+2}=\dfrac{11}{4}$

b). $\lim\limits_{x\to -2}\dfrac{x^4-16}{x^2+6x+8}=\lim\limits_{x\to -2}\dfrac{\left(x^2-4\right)\left(x^2+4\right)}{\left(x+2\right)\left(x+4\right)}=\lim\limits_{x\to -2}\dfrac{(x-2)(x+2)(x^2+4)}{(x+2)(x+4)}=\lim\limits_{x\to -2}\dfrac{(x-2)(x^2+4)}{x+4}=-16$

c). $\lim\limits_{x\to 2}\dfrac{x^5-2x^4+x-2}{x^2-4}$

Phân tích x^5-2x^4+x-2 thành nhân tử bằng Hoocner:

	1	-2	0	0	1	-2
2	1	0	0	0	1	0

$\Rightarrow x^5-2x^4+x-2=(x-2)(x^4+1)$

Vậy $\lim\limits_{x\to 2}\dfrac{(x-2)(x^4+1)}{(x-2)(x+2)}=\lim\limits_{x\to 2}\dfrac{x^4+1}{x+2}=\dfrac{17}{4}$

d). $\lim\limits_{x\to 1}\dfrac{x^4-x^3-x+1}{x^3-5x^2+7x-3}$

Phân tích x^4-x^3-x+1 thành nhân tử bằng Hoocner:

	1	-1	0	-1	1
1	1	0	0	-1	0

$\Rightarrow x^4-x^3-x+1=(x-1)(x^3-1)$

Phân tích x^3-5x^2+7x-3 thành nhân tử bằng Hoocner:

	1	-5	7	-3
1	1	-4	3	0

$\Rightarrow x^3-5x^2+7x-3=(x-1)(x^2-4x+3)$

$\lim\limits_{x\to 1}\dfrac{(x-1)(x^3-1)}{(x-1)(x^2-4x+3)}=\lim\limits_{x\to 1}\dfrac{x^3-1}{x^2-4x+3}=\lim\limits_{x\to 1}\dfrac{(x-1)(x^2+x+1)}{(x-1)(x+3)}=\lim\limits_{x\to 1}\dfrac{x^2+x+1}{x+3}=\dfrac{3}{4}$.

e). $\lim\limits_{x\to 1}\dfrac{x+2\sqrt{x}-3}{x-5\sqrt{x}+4}=\lim\limits_{x\to 1}\dfrac{(\sqrt{x}-1)(\sqrt{x}+3)}{(\sqrt{x}-1)(\sqrt{x}-4)}=\lim\limits_{x\to 1}\dfrac{\sqrt{x}+3}{\sqrt{x}-4}=\dfrac{4}{-3}$.

f). $\lim\limits_{x\to1}\dfrac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}=\lim\limits_{x\to1}\dfrac{\left(\sqrt[3]{x}-1\right)^2}{\left(\left(\sqrt{x}\right)^2-1\right)^2}=\lim\limits_{x\to1}\dfrac{\left(\sqrt[3]{x}-1\right)^2}{\left[(\sqrt{x}-1)(\sqrt{x}+1)\right]}=\lim\limits_{x\to1}\dfrac{1}{(\sqrt{x}+1)^2}=\dfrac{1}{4}$.

Câu 4: Tìm các giới hạn sau:

a). $\lim\limits_{x\to3}\dfrac{x^3-5x^2+3x+9}{x^4-8x^2-9}$ b). $\lim\limits_{x\to-2}\dfrac{2x^4+8x^3+7x^2-4x-4}{3x^3+14x^2+20x+8}$

c). $\lim\limits_{x\to1}\dfrac{x^4-5x^3+9x^2-7x+2}{x^4-3x^3+x^2+3x-2}$ d). $\lim\limits_{x\to1}\dfrac{x^5+x^4+x^3+x^2+x-5}{x^2-1}$ e). $\lim\limits_{x\to1}\dfrac{4x^6-5x^5+x}{(1-x)^2}$

LỜI GIẢI

a). $\lim\limits_{x\to3}\dfrac{x^3-5x^2+3x+9}{x^4-8x^2-9}$

Phân tích x^3-5x^2+3x+9 thành nhân tử bằng Hoocner:

	1	-5	3	9
3	1	-2	-3	0

$\Rightarrow x^3-5x^2+3x+9=(x-3)(x^2-2x+3)$

$\lim\limits_{x\to3}\dfrac{(x-3)(x^2-2x+3)}{(x^2+1)(x^2-9)}=\lim\limits_{x\to3}\dfrac{(x-3)(x^2-2x-3)}{(x^2+1)(x-3)(x+3)}=\lim\limits_{x\to3}\dfrac{x^2-2x-3}{(x^2+1)(x+3)}=0$

b). $\lim\limits_{x\to-2}\dfrac{2x^4+8x^3+7x^2-4x-4}{3x^3+14x^2+20x+8}$

Phân tích $2x^4+8x^3+7x^2-4x-4$ thành nhân tử bằng Hoocner:

	2	8	7	-4	-4
-2	2	4	-1	-2	0

$\Rightarrow 2x^4+8x^3+7x^2-4x-4=(x+2)(2x^3+4x^2-x-2)$

Phân tích $3x^3+14x^2+20x+8$ thành nhân tử bằng Hoocner:

	3	14	20	8
-2	3	8	4	0

$\Rightarrow 3x^3+14x^2+20x+8=(x+2)(3x^2+8x+4)$

$\lim\limits_{x\to-2}\dfrac{(x+2)(2x^3+4x^2-x-2)}{(x+2)(3x^2+8x+4)}=\lim\limits_{x\to-2}\dfrac{2x^3+4x^2-x-2}{3x^2+8x+4}$ (Khi $x\to-2$ ta thấy cả tử và mẫu đều dần về 0, nên vẫn còn vô định. Do đó ta phân tích thành nhân tử cả tử và mẫu tiếp để khử dạng vô định).

Phân tích $2x^3+4x^2-x-2$ thành nhân tử bằng Hoocner:

	2	4	-1	-2
-2	2	0	-1	0

$\Rightarrow 2x^3+4x^2-x-2=(x+2)(2x^2-1)$

Phân tích $3x^2+8x+4$ thành nhân tử bằng Hoocner:

	3	8	4
-2	3	2	0

$\Rightarrow 3x^2+8x+4=(x+2)(3x+2)$

$\lim_{x\to -2}\frac{(x+2)(2x^2-1)}{(x+2)(3x+2)}=\lim_{x\to -2}\frac{2x^2-1}{3x+2}=-\frac{7}{4}$

c). $L=\lim_{x\to 1}\frac{x^4-5x^3+9x^2-7x+2}{x^4-3x^3+x^2+3x-2}$

Phân tích $x^4-5x^3+9x^2-7x+2$ thành nhân tử bằng Hoocner:

	1	-5	9	-7	2
1	1	-4	5	-2	0

$\Rightarrow x^4-5x^3+9x^2-7x+2=(x-1)(x^3-4x^2+5x-2)$

Phân tích $x^4-3x^3+x^2+3x-2$ thành nhân tử bằng Hoocner:

	1	-3	1	3	-2
1	1	-2	-1	2	0

$\Rightarrow x^4-3x^3+x^2+3x-2=(x-1)(x^3-2x^2-x+2)$

$L=\lim_{x\to 1}\frac{(x-1)(x^3-4x^2+5x-2)}{(x-1)(x^3-2x^2-x+2)}=\lim_{x\to 1}\frac{x^3-4x^2+5x-2}{x^3-2x^2-x+2}$ (Khi $x\to 1$ ta thấy cả tử và mẫu đều dần về 0, nên vẫn còn vô định. Do đó ta phân tích thành nhân tử cả tử và mẫu tiếp để khử dạng vô định).

Phân tích x^3-4x^2+5x-2 thành nhân tử bằng Hoocner:

	1	-4	5	-2
1	1	-3	2	0

$\Rightarrow x^3-4x^2+5x-2=(x-1)(x^2-3x+2)$

Phân tích x^3-2x^2-x+2 thành nhân tử bằng Hoocner:

	1	-2	-1	2
1	1	-1	-2	0

$\Rightarrow x^3-2x^2-x+2=(x-1)(x^2-x-2)$

$L=\lim_{x\to 1}\frac{(x-1)(x^2-3x+2)}{(x-1)(x^2-x-2)}=\lim_{x\to 1}\frac{x^2-3x+2}{x^2-x-2}=0$

d). $\lim_{x\to 1}\frac{x^5+x^4+x^3+x^2+x-5}{x^2-1}$

Phân tích $x^5+x^4+x^3+x^2+x-5$ thành nhân tử bằng Hoocner:

	1	1	1	1	1	-5
1	1	2	3	4	5	0

$\lim_{x\to 1}\frac{(x-1)(x^4+2x^3+3x^2+4x+5)}{(x-1)(x+1)}=\lim_{x\to 1}\frac{x^4+2x^3+3x^2+4x+5}{x+1}=\frac{15}{2}$

e). $\lim_{x\to 1}\frac{4x^6-5x^5+x}{(1-x)^2}=\lim_{x\to 1}\frac{x(4x^5-5x^4+1)}{(x-1)^2}$

Phân tích $4x^5-5x^4+1$ thành nhân tử bằng Hoocner:

	4	-5	0	0	0	1
1	4	-1	-1	-1	-1	0

$\Rightarrow 4x^5-5x^4+1=(x-1)(4x^4-x^3-x^2-x-1)$

$\lim\limits_{x\to1}\dfrac{x(x-1)(4x^4-x^3-x^2-x-1)}{(x-1)^2}=\lim\limits_{x\to1}\dfrac{x(4x^4-x^3-x^2-x-1)}{(x-1)}$

Phân tích $4x^4-x^3-x^2-x-1$ thành nhân tử bằng Hoocner:

	4	-1	-1	-1	-1
1	4	3	2	1	0

$\Rightarrow 4x^4-x^3-x^2-x-1=(x-1)(4x^3+3x^2+2x+1)$

$\lim\limits_{x\to1}\dfrac{x(x-1)(4x^3+3x^2+2x+1)}{x-1}=\lim\limits_{x\to1}x(4x^3+3x^2+2x+1)=10$.

Câu 5: Tìm các giới hạn sau:

a). $\lim\limits_{x\to1}\left(\dfrac{1}{x-1}-\dfrac{2}{x^2-1}\right)$ b). $\lim\limits_{x\to2}\left(\dfrac{1}{x^2-5x+6}+\dfrac{1}{x^2-3x+2}\right)$

c). $\lim\limits_{x\to-2}\left(\dfrac{2x-3}{x+2}-\dfrac{x-26}{4-x^2}\right)$ d). $\lim\limits_{x\to1}\left(\dfrac{1}{x^2+x-2}-\dfrac{1}{x^3-1}\right)$

LỜI GIẢI

a). $\lim\limits_{x\to1}\left(\dfrac{1}{x-1}-\dfrac{2}{x^2-1}\right)$

$=\lim\limits_{x\to1}\left(\dfrac{1}{x-1}-\dfrac{2}{(x-1)(x+1)}\right)=\lim\limits_{x\to1}\dfrac{x+1-2}{(x-1)(x+1)}=\lim\limits_{x\to1}\dfrac{x-1}{(x-1)(x+1)}=\lim\limits_{x\to1}\dfrac{1}{x+1}=\dfrac{1}{2}$

b). $\lim\limits_{x\to2}\left(\dfrac{1}{x^2-5x+6}+\dfrac{1}{x^2-3x+2}\right)$

$=\lim\limits_{x\to2}\left(\dfrac{1}{(x-2)(x-3)}+\dfrac{1}{(x-1)(x-3)}\right)=\lim\limits_{x\to2}\left(\dfrac{x-1+x-3}{(x-2)(x-3)(x-1)}\right)$

$=\lim\limits_{x\to2}\dfrac{2(x-2)}{(x-2)(x-3)(x-1)}=\lim\limits_{x\to2}\dfrac{2}{(x-3)(x-1)}=-2$.

c). $\lim\limits_{x\to-2}\left(\dfrac{2x-3}{x+2}-\dfrac{x-26}{4-x^2}\right)$

$=\lim\limits_{x\to-2}\left(\dfrac{2x-3}{x+2}+\dfrac{x-26}{(x-2)(x+2)}\right)=\lim\limits_{x\to-2}\dfrac{(2x-3)(x-2)+x-26}{(x-2)(x+2)}$

$=\lim\limits_{x\to-2}\dfrac{2x^2-6x-20}{(x-2)(x+2)}=\lim\limits_{x\to-2}\dfrac{2(x+2)(x-5)}{(x-2)(x+2)}=\lim\limits_{x\to-2}\dfrac{2(x-5)}{x-2}=\dfrac{7}{2}$.

d). $\lim\limits_{x\to1}\left(\dfrac{1}{x^2+x-2}-\dfrac{1}{x^3-1}\right)$

$=\lim\limits_{x\to1}\left(\dfrac{1}{(x-1)(x+2)}-\dfrac{1}{(x-1)(x^2+x+1)}\right)=\lim\limits_{x\to1}\dfrac{x^2+x+1-x-2}{(x-1)(x+2)(x^2+x+1)}$

$=\lim\limits_{x\to1}\dfrac{x^2-1}{(x-1)(x+2)(x^2+x+1)}=\lim\limits_{x\to1}\dfrac{(x-1)(x+1)}{(x-1)(x+2)(x^2+x+1)}=\lim\limits_{x\to1}\dfrac{x+1}{(x+2)(x^2+x+1)}=\dfrac{1}{9}$

Câu 6: Tính các giới hạn sau:

a). $\lim\limits_{x\to 9}\frac{\sqrt{x}-3}{9x-x^2}$ b) $\lim\limits_{x\to 6}\frac{\sqrt{x+3}-3}{x-6}$ c). $\lim\limits_{x\to 0}\frac{\sqrt{x^3+1}-1}{x^2+x}$

d). $\lim\limits_{x\to 1}\frac{\sqrt{2x-x^2}-1}{x^2-x}$ e). $\lim\limits_{x\to 4}\frac{\sqrt{x+5}-3}{x^2-3x-4}$ f). $\lim\limits_{x\to 3}\frac{x^2-9}{\sqrt{x+1}-2}$

LỜI GIẢI

a). $\lim\limits_{x\to 9}\frac{\sqrt{x}-3}{9x-x^2}=\lim\limits_{x\to 9}\frac{x-9}{-x(x-9)(\sqrt{x}+3)}\lim\limits_{x\to 9}\frac{1}{-x(\sqrt{x}+3)}=-\frac{5}{4}$

b). $\lim\limits_{x\to 6}\frac{\sqrt{x+3}-3}{x-6}=\lim\limits_{x\to 6}\frac{(x+3-9)}{(x-6)(\sqrt{x+3}+3)}=\lim\limits_{x\to 6}\frac{x-6}{(x-6)(\sqrt{x+3}+3)}=\lim\limits_{x\to 6}\frac{1}{\sqrt{x+3}+3}=\frac{1}{6}$

c). $\lim\limits_{x\to 0}\frac{\sqrt{x^3+1}-1}{x^2+x}$

$=\lim\limits_{x\to 0}\frac{x^3+1-1}{(x^2+x)\left(\sqrt{x^3+1}+1\right)}=\lim\limits_{x\to 0}\frac{x^3}{x(x+1)\left(\sqrt{x^3+1}+1\right)}=\lim\limits_{x\to 0}\frac{x^2}{(x+1)\left(\sqrt{x^3+1}+1\right)}=0$

d). $\lim\limits_{x\to 1}\frac{\sqrt{2x-x^2}-1}{x^2-x}$

$=\lim\limits_{x\to 1}\frac{2x-x^2-1}{(x^2-x)\left(\sqrt{2x-x^2}+1\right)}=\lim\limits_{x\to 1}\frac{-(x-1)^2}{x(x-1)\left(\sqrt{2x-x^2}+1\right)}=\lim\limits_{x\to 1}\frac{-(x-1)}{x\left(\sqrt{2x-x^2}+1\right)}=0$

e). $\lim\limits_{x\to 4}\frac{\sqrt{x+5}-3}{x^2-3x-4}=\lim\limits_{x\to 4}\frac{x+5-9}{(x^2-3x-4)\left(\sqrt{x+5}+3\right)}=\lim\limits_{x\to 4}\frac{x-4}{(x+1)(x-4)\left(\sqrt{x+5}+3\right)}$

$=\lim\limits_{x\to 4}\frac{1}{(x+1)\left(\sqrt{x+5}+3\right)}=\frac{1}{30}$

f). $\lim\limits_{x\to 3}\frac{x^2-9}{\sqrt{x+1}-2}=\lim\limits_{x\to 3}\frac{(x^2-9)\left(\sqrt{x+1}+2\right)}{x+1-4}=\lim\limits_{x\to 3}\frac{(x+3(x-3)\left(\sqrt{x+1}+2\right)}{x-3}$

$=\lim\limits_{x\to 3}(x+3)\left(\sqrt{x+1}+2\right)=24$.

Câu 7: Tìm các giới hạn sau:

a). $\lim\limits_{x\to 1}\frac{\sqrt{3x+1}-\sqrt{x+3}}{\sqrt{x+8}-3}$ b). $\lim\limits_{x\to 9}\frac{3-\sqrt{x}}{\sqrt{x-5}-2}$ c). $\lim\limits_{x\to -1}\frac{\sqrt{3+2x}-\sqrt{x+2}}{3x+3}$

d). $\lim\limits_{x\to -1}\frac{\sqrt{4+x+x^2}-2}{x+1}$ e). $\lim\limits_{x\to -1}\frac{\sqrt{7-2x}+x-2}{x^2-1}$

LỜI GIẢI

a). $\lim\limits_{x\to 1}\frac{\sqrt{3x+1}-\sqrt{x+3}}{\sqrt{x+8}-3}=\lim\limits_{x\to 1}\frac{(3x+1-x-3)\left(\sqrt{x+8}+3\right)}{(x+8-9)\left(\sqrt{3x+1}+\sqrt{x+3}\right)}$

$=\lim\limits_{x\to 1}\frac{2(x-1)\left(\sqrt{x+8}+3\right)}{(x-1)\left(\sqrt{3x+1}+\sqrt{x+3}\right)}=\lim\limits_{x\to 1}\frac{2\left(\sqrt{x+8}+3\right)}{\sqrt{3x+1}+\sqrt{x+3}}=3$

b). $\lim\limits_{x\to 9}\frac{3-\sqrt{x}}{\sqrt{x-5}-2}=\lim\limits_{x\to 9}\frac{(9-x)\left(\sqrt{x-5}+2\right)}{(x-5-4)\left(3+\sqrt{x}\right)}=\lim\limits_{x\to 9}\frac{-(x-9)\left(\sqrt{x-5}+2\right)}{(x-9)\left(3+\sqrt{x}\right)}=\lim\limits_{x\to 9}\frac{-\left(\sqrt{x-5}+2\right)}{3+\sqrt{x}}=-\frac{2}{3}$.

c). $\lim\limits_{x\to -1}\frac{\sqrt{3+2x}-\sqrt{x+2}}{3x+3}=\lim\limits_{x\to -1}\frac{3+2x-(x+2)}{(3x+3)\left(\sqrt{3+2x}+\sqrt{x+2}\right)}$

$=\lim\limits_{x\to -1}\frac{x+1}{3(x+1)\left(\sqrt{3+2x}+\sqrt{x+2}\right)}=\lim\limits_{x\to -1}\frac{1}{3\left(\sqrt{3+2x}+\sqrt{x+2}\right)}=\frac{1}{6}$.

d). $\lim\limits_{x\to -1}\frac{\sqrt{4+x+x^2}-2}{x+1}=\lim\limits_{x\to -1}\frac{4+x+x^2-4}{(x+1)\left(\sqrt{4+x+x^2}+2\right)}=\lim\limits_{x\to -1}\frac{x(x+1)}{(x+1)\left(\sqrt{4+x+x^2}+2\right)}$

$=\lim\limits_{x\to -1}\frac{x}{\sqrt{4+x+x^2}+2}=-\frac{1}{4}$.

e). $\lim\limits_{x\to -1}\frac{\sqrt{7-2x}+x-2}{x^2-1}=\lim\limits_{x\to -1}\frac{\left(\sqrt{7-2x}\right)^2-(x-2)^2}{(x^2-1)\left[\sqrt{7-2x}-(x-2)\right]}=\lim\limits_{x\to -1}\frac{7-2x-(x^2-4x+4)}{(x^2-1)\left[\sqrt{7-2x}-(x-2)\right]}$

$=\lim\limits_{x\to -1}\frac{-x^2+2x+3}{(x^2-1)\left[\sqrt{7-2x}-(x-2)\right]}=\lim\limits_{x\to -1}\frac{-(x+1)(x-3)}{(x-1)(x+1)\left(\sqrt{7-2x}-x+2\right)}=\lim\limits_{x\to -1}\frac{-(x-3)}{(x-1)\left(\sqrt{7-2x}-x+2\right)}=-\frac{1}{3}$

Câu 8: Tìm các giới hạn sau:

a). $\lim\limits_{x\to -1}\frac{\sqrt[3]{5x-3}+2}{x+1}$ b). $\lim\limits_{x\to 0}\frac{1-\sqrt[3]{1-x}}{x}$ c). $\lim\limits_{x\to 3}\frac{\sqrt[3]{x^2-1}-2}{x-3}$

d). $\lim\limits_{x\to 1}\frac{\sqrt[3]{x+7}-2}{\sqrt{x}-1}$ e). $\lim\limits_{x\to 8}\frac{\sqrt[3]{x}-2}{\sqrt{2x+9}-5}$ f). $\lim\limits_{x\to -1}\frac{x^2-1}{2x+\sqrt{3x^2+1}}$

LỜI GIẢI

a). $\lim\limits_{x\to -1}\frac{\sqrt[3]{5x-3}+2}{x+1}=\lim\limits_{x\to -1}\frac{5x-3+8}{(x+1)\left[\left(\sqrt[3]{5x-3}\right)^2-2\sqrt[3]{5x-3}+4\right]}$

$=\lim\limits_{x\to -1}\frac{5(x+1)}{(x+1)\left[\left(\sqrt[3]{5x-3}-2.\sqrt[3]{5x-3}+4\right)\right]}=\lim\limits_{x\to -1}\frac{5}{\left[\left(\sqrt[3]{5x-3}\right)^2-2\sqrt[3]{5x-3}+4\right]}=\frac{5}{12}$

b). $\lim\limits_{x\to 0}\frac{1-\sqrt[3]{1-x}}{x}=\lim\limits_{x\to 0}\frac{1-(1-x)}{x\left[1+\sqrt[3]{1-x}+\left(\sqrt[3]{1-x}\right)^2\right]}=\lim\limits_{x\to 0}\frac{1}{1+\sqrt[3]{1-x}+\left(\sqrt[3]{1-x}\right)^2}=\frac{1}{3}$

c). $\lim\limits_{x\to 3}\frac{\sqrt[3]{x^2-1}-2}{x-3}=\lim\limits_{x\to 3}\frac{x^2-1-8}{(x-3)\left[\left(\sqrt[3]{x^2-1}\right)^2+2\sqrt[3]{x^2-1}+4\right]}$

$=\lim\limits_{x\to 3}\frac{(x-3)(x+3)}{\left(\sqrt[3]{x^2-1}\right)^2+2\sqrt[3]{x^2-1}+4}=\lim\limits_{x\to 3}\frac{x+3}{\left(\sqrt[3]{x^2-1}\right)^2+2\sqrt[3]{x^2-1}+4}=\frac{1}{2}$

d). $\lim\limits_{x\to 1}\frac{\sqrt[3]{x+7}-2}{\sqrt{x}-1}$

Ta có : $\sqrt[3]{x+7}-2=\dfrac{x+7-8}{\left(\sqrt[3]{x+7}\right)^2+2\sqrt[3]{x+7}+4}=\dfrac{x-1}{\left(\sqrt[3]{x+7}\right)^2+2\sqrt[3]{x+7}+4}$

Và $\dfrac{1}{\sqrt{x}-1}=\dfrac{\sqrt{x}+1}{x-1}$

Vậy $\lim\limits_{x\to1}\dfrac{x-1}{\left(\sqrt[3]{x+7}\right)^2+2\sqrt[3]{x+7}+4}.\dfrac{\sqrt{x}+1}{x-1}=\lim\limits_{x\to1}\dfrac{\sqrt{x}+1}{\left(\sqrt[3]{x+7}\right)^2+2\sqrt[3]{x+7}+4}=\dfrac{1}{6}$

e). $\lim\limits_{x\to8}\dfrac{\sqrt[3]{x}-2}{\sqrt{2x+9}-5}=\lim\limits_{x\to8}\left(\dfrac{x-8}{\left(\sqrt[3]{x}\right)^2+2\sqrt[3]{x}+4}.\dfrac{\sqrt{2x+9}+5}{2x+9-25}\right)$

$=\lim\limits_{x\to8}\left(\dfrac{x-8}{\left(\sqrt[3]{x}\right)^2+2\sqrt[3]{x}+4}.\dfrac{\sqrt{2x+9}+5}{2(x-8)}\right)=\lim\limits_{x\to8}\dfrac{\sqrt{2x+9}+5}{2\left[\left(\sqrt[3]{x}\right)^2+2\sqrt[3]{x}+4\right]}=\dfrac{10}{24}$

f). $\lim\limits_{x\to-1}\dfrac{x^2-1}{2x+\sqrt{3x^2+1}}$

$\lim\limits_{x\to-1}\dfrac{\left(x^2-1\right)\left(2x-\sqrt{3x^2+1}\right)}{4x^2-\left(3x^2+1\right)}=\lim\limits_{x\to-1}\dfrac{\left(x^2-1\right)\left(2x-\sqrt{3x^2+1}\right)}{x^2-1}=\lim\limits_{x\to-1}\left(2x-\sqrt{3x^2+1}\right)=-4$

Câu 9: Tìm các giới hạn sau:

a). $\lim\limits_{x\to0}\dfrac{\sqrt{x+9}+\sqrt{x+16}-7}{x}$ b). $\lim\limits_{x\to1}\dfrac{\sqrt[3]{x^3+7}-\sqrt{x^2+3}}{x-1}$ c). $\lim\limits_{x\to a}\dfrac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}},(a>0)$ d). $\lim\limits_{x\to2}\dfrac{\sqrt[3]{8x+11}-\sqrt{x+7}}{x^2-3x+2}$ e). $\lim\limits_{x\to0}\dfrac{\sqrt{1+2x}-\sqrt[3]{1+3x}}{x^2}$

LỜI GIẢI

a). $\lim\limits_{x\to0}\dfrac{\sqrt{x+9}+\sqrt{x+16}-7}{x}=\lim\limits_{x\to0}\dfrac{\sqrt{x+9}-3+\sqrt{x+16}-7}{x}$

$=\lim\limits_{x\to0}\dfrac{\sqrt{x+9}-3}{x}+\lim\limits_{x\to0}\dfrac{\sqrt{x+16}-4}{x}=\lim\limits_{x\to0}\dfrac{x+9-9}{\left(\sqrt{x+9}+3\right)x}+\lim\limits_{x\to0}\dfrac{x+16-16}{\left(\sqrt{x+16}+4\right)x}$

$=\lim\limits_{x\to0}\dfrac{x}{\left(\sqrt{x+9}+3\right)x}+\lim\limits_{x\to0}\dfrac{x}{\left(\sqrt{x+16}+4\right)x}=\lim\limits_{x\to0}\dfrac{1}{\sqrt{x+9}+3}+\lim\limits_{x\to0}\dfrac{1}{\sqrt{x+16}+4}=\dfrac{7}{24}$

b). $\lim\limits_{x\to1}\dfrac{\sqrt[3]{x^3+7}-\sqrt{x^2+3}}{x-1}=\lim\limits_{x\to1}\dfrac{\sqrt[3]{x^3+7}-2+2-\sqrt{x^2+3}}{x-1}$

$=\lim\limits_{x\to1}\dfrac{\sqrt[3]{x^3+7}-2}{x-1}+\lim\limits_{x\to1}\dfrac{2-\sqrt{x^2+3}}{x-1}$

$$= \lim_{x\to 1} \frac{x^3+7-8}{\left[\left(\sqrt[3]{x^3+7}\right)^2 + 2\sqrt[3]{x^3+7} + 4\right](x-1)} + \lim_{x\to 1} \frac{2-x^2-3}{(2+\sqrt{x^2+3})(x-1)}$$

$$= \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{\left[\left(\sqrt[3]{x^3+7}\right)^2 + 2\sqrt[3]{x^3+7} + 4\right](x-1)} + \lim_{x\to 1} \frac{-(x-1)(x+1)}{(2+\sqrt{x^2+3})(x-1)}$$

$$= \lim_{x\to 1} \frac{x^2+x+4}{\left(\sqrt[3]{x^3+7}\right)^2 + 2\sqrt[3]{x^3+7} + 4} + \lim_{x\to 1} \frac{x+1}{2+\sqrt{x^2+3}} = \frac{3}{4}$$

c). $\lim_{x\to a} \dfrac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}}$ $(a>0)$

$$= \lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{\sqrt{x^2-a^2}} + \lim_{x\to a} \frac{\sqrt{x-a}}{\sqrt{x^2-a^2}} = \lim_{x\to a} \frac{x-a}{\sqrt{(x-a)(x+a)}\left(\sqrt{x}+\sqrt{a}\right)} + \lim_{x\to a} \frac{\sqrt{x-a}}{\sqrt{(x-a)(x+a)}}$$

$$= \lim_{x\to a} \frac{\left(\sqrt{x-a}\right)^2}{\sqrt{x-a}.\sqrt{x+a}.\left(\sqrt{x}+\sqrt{a}\right)} + \lim_{x\to a} \frac{1}{\sqrt{x+a}} = \lim_{x\to a} \frac{\sqrt{x-a}}{\sqrt{x+a}\left(\sqrt{x}+\sqrt{a}\right)} + \lim_{x\to a} \frac{1}{\sqrt{x+a}} = \frac{1}{\sqrt{2a}}$$

d). $\lim_{x\to 2} \dfrac{\sqrt[3]{8x+11}-\sqrt{x+7}}{x^2-3x+2} = \lim_{x\to 2} \dfrac{\sqrt[3]{8x+11}-3+3-\sqrt{x+7}}{x^2-3x+2}$

$$= \lim_{x\to 2} \frac{\sqrt[3]{8x+11}-3}{x^2-3x+2} + \lim_{x\to 2} \frac{3-\sqrt{x+7}}{x^2-3x+2}$$

$$= \lim_{x\to 2} \frac{8x+11-27}{\left[\left(\sqrt[3]{8x+11}\right)^2 + 3\sqrt[3]{8x+11} + 9\right](x^2-3x+2)} + \lim_{x\to 2} \frac{9-x-7}{\left(3+\sqrt{x+7}\right)(x^2-3x+2)}$$

$$= \lim_{x\to 2} \frac{8(x-2)}{\left[\left(\sqrt[3]{8x+11}\right)^2 + 3\sqrt[3]{8x+11} + 9\right](x-2)(x-1)} + \lim_{x\to 2} \frac{-(x-2)}{(3+\sqrt{x+7})(x-2)(x-1)}$$

$$= \lim_{x\to 2} \frac{8}{\left[\left(\sqrt[3]{8x+11}\right)^2 + 3\sqrt[3]{8x+11} + 9\right](x-1)} + \lim_{x\to 2} \frac{-1}{\left(3+\sqrt{x+7}\right)(x-1)} = \frac{7}{54}$$

e). $L = \lim_{x\to 0} \dfrac{\sqrt{1+2x}-\sqrt[3]{1+3x}}{x^2}$

$$= \lim_{x\to 0} \left[\frac{1}{x}\left(\frac{\sqrt{1+2x}-1}{x} - \frac{\sqrt[3]{1+3x}-1}{x}\right)\right] = \lim_{x\to 0} \left[\frac{1}{x}\left(\frac{2}{\sqrt{1+2x}+1} - \frac{3}{\left(\sqrt[3]{1+3x}\right)^2 + \sqrt[3]{1+3x}+1}\right)\right]$$

$$= \lim_{x\to 0} \left[\frac{1}{x} \frac{2\left(\sqrt[3]{1+3x}\right)^2 + 2\sqrt[3]{1+3x} + 2 - 3\sqrt{1+2x} - 3}{\underbrace{\left(\sqrt{1+2x}+1\right)\left(\left(\sqrt[3]{1+3x}\right)^2 + \sqrt[3]{1+3x}+1\right)}_{A}}\right]$$

$$= \lim_{x\to 0} \left[\frac{1}{A}\left(2\frac{\left(\sqrt[3]{1+3x}\right)^2-1}{x} + 2\frac{\sqrt[3]{1+3x}-1}{x} - 3\frac{\sqrt{1+2x}-1}{x}\right)\right]$$

$$=\lim_{x\to 0}\left[\frac{1}{A}\left(2\left(\sqrt[3]{1+3x}+1\right)\frac{\sqrt[3]{1+3x}-1}{x}+2\frac{\sqrt[3]{1+3x}-1}{x}-3\frac{\sqrt{1+2x}-1}{x}\right)\right]$$

$$=\lim_{x\to 0}\left[\frac{1}{A}\left(\frac{6\left(\sqrt[3]{1+3x}+1\right)}{\left(\sqrt[3]{1+3x}\right)^2+\sqrt[3]{1+3x}+1}+\frac{6}{\left(\sqrt[3]{1+3x}\right)^2+\sqrt[3]{1+3x}+1}-\frac{6}{\sqrt{1+2x}+1}\right)\right]=\frac{1}{6}(4+2-3)=\frac{1}{2}.$$

CÁCH 2:

$$L=\lim_{x\to 0}\frac{\sqrt{1+2x}-(1+x)+(1+x)-\sqrt[3]{1+3x}}{x^2}=\lim_{x\to 0}\frac{\sqrt{1+2x}-(1+x)}{x^2}+\lim_{x\to 0}\frac{(1+x)-\sqrt[3]{1+3x}}{x^2}$$

$$=\lim_{x\to 0}\frac{-x^2}{x^2\left[\sqrt{1+2x}+(1+x)\right]}+\lim_{x\to 0}\frac{3x^3+3x^2}{x^2\left[(1+x)^2+(1+x)\sqrt[3]{1+3x}+\left(\sqrt[3]{1+3x}\right)^2\right]}$$

$$=\lim_{x\to 0}\frac{-1}{\sqrt{1+2x}+(1+x)}+\lim_{x\to 0}\frac{3(x+1)}{(1+x)^2+(1+x)\sqrt[3]{1+3x}+\left(\sqrt[3]{1+3x}\right)^2}=-\frac{1}{2}+1=\frac{1}{2}$$

Câu 10: Tính các giới hạn sau:

a). $\lim\limits_{x\to 1}\frac{x+x^2+...+x^n-n}{x-1}$ b). $\lim\limits_{x\to 1}\frac{x^n-1}{x^m-1}$ c). $\lim\limits_{x\to 1}\frac{x^n-nx+n-1}{(x-1)^2}$

LỜI GIẢI

a). $\lim\limits_{x\to 1}\frac{x+x^2+...+x^n-n}{x-1}=\lim\limits_{x\to 1}\frac{(x-1)+(x^2-1)+...+x^n-1}{x-1}$

$$=\lim_{x\to 1}\frac{(x-1)+(x-1)(x+1)+...+(x-1)(x^{n-1}+x^{n-x}+...+1)}{x-1}$$

$$=\lim_{x\to 1}\frac{(x-1)\left[1+(x+1)+...+(x^{n-1}+x^{n-x}+...+1)\right]}{x-1}=1+2+3+...+n=\frac{n(n+1)}{2}$$

b). $\lim\limits_{x\to 1}\frac{x^n-1}{x^m-1}=\lim\limits_{x\to 1}\frac{(x-1)(x^{n-1}+x^{n-2}+x^{n-3}+...+1)}{(x-1)(x^{m-1}+x^{m-2}+x^{m-3}+...+1)}=\frac{1+1+1+...+1}{1+1+1+...+1}=\frac{n}{m}$

c). $\lim\limits_{x\to 1}\frac{x^n-nx+n-1}{(x-1)^2}=\lim\limits_{x\to 1}\frac{(x^n-1)-n(x-1)}{(x-1)^2}$

$$=\lim_{x\to 1}\frac{(x-1)(x^{n-1}+x^{n-2}+x^{n-3}+...+1)-n(x-1)}{(x-1)^2}$$

$$=\lim_{x\to 1}\frac{(x-1)(x^{n-1}+x^{n-2}+x^{n-3}+...+1-n)}{(x-1)^2}=\lim_{x\to 1}\frac{x^{n-1}+x^{n-2}+x^{n-3}+...+1-n}{x-1}$$

$$=\lim_{x\to 1}\frac{(x^{n-1}-1)+(x^{n-2}-1)+(x^{n-3}-1)+...+(x-1)+(1-1)}{x-1}$$

$$=\lim_{x\to 1}\frac{(x-1)(x^{n-2}+x^{n-3}+...+1)+(x-1)(x^{n-3}+x^{n-4}+...+1)+...+(x-1)}{x-1}$$

$$=\lim_{x\to 1}\frac{(x-1)\left[(x^{n-2}+x^{n-3}+...+1)+(x^{n-3}+x^{n-4}+...+1)+...+1\right]}{x-1}$$

$$=(n-1)+(n-2)+...+1=\frac{(n-1).n}{2}.$$

Câu 10: Tìm các giới hạn sau:

1). $\lim\limits_{x\to 0}\dfrac{2\sqrt{1-x}-\sqrt[3]{8-x}}{x}$

2). $\lim\limits_{x\to 1}\dfrac{\sqrt{3x-2}-\sqrt[3]{4x^2-x-2}}{x^2-3x+2}$

3). $\lim\limits_{x\to 1}\dfrac{\sqrt{5-x^3}-\sqrt[3]{x^2+7}}{x^2-1}$

4). $\lim\limits_{x\to 4}\dfrac{2-\sqrt{x^2-12}}{\left(\sqrt{x^2+x-19}-1\right)\left(\sqrt{x+12}-2\right)}$

5). $\lim\limits_{x\to 2}\dfrac{3\sqrt[3]{4x^3-24}+\sqrt{x+2}-8\sqrt{2x-3}}{4-x^2}$

7). $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2+2x+6}-4x+1}{x^3-2x+1}$

6). $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2+3}+\sqrt{2x^2+4x+19}-\sqrt{3x^2+46}}{x^2-1}$

8). $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{x+6}-\sqrt[4]{7x+2}}{x-2}$

9). $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{3x+2}-\sqrt{3x-2}}{x-2}$

10). $\lim\limits_{x\to 1}\dfrac{\sqrt{6x+3}+2x^2-5x}{(x-1)^2}$

LỜI GIẢI

1). $\lim\limits_{x\to 0}\dfrac{2\sqrt{1-x}-\sqrt[3]{8-x}}{x}=\lim\limits_{x\to 0}\dfrac{2\sqrt{1-x}-2+2-\sqrt[3]{8-x}}{x}=\lim\limits_{x\to 0}\dfrac{2\sqrt{1-x}-2}{x}$

$+\lim\limits_{x\to 0}\dfrac{2-\sqrt[3]{8-x}}{x}=\lim\limits_{x\to 0}\dfrac{4(1-x)-4}{x\left(2\sqrt{1-x}+2\right)}+\lim\limits_{x\to 0}\dfrac{8-(8-x)}{x\left[4+2\sqrt[3]{8-x}+\left(\sqrt[3]{8-x}\right)^2\right]}$

$=\lim\limits_{x\to 0}\dfrac{-4}{2\sqrt{1-x}+2}+\lim\limits_{x\to 0}\dfrac{1}{4+2\sqrt[3]{8-x}+\left(\sqrt[3]{8-x}\right)^2}=\dfrac{-4}{4}+\dfrac{1}{12}=-\dfrac{11}{12}$

2). $\lim\limits_{x\to 1}\dfrac{\sqrt{3x-2}-\sqrt[3]{4x^2-x-2}}{x^2-3x+2}$

$\lim\limits_{x\to 1}\dfrac{\sqrt{3x-2}-1+1-\sqrt[3]{4x^2-x-2}}{x^2-3x+2}=\lim\limits_{x\to 1}\dfrac{\sqrt{3x-2}-1}{x^2-3x+2}+\lim\limits_{x\to 1}\dfrac{1-\sqrt[3]{4x^2-x-2}}{x^2-3x+2}$

- Tính $\lim\limits_{x\to 1}\dfrac{\sqrt{3x-2}-1}{x^2-3x+2}=\lim\limits_{x\to 1}\dfrac{3x-2-1}{(x-1)(x-2)\left(\sqrt{3x-2}+1\right)}$

$=\lim\limits_{x\to 1}\dfrac{3(x-1)}{(x-1)(x-2)\left(\sqrt{3x-2}+1\right)}=\lim\limits_{x\to 1}\dfrac{3}{(x-2)\left(\sqrt{3x-2}+1\right)}=\dfrac{3}{-1.2}=-\dfrac{3}{2}$

- Tính $\lim\limits_{x\to 1}\dfrac{1-\sqrt[3]{4x^2-x-2}}{x^2-3x+2}=\lim\limits_{x\to 1}\dfrac{1-(4x^2-x-2)}{\left(x^2-3x+2\right)\left[1+\sqrt[3]{4x^2-x-2}+\left(\sqrt[3]{4x^2-x-2}\right)^2\right]}$

$=\lim\limits_{x\to 1}\dfrac{-(x-1)(4x+3)}{(x-1)(x-2)\left[1+\sqrt[3]{4x^2-x-2}+\left(\sqrt[3]{4x^2-x-2}\right)^2\right]}$

$=\lim\limits_{x\to 1}\dfrac{-(4x+3)}{(x-2)\left[1+\sqrt[3]{4x^2-x-2}+\left(\sqrt[3]{4x^2-x-2}\right)^2\right]}=\dfrac{-7}{-1.3}=\dfrac{7}{3}$

Vậy giới hạn cần tìm: $-\dfrac{3}{2}+\dfrac{7}{3}=\dfrac{5}{6}$

CÁCH 2:

$$\lim_{x\to1}\frac{\sqrt{3x-2}-1+1-\sqrt[3]{4x^2-x-2}}{x^2-3x+2}=\lim_{x\to1}\frac{\frac{\sqrt{3x-2}-1}{x-1}+\frac{1-\sqrt[3]{4x^2-x-2}}{x-1}}{\frac{x^2-3x+2}{x-1}}$$

$$=\lim_{x\to1}\frac{\frac{2}{\sqrt{3x-2}+1}+\frac{-4x-3}{1+\sqrt[3]{4x^2-x-2}+\left(\sqrt[3]{4x^2-x-2}\right)^2}}{x-2}=\frac{5}{6}$$

3) $L=\lim_{x\to1}\frac{\sqrt{5-x^3}-\sqrt[3]{x^2+7}}{x^2-1}$

$$L=\lim_{x\to1}\frac{\sqrt{5-x^3}-2+2-\sqrt[3]{x^2+7}}{x^2-1}=\lim_{x\to1}\frac{\sqrt{5-x^3}-2}{x^2-1}+\lim_{x\to1}\frac{2-\sqrt[3]{x^2+7}}{x^2-1}$$

g Tính $\lim_{x\to1}\frac{\sqrt{5-x^3}-2}{x^2-1}=\lim_{x\to1}\frac{5-x^3-4}{(x^2-1)\left(\sqrt{5-x^3}+2\right)}=\lim_{x\to1}\frac{(1-x)(1+x+x^2)}{(x-1)(x+1)\left(\sqrt{5-x^3}+2\right)}$

$$=\lim_{x\to1}\frac{-(1+x+x^2)}{(x+1)\left(\sqrt{5-x^3}+2\right)}=\frac{-3}{2.4}=-\frac{3}{8}$$

g Tính $\lim_{x\to1}\frac{2-\sqrt[3]{x^2+7}}{x^2-1}=\lim_{x\to1}\frac{8-(x^2+7)}{(x^2-1)\left(4+\sqrt[3]{x^2+7}+\left(\sqrt[3]{x^2+7}\right)^2\right)}$

$$=\lim_{x\to1}\frac{1-x^2}{(x^2-1)\left(4+\sqrt[3]{x^2+7}+\left(\sqrt[3]{x^2+7}\right)^2\right)}=\lim_{x\to1}\frac{-1}{\left(4+\sqrt[3]{x^2+7}+\left(\sqrt[3]{x^2+7}\right)^2\right)}=-\frac{1}{12}$$

Kết luận $L=-\frac{3}{8}-\frac{1}{12}=-\frac{11}{24}$

4). $\lim_{x\to4}\frac{2-\sqrt{x^2-12}}{\left(\sqrt{x^2+x-19}-1\right)\left(\sqrt{x+12}-2\right)}$

Ta có $2-\sqrt{x^2-12}=\frac{4-(x^2-12)}{2+\sqrt{x^2-12}}=\frac{16-x^2}{2+\sqrt{x^2-12}}=\frac{(4-x)(4+x)}{2+\sqrt{x^2-12}}$

Ta có $\frac{1}{\sqrt{x^2+x-19}-1}=\frac{\sqrt{x^2+x-19}+1}{x^2+x-19-1}=\frac{\sqrt{x^2+x-19}+1}{x^2+x-20}=\frac{\sqrt{x^2+x-19}+1}{(x-4)(x+5)}$

$$=\lim_{x\to4}\frac{(4-x)(4+x)}{2+\sqrt{x^2-12}}\cdot\frac{\sqrt{x^2+x-19}+1}{(x-4)(x+5)}\cdot\frac{1}{\sqrt{x+12}-2}$$

$$=\lim_{x\to4}\frac{-(4+x)}{2+\sqrt{x^2-12}}\cdot\frac{\sqrt{x^2+x-19}+1}{x+5}\cdot\frac{1}{\sqrt{x+12}-2}=\frac{-8}{4}\cdot\frac{2}{9}\cdot\frac{1}{2}=-\frac{2}{9}$$

5). $\lim_{x\to2}\frac{3\sqrt[3]{4x^3-24}+\sqrt{x+2}-8\sqrt{2x-3}}{4-x^2}$. Đặt $f(x)=\frac{3\sqrt[3]{4x^3-24}+\sqrt{x+2}-8\sqrt{2x-3}}{4-x^2}$

$$\lim_{x\to2}f(x)=\lim_{x\to2}\frac{3\sqrt[3]{4x^3-24}-6+\sqrt{x+2}-2+8-8\sqrt{2x-3}}{4-x^2}$$

$$= \lim_{x\to 2}\frac{3\sqrt[3]{4x^3-24}-6}{4-x^2}+\lim_{x\to 2}\frac{\sqrt{x+2}-2}{4-x^2}+\lim_{x\to 2}\frac{8-8\sqrt{2x-3}}{4-x^2}$$

• Tính : $\lim_{x\to 2}\frac{3\sqrt[3]{4x^3-24}-6}{4-x^2}=\lim_{x\to 2}3\cdot\frac{\sqrt[3]{4x^3-24}-2}{4-x^2}$

$$=3\lim_{x\to 2}\frac{4x^3-24-8}{\left(4-x^2\right)\underbrace{\left[\left(\sqrt[3]{4x^3-24}\right)+2\sqrt[3]{4x^3-24}+2\right]}_{A}}=3\lim_{x\to 2}\frac{4x^3-24-8}{\left(4-x^2\right).A}=3\lim_{x\to 2}\frac{4\left(x^3-8\right)}{\left(4-x^2\right).A}$$

$$=3\lim_{x\to 2}\frac{4\left(x-2\right)\left(x^2+2x+4\right)}{\left(2-x\right)\left(2+x\right).A}=3\lim_{x\to 2}\frac{-4\left(x^2+2x+4\right)}{\left(2+x\right).A}=3\lim_{x\to 2}\frac{-4.12}{4.8}=-\frac{9}{2}$$

• Tính $\lim_{x\to 2}\frac{\sqrt{x+2}-2}{4-x^2}=\lim_{x\to 2}\frac{x+2-4}{\left(4-x^2\right)\left(\sqrt{x+2}+2\right)}=\lim_{x\to 2}\frac{x-2}{\left(2+x\right)\left(2-x\right)\left(\sqrt{x+2}+2\right)}$

$$=\lim_{x\to 2}\frac{-1}{\left(2+x\right)\left(\sqrt{x+2}+2\right)}=-\frac{1}{4}.$$

• Tính $\lim_{x\to 2}\frac{8-8\sqrt{2x-3}}{4-x^2}=\lim_{x\to 2}\frac{8\left(1-\sqrt{2x-3}\right)}{4-x^2}=8\lim_{x\to 2}\frac{1-\left(2x-3\right)}{\left(4-x^2\right)\left(1+\sqrt{2x-3}\right)}$

$$=8\lim_{x\to 2}\frac{2\left(2-x\right)}{\left(2-x\right)\left(2+x\right)\left(1+\sqrt{2x-3}\right)}=8\lim_{x\to 2}\frac{2}{\left(2+x\right)\left(1+\sqrt{2x-3}\right)}=8\cdot\frac{2}{4.2}=2$$

Vậy giới hạn cần tìm : $\lim_{x\to 2}f\left(x\right)=-\frac{9}{2}-\frac{1}{4}+2=-\frac{11}{4}$

7). $\lim_{x\to 1}\frac{\sqrt{x^2+2x+6}-4x+1}{x^3-2x+1}$

$$\lim_{x\to 1}\frac{\sqrt{x^2+2x+6}-\left(4x-1\right)}{x^3-2x+1}=\lim_{x\to 1}\frac{x^2+2x+6-\left(4x-1\right)^2}{\left(x^3-2x+1\right)\left(\sqrt{x^2+2x+6}+\left(4x-1\right)\right)}$$

$$=\lim_{x\to 1}\frac{-15x^2+10x+5}{\left(x^3-2x+1\right)\left(\sqrt{x^2+2x+6}+4x-1\right)}=\lim_{x\to 1}\frac{-5\left(x-1\right)\left(3x+1\right)}{\left(x-1\right)\left(x^2+x-1\right)\left(\sqrt{x^2+2x+6}+4x-1\right)}$$

Phân tích $x^3-2x+1=(x-1)(x^2+x-1)$, bằng sơ đồ Hoocne sau:

	1	0	-2	1
1	1	1	-1	0

$$=\lim_{x\to 1}\frac{-5\left(3x+1\right)}{\left(x^2+x-1\right)\left(\sqrt{x^2+2x+6}+4x-1\right)}=\frac{-20}{1.6}=-\frac{10}{3}$$

6). $L=\lim_{x\to 1}\frac{\sqrt{x^2+3}+\sqrt{2x^2+4x+19}-\sqrt{3x^2+46}}{x^2-1}$

$$L=\lim_{x\to 1}\frac{\sqrt{x^2+3}-2+\sqrt{2x^2+4x+19}-5+7-\sqrt{3x^2+46}}{x^2-1}$$

$$=\lim_{x\to 1}\frac{\sqrt{x^2+3}-2}{x^2-1}+\lim_{x\to 1}\frac{\sqrt{2x^2+4x+19}-5}{x^2-1}+\lim_{x\to 1}\frac{7-\sqrt{3x^2+46}}{x^2-1}$$

• Tính $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2+3}-2}{x^2-1}=\lim\limits_{x\to 1}\dfrac{x^2+3-4}{(x^2-1)\left(\sqrt{x^2+3}+2\right)}$

$$=\lim_{x\to 1}\frac{x^2-1}{(x^2-1)\left(\sqrt{x^2+3}+2\right)}=\lim_{x\to 1}\frac{1}{\sqrt{x^2+3}+2}=\frac{1}{4}$$

• Tính $\lim\limits_{x\to 1}\dfrac{\sqrt{2x^2+4x+19}-5}{x^2-1}=\lim\limits_{x\to 1}\dfrac{2x^2+4x+19-25}{(x^2-1)\left(\sqrt{2x^2+4x+19}+5\right)}$

$$=\lim_{x\to 1}\frac{2x^2+4x-6}{(x^2-1)\left(\sqrt{2x^2+4x+19}+5\right)}=\lim_{x\to 1}\frac{2(x-1)(x+3)}{(x-1)(x+1)\left(\sqrt{2x^2+4x+19}+5\right)}$$

$$=\lim_{x\to 1}\frac{2(x+3)}{(x+1)\left(\sqrt{2x^2+4x+19}+5\right)}=\frac{2.4}{2.10}=\frac{2}{5}$$

• Tính $\lim\limits_{x\to 1}\dfrac{7-\sqrt{3x^2+46}}{x^2-1}=\lim\limits_{x\to 1}\dfrac{49-(3x^2+46)}{(x^2-1)\left(7+\sqrt{3x^2+46}\right)}=\lim\limits_{x\to 1}\dfrac{-3(x^2-1)}{(x^2-1)\left(7+\sqrt{3x^2+46}\right)}$

$$=\lim_{x\to 1}\frac{-3}{\left(7+\sqrt{3x^2+46}\right)}=-\frac{3}{14}$$

Kết luận $L=\dfrac{1}{4}+\dfrac{2}{5}-\dfrac{3}{14}=\dfrac{61}{140}$

8). $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{x+6}-\sqrt[4]{7x+2}}{x-2}$. Đặt $f(x)=\dfrac{\sqrt[3]{x+6}-\sqrt[4]{7x+2}}{x-2}$

$$\lim_{x\to 2}f(x)=\lim_{x\to 2}\frac{\sqrt[3]{x+6}-2+2-\sqrt[4]{7x+2}}{x-2}=\lim_{x\to 2}\frac{\sqrt[3]{x+6}-2}{x-2}+\lim_{x\to 2}\frac{2-\sqrt[4]{7x+2}}{x-2}$$

• Tính $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{x+6}-2}{x-2}=\lim\limits_{x\to 2}\dfrac{x+6-8}{(x-2)\left[\left(\sqrt[3]{x+6}\right)^2+2.\sqrt[3]{x+6}+4\right]}$

$$=\lim_{x\to 2}\frac{1}{\left(\sqrt[3]{x+6}\right)^2+2.\sqrt[3]{x+6}+4}=\frac{1}{12}$$

• Tính $\lim\limits_{x\to 2}\dfrac{2-\sqrt[4]{7x+2}}{x-2}=\lim\limits_{x\to 2}\dfrac{4-\sqrt{7x+2}}{(x-2)\left(2+\sqrt[4]{7x+2}\right)}=\lim\limits_{x\to 2}\dfrac{16-(7x+2)}{(x-2)\left(2+\sqrt[4]{7x+2}\right)\left(4+\sqrt{7x+2}\right)}$

$$=\lim_{x\to 2}\frac{-7(x-2)}{(x-2)\left(2+\sqrt[4]{7x+2}\right)\left(4+\sqrt{7x+2}\right)}=\lim_{x\to 2}\frac{-7}{\left(2+\sqrt[4]{7x+2}\right)\left(4+\sqrt{7x+2}\right)}=-\frac{7}{32}$$

Vậy $\lim\limits_{x\to 2}f(x)=\dfrac{1}{12}-\dfrac{7}{32}=-\dfrac{13}{96}$.

9). $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{3x+2}-\sqrt{3x-2}}{x-2}$. Đặt $f(x)=\dfrac{\sqrt[3]{3x+2}-\sqrt{3x-2}}{x-2}$

Có $\lim\limits_{x\to 2}f(x)=\lim\limits_{x\to 2}\dfrac{\sqrt[3]{3x+2}-2+2-\sqrt{3x-2}}{x-2}=\lim\limits_{x\to 2}\dfrac{\sqrt[3]{3x+2}-2}{x-2}+\lim\limits_{x\to 2}\dfrac{2-\sqrt{3x-2}}{x-2}$

• Tính $\lim\limits_{x\to 2}\frac{\sqrt[3]{3x+2}-2}{x-2}=\lim\limits_{x\to 2}\frac{3x+2-8}{(x-2)\left[\left(\sqrt[3]{3x+2}\right)^2+2.\sqrt[3]{3x+2}+4\right]}$

$=\lim\limits_{x\to 2}\frac{3(x-2)}{(x-2)\left[\left(\sqrt[3]{3x+2}\right)^2+2.\sqrt[3]{3x+2}+4\right]}=\lim\limits_{x\to 2}\frac{3}{\left[\left(\sqrt[3]{3x+2}\right)^2+2.\sqrt[3]{3x+2}+4\right]}=\frac{1}{4}$

• Tính $\lim\limits_{x\to 2}\frac{2-\sqrt{3x-2}}{x-2}=\lim\limits_{x\to 2}\frac{4-(3x-2)}{(x-2)(2+\sqrt{3x-2})}$

$=\lim\limits_{x\to 2}\frac{-3(x-2)}{(x-2)(2+\sqrt{3x-2})}=\lim\limits_{x\to 2}\frac{-3}{2+\sqrt{3x-2}}=-\frac{3}{4}$

Vậy $\lim\limits_{x\to 2}f(x)=\frac{1}{4}-\frac{3}{4}=-\frac{1}{2}$.

Tương tự: Tìm $\lim\limits_{x\to 2}\frac{\sqrt{6-x}-\sqrt[3]{x^2+4}}{x^2-4}$; $\lim\limits_{x\to 7}\frac{\sqrt{x+2}-\sqrt[3]{x+20}}{\sqrt[4]{x+9}-2}$; $\lim\limits_{x\to 0}\frac{\sqrt{1+4x}-\sqrt[3]{1+6x}}{x}$.

10). $\lim\limits_{x\to 1}\frac{\sqrt{6x+3}+2x^2-5x}{(x-1)^2}=\lim\limits_{x\to 1}\frac{\sqrt{6x+3}-(x+2)+2(x^2-2x+1)}{(x-1)^2}$

$=\lim\limits_{x\to 1}\frac{\sqrt{6x+3}-(x+2)}{(x-1)^2}+\lim\limits_{x\to 1}\frac{2(x^2-2x+1)}{(x-1)^2}$

$=\lim\limits_{x\to 1}\frac{6x+3-(x+2)^2}{(x-1)^2}+2=\lim\limits_{x\to 1}\frac{-x^2+2x-1}{(x-1)^2}+2=\lim\limits_{x\to 1}\frac{-(x-1)^2}{(x-1)^2}+2=-1+2=1$.

Câu 10: Tìm các giới hạn sau:

1). $\lim\limits_{x\to 1}\frac{2x^4-5x^3+3x^2+x-1}{3x^4-8x^3+6x^2-1}$

2). $\lim\limits_{x\to 0}\frac{(1+x)(1+2x)(1+3x)-1}{x}$

3). $\lim\limits_{x\to 0}\frac{\sqrt[n]{a+x}-\sqrt[n]{a}}{x}$

4). $\lim\limits_{x\to 0}\frac{\sqrt[n]{(1+2x)(1+3x)(1+4x)}-1}{x}$

5). $\lim\limits_{x\to 2}\frac{\sqrt{x+2}-\sqrt{2x}}{\sqrt{x-1}-\sqrt{3-x}}$

6). $\lim\limits_{x\to 1}\frac{x-1}{\sqrt{x^2+3}+x^3-3x}$

7). $\lim\limits_{x\to 2}\frac{\sqrt{x-1}+x^4-3x^3+x^2+3}{\sqrt{2x}-2}$

8). $\lim\limits_{x\to 1}\frac{\sqrt{2x-1}+x^2-3x+1}{\sqrt[3]{x-2}+x^2-x+1}$

9). $\lim\limits_{x\to 0}\frac{\sqrt[5]{1+5x}-1}{x}$

20). $\lim\limits_{x\to 1}\frac{\sqrt[4]{4x-3}-1}{x-1}$

LỜI GIẢI

1). $L=\lim\limits_{x\to 1}\frac{2x^4-5x^3+3x^2+x-1}{3x^4-8x^3+6x^2-1}$

Phân tích $2x^4-5x^3+3x^2+x-1=(x-1)(2x^3-3x^2+1)$, bằng sơ đồ Hoocne sau:

	2	-5	3	1	-1
1	2	-3	0	1	0

Phân tích $3x^4-8x^3+6x^2-1=(x-1)(3x^3-5x^2+x+1)$, bằng sơ đồ Hoocne sau:

	3	-8	6	0	-1

1	3	-5	1	1	0

$L=\lim\limits_{x\to 1}\dfrac{(x-1)(2x^3-3x^2+1)}{(x-1)(3x^3-5x^2+x+1)}=\lim\limits_{x\to 1}\dfrac{2x^3-3x^2+1}{3x^3-5x^2+x+1}$ (thay $x=0$ vào tử và mẫu vẫn còn dạng vô định $\dfrac{0}{0}$, nên tiếp tục phân tích đa thức thành nhân tử, cả tử và mẫu).

Phân tích $2x^3-3x^2+1=(x-1)(2x^2-x-1)$, bằng sơ đồ Hoocne sau:

	2	-3	0	1
1	2	-1	-1	0

Phân tích $3x^3-5x^2+x+1=(x-1)(3x^2-2x-1)$, bằng sơ đồ Hoocne sau:

	3	-5	1	1
1	3	-2	-1	0

$L=\lim\limits_{x\to 1}\dfrac{(x-1)(2x^2-x-1)}{(x-1)(3x^2-2x-1)}=\lim\limits_{x\to 1}\dfrac{2x^2-x-1}{3x^2-2x-1}$ (thay $x=0$ vào tử và mẫu vẫn còn dạng vô định $\dfrac{0}{0}$, nên tiếp tục phân tích đa thức thành nhân tử, cả tử và mẫu).

$L=\lim\limits_{x\to 1}\dfrac{(x-1)(2x+1)}{(x-1)(3x+1)}=\lim\limits_{x\to 1}\dfrac{2x+1}{3x+1}=\dfrac{3}{4}$

2). $L=\lim\limits_{x\to 0}\dfrac{(1+x)(1+2x)(1+3x)-1}{x}=\lim\limits_{x\to 0}\dfrac{x(1+2x)(1+3x)}{x}+\lim\limits_{x\to 0}\dfrac{2x(1+3x)}{x}+\lim\limits_{x\to 0}\dfrac{3x}{x}$

$=\lim\limits_{x\to 0}(1+2x)(1+3x)+\lim\limits_{x\to 0}2(1+3x)+3=1+2+3=6$

Tương tự: Tìm $L=\lim\limits_{x\to 0}\dfrac{(1+x)(1+2x)(1+3x)(1+4x)-1}{x}$

3). $L=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{a+x}-\sqrt[n]{a}}{x}=\lim\limits_{x\to 0}\dfrac{x}{x\left(\sqrt[n]{(a+x)^{n-1}}+\sqrt[n]{(a+x)^{n-2}}.\sqrt[n]{a}+\cdots+\sqrt[n]{a^{n-1}}\right)}$

$=\lim\limits_{x\to 0}\dfrac{1}{\sqrt[n]{(a+x)^{n-1}}+\sqrt[n]{(a+x)^{n-2}}.\sqrt[n]{a}+\cdots+\sqrt[n]{a^{n-1}}}=\dfrac{1}{n\sqrt[n]{a^{n-1}}}$

4). $L=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{(1+2x)(1+3x)(1+4x)}-1}{x}$.

Đặt $t=\sqrt[n]{(1+2x)(1+3x)(1+4x)}$. Ta có $x\to 0\Rightarrow t\to 1$

Và $\lim\limits_{x\to 0}\dfrac{t^n-1}{x}=\lim\limits_{x\to 0}\dfrac{(1+2x)(1+3x)(1+4x)-1}{x}=9$

Vậy $L=\lim\limits_{x\to 0}\dfrac{t-1}{x}=\lim\limits_{x\to 0}\dfrac{t^n-1}{x\left(t^{n-1}+t^{n-2}+\cdots+t+1\right)}=\dfrac{9}{n}$

5). $L=\lim\limits_{x\to 2}\dfrac{\sqrt{x+2}-\sqrt{2x}}{\sqrt{x-1}-\sqrt{3-x}}=\lim\limits_{x\to 2}\dfrac{x+2-2x}{\sqrt{x+2}+\sqrt{2x}}\cdot\dfrac{\sqrt{x-1}+\sqrt{3-x}}{x-1-(3-x)}$

$=\lim\limits_{x\to 2}\dfrac{-(x-2)\left(\sqrt{x-1}+\sqrt{3-x}\right)}{2(x-2)\left(\sqrt{x+2}+\sqrt{2x}\right)}=\lim\limits_{x\to 2}\dfrac{-\left(\sqrt{x-1}+\sqrt{3-x}\right)}{2\left(\sqrt{x+2}+\sqrt{2x}\right)}=-\dfrac{1}{4}$

6). $\lim\limits_{x\to 1}\dfrac{x-1}{\sqrt{x^2+3}+x^3-3x}=\lim\limits_{x\to 1}\dfrac{x-1}{\sqrt{x^2+3}-2+x^3-3x+2}$

$$= \lim_{x\to1}\frac{\frac{x-1}{x-1}}{\frac{\sqrt{x^2+3}-2}{x-1}+\frac{x^3-3x+2}{x-1}} = \lim_{x\to1}\frac{1}{\frac{x+1}{\sqrt{x^2+3}+2}+x^2+x-2} = 2$$

7). $L = \lim_{x\to2}\frac{\sqrt{x-1}+x^4-3x^3+x^2+3}{\sqrt{2x}-2}$

$$= \lim_{x\to2}\frac{\sqrt{x-1}-1+x^4-3x^3+x^2+4}{\sqrt{2x}-2} = \lim_{x\to2}\frac{\sqrt{x-1}-1}{\sqrt{2x}-2}+\lim_{x\to2}\frac{x^4-3x^3+x^2+4}{\sqrt{2x}-2} = M+N$$

Tính M: $\lim_{x\to2}\frac{\sqrt{x-1}-1}{\sqrt{2x}-2} = \lim_{x\to2}\frac{x-1-1}{\sqrt{x-1}+1}\cdot\frac{\sqrt{2x}+2}{2x-4} = \lim_{x\to2}\frac{\sqrt{2x}+2}{2\left(\sqrt{x-1}+1\right)} = 1$

Tính N: $\lim_{x\to2}\frac{x^4-3x^3+x^2+4}{\sqrt{2x}-2}$

$$= \lim_{x\to2}\frac{(x-1)\left(x^3-x^2-x-2\right)\left(\sqrt{2x}+2\right)}{2x-4} = \lim_{x\to2}\frac{\left(x^3-x^2-x-2\right)\left(\sqrt{2x}+2\right)}{2} = 0$$

Vậy $L = 1+0 = 1$

Tương tự: Tìm $\lim_{x\to-1}\frac{\sqrt[3]{x}+x^2+x+1}{x+1}$, $\lim_{x\to1}\frac{\sqrt[3]{2x-1}-\sqrt[3]{x}}{\sqrt{x}-1}$

8). $L = \lim_{x\to1}\frac{\sqrt{2x-1}+x^2-3x+1}{\sqrt[3]{x-2}+x^2-x+1} = \lim_{x\to1}\frac{\sqrt{2x-1}-1+x^2-3x+2}{\sqrt[3]{x-2}+1+x^2-x}$

$$= \lim_{x\to1}\frac{\frac{\sqrt{2x-1}-1}{x-1}+\frac{x^2-3x+2}{x-1}}{\frac{\sqrt[3]{x-2}+1}{x-1}+\frac{x^2-x}{x-1}} = \lim_{x\to1}\frac{\frac{2}{\sqrt{2x-1}+1}+x-2}{\frac{1}{\left(\sqrt[3]{x-2}\right)^2-\sqrt[3]{x-2}+1}+x} = 0$$. Vậy $L = 0$

9). $L = \lim_{x\to0}\frac{\sqrt[5]{1+5x}-1}{x}$

Đặt $t = \sqrt[5]{1+5x} \Rightarrow t^5 = 1+5x \Rightarrow x = \frac{t^5-1}{5}$. Ta có khi $x\to0$ thì $t\to1$

Vậy $L = \lim_{t\to1}\frac{5(t-1)}{t^5-1} = \lim_{t\to1}\frac{5(t-1)}{(t-1)(t^4+t^3+t^2+t+1)} = \lim_{t\to1}\frac{5}{(t^4+t^3+t^2+t+1)} = 1$

10). $\lim_{x\to1}\frac{\sqrt[4]{4x-3}-1}{x-1} = \lim_{x\to1}\frac{\sqrt{4x-3}-1}{(x-1)\left(\sqrt[4]{4x-3}+1\right)} = \lim_{x\to1}\frac{4(x-1)}{(x-1)\left(\sqrt[4]{4x-3}+1\right)\left(\sqrt{4x-3}+1\right)}$

$$= \lim_{x\to1}\frac{4}{\left(\sqrt[4]{4x-3}+1\right)\left(\sqrt{4x-3}+1\right)} = \frac{4}{4} = 1$$

Câu 10: Tìm các giới hạn sau:

1). $L = \lim_{x\to1}\frac{\sqrt[7]{2-x}-1}{x-1}$ 2). $\lim_{x\to0}\frac{\left(x^2+2004\right)\sqrt[7]{1-2x}-2004}{x}$

3). $L = \lim_{x\to0}\frac{\sqrt[3]{1+x^2}-\sqrt[4]{1-2x}}{x+x^2}$ 4). $L = \lim_{x\to1}\frac{\sqrt[4]{2x-1}+\sqrt[5]{x-2}}{x-1}$

5). $L = \lim_{x\to0}\frac{\sqrt{1+4x}-\sqrt[3]{1+6x}}{x^2}$ 6). $\lim_{x\to1}\frac{x\sqrt{2x-1}+\sqrt[3]{3x-2}-2}{x^2-1}$

7). $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x^2}-\sqrt[4]{1-2x}}{x^2+x}$	8). $\lim\limits_{x\to 0}\dfrac{\sqrt{4x+4}+\sqrt{9-6x}-5}{x^2}$

LỜI GIẢI

1). $L=\lim\limits_{x\to 1}\dfrac{\sqrt[7]{2-x}-1}{x-1}$. Đặt $t=\sqrt[7]{2-x}\Rightarrow t^7=2-x\Rightarrow x=2-t^7$

Ta có $x\to 1\Rightarrow t\to 1$

Vậy $L=\lim\limits_{t\to 1}\dfrac{t-1}{1-t^7}=\lim\limits_{t\to 1}\dfrac{t-1}{(1-t)\left(1+t+t^2+t^3+\cdots+t^7\right)}=\lim\limits_{t\to 1}\dfrac{-1}{1+t+t^2+t^3+\cdots+t^7}=-\dfrac{1}{8}$

2). $\lim\limits_{x\to 0}\dfrac{\left(x^2+2004\right)\sqrt[7]{1-2x}-2004}{x}$

$=\lim\limits_{x\to 0}\dfrac{x^2.\sqrt[7]{1-2x}+2004\left(\sqrt[7]{1-2x}-1\right)}{x}=\lim\limits_{x\to 0}\dfrac{x^2.\sqrt[7]{1-2x}}{x}+\lim\limits_{x\to 0}\dfrac{2004\left(\sqrt[7]{1-2x}-1\right)}{x}$

$=\lim\limits_{x\to 0}x.\sqrt[7]{1-2x}+\lim\limits_{x\to 0}\dfrac{2004\left(\sqrt[7]{1-2x}-1\right)}{x}=\lim\limits_{x\to 0}\dfrac{2004\left(\sqrt[7]{1-2x}-1\right)}{x}$

Đặt $t=\sqrt[7]{1-2x}\Rightarrow t^7=1-2x\Rightarrow x=\dfrac{1-t^7}{2}$

Ta có khi $x\to 0$ thì $t\to 1$

Vậy $\lim\limits_{x\to 0}\dfrac{2004\left(\sqrt[7]{1-2x}-1\right)}{x}=\lim\limits_{t\to 1}\dfrac{2.2004(t-1)}{1-t^7}=\lim\limits_{t\to 1}\dfrac{4008(t-1)}{(1-t)(1+t+t^2+\cdots+t^7)}$

$=\lim\limits_{t\to 1}\dfrac{-4008}{1+t+t^2+\cdots+t^7}=\dfrac{-4008}{8}=-501$

Tương tự: $\lim\limits_{x\to 0}\dfrac{\left(x^2+2001\right)\sqrt[9]{1-5x}-2001}{x}$

3). $L=\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x^2}-\sqrt[4]{1-2x}}{x+x^2}$

$L=\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x^2}-1+1-\sqrt[4]{1-2x}}{x+x^2}=\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x^2}-1}{x+x^2}+\lim\limits_{x\to 0}\dfrac{1-\sqrt[4]{1-2x}}{x+x^2}$

- Tính $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x^2}-1}{x+x^2}$

$=\lim\limits_{x\to 0}\dfrac{x^2}{x(1+x)\left[\left(\sqrt[3]{1+x^2}\right)^2+\sqrt[3]{1+x^2}+1\right]}=\lim\limits_{x\to 0}\dfrac{x}{(1+x)\left[\left(\sqrt[3]{1+x^2}\right)^2+\sqrt[3]{1+x^2}+1\right]}=0$

- Tính $\lim\limits_{x\to 0}\dfrac{1-\sqrt[4]{1-2x}}{x+x^2}$

$=\lim\limits_{x\to 0}\dfrac{1-\sqrt{1-2x}}{\left(x+x^2\right)\left(1+\sqrt[4]{1-2x}\right)}=\lim\limits_{x\to 0}\dfrac{2x}{x(1+x)\left(1+\sqrt[4]{1-2x}\right)\left(1+\sqrt{1-2x}\right)}$

$=\lim\limits_{x\to 0}\dfrac{2}{(1+x)\left(1+\sqrt[4]{1-2x}\right)\left(1+\sqrt{1-2x}\right)}=\dfrac{1}{2}$

Vậy $L=0+\dfrac{1}{2}=\dfrac{1}{2}$

4). $L=\lim\limits_{x\to 1}\dfrac{\sqrt[4]{2x-1}+\sqrt[5]{x-2}}{x-1}$

$L=\lim\limits_{x\to 1}\dfrac{\sqrt[4]{2x-1}-1+1+\sqrt[5]{x-2}}{x-1}=\lim\limits_{x\to 1}\dfrac{\sqrt[4]{2x-1}-1}{x-1}+\lim\limits_{x\to 1}\dfrac{1+\sqrt[5]{x-2}}{x-1}=M+N$

Tính M: $\lim\limits_{x\to 1}\dfrac{\sqrt[4]{2x-1}-1}{x-1}=\lim\limits_{x\to 1}\dfrac{\sqrt{2x-1}-1}{(x-1)\left(\sqrt[4]{2x-1}+1\right)}=\lim\limits_{x\to 1}\dfrac{2(x-1)}{(x-1)\left(\sqrt[4]{2x-1}+1\right)\left(\sqrt{2x-1}+1\right)}$

$=\lim\limits_{x\to 1}\dfrac{2}{\left(\sqrt[4]{2x-1}+1\right)\left(\sqrt{2x-1}+1\right)}=\dfrac{1}{2}$

Tính N: $\lim\limits_{x\to 1}\dfrac{1+\sqrt[5]{x-2}}{x-1}$

Đặt $t=\sqrt[5]{x-2}\Rightarrow t^5=x-2\Rightarrow x=t^5+2$

Ta có $x\to 1\Rightarrow t\to -1$

$N=\lim\limits_{t\to -1}\dfrac{1+t}{t^5+1}=\lim\limits_{t\to -1}\dfrac{t+1}{(t+1)(t^4-t^3+t^2-t+1)}=\lim\limits_{t\to -1}\dfrac{1}{t^4-t^3+t^2-t+1}=\dfrac{1}{5}$

Vậy $L=\dfrac{1}{2}+\dfrac{1}{5}=\dfrac{7}{10}$

Tương tự tính: $\lim\limits_{x\to 0}\dfrac{\sqrt{x+1}+\sqrt[3]{x-1}}{x}$, $\lim\limits_{x\to 1}\dfrac{x^6-6x+5}{(x-1)^2}$

5). $L=\lim\limits_{x\to 0}\dfrac{\sqrt{1+4x}-\sqrt[3]{1+6x}}{x^2}=\lim\limits_{x\to 0}\dfrac{\sqrt{1+4x}-(1+2x)+(1+2x)-\sqrt[3]{1+6x}}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{\sqrt{1+4x}-(1+2x)}{x^2}+\lim\limits_{x\to 0}\dfrac{(1+2x)-\sqrt[3]{1+6x}}{x^2}$

- Tính $\lim\limits_{x\to 0}\dfrac{\sqrt{1+4x}-(1+2x)}{x^2}=\lim\limits_{x\to 0}\dfrac{1+4x-\left(1+4x+4x^2\right)}{x^2\left(\sqrt{1+4x}+1+2x\right)}=\lim\limits_{x\to 0}\dfrac{-4}{\sqrt{1+4x}+1+2x}=-2$

- Tính $\lim\limits_{x\to 0}\dfrac{(1+2x)-\sqrt[3]{1+6x}}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{(1+2x)^3-(1+6x)}{x^2\left[(1+2x)^2+(1+2x)\sqrt[3]{1+6x}+\left(\sqrt[3]{1+6x}\right)^2\right]}=\lim\limits_{x\to 0}\dfrac{8x^3+12x^2}{x^2\left[(1+2x)^2+(1+2x)\sqrt[3]{1+6x}+\left(\sqrt[3]{1+6x}\right)^2\right]}$

$=\lim\limits_{x\to 0}\dfrac{8x+12}{(1+2x)^2+(1+2x)\sqrt[3]{1+6x}+\left(\sqrt[3]{1+6x}\right)^2}=4$

Vậy $L=-2+4=2$

6). Ta có $\lim\limits_{x\to 1}\dfrac{x\sqrt{2x-1}+\sqrt[3]{3x-2}-2}{x^2-1}=\lim\limits_{x\to 1}\dfrac{x\sqrt{2x-1}-1+\sqrt[3]{3x-2}-1}{x^2-1}$

$=\lim\limits_{x\to 1}\left[\dfrac{x\sqrt{2x-1}-1}{x^2-1}+\dfrac{\sqrt[3]{3x-2}-1}{x^2-1}\right]$

$=\lim\limits_{x\to 1}\left[\dfrac{2x^3-x^2-1}{\left(x^2-1\right)\left(x\sqrt{2x-1}+1\right)}+\dfrac{3x-3}{\left(x^2-1\right)\left[\sqrt[3]{(3x-2)^2}+\sqrt[3]{3x-2}+1\right]}\right]$

$$=\lim_{x\to 1}\left[\frac{2x^2+x+1}{(x+1)\left(x\sqrt{2x-1}+1\right)}+\frac{3}{(x+1)\left[\sqrt[3]{(3x-2)^2}+\sqrt[3]{3x-2}+1\right]}\right]$$

$$=\frac{4}{4}+\frac{3}{6}=\frac{3}{2}.$$

7). $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x^2}-\sqrt[4]{1-2x}}{x^2+x}$

$$=\lim_{x\to 0}\frac{\sqrt[3]{1+x^2}-1+1-\sqrt[4]{1-2x}}{x^2+x}=\lim_{x\to 0}\frac{\dfrac{\sqrt[3]{1+x^2}-1}{x}+\dfrac{1-\sqrt[4]{1-2x}}{x}}{\dfrac{x^2+x}{x}}$$

$$=\frac{\lim\limits_{x\to 0}\dfrac{x}{\left(\sqrt[3]{1+x^2}\right)^2+\sqrt[3]{1+x^2}+1}+\lim\limits_{x\to 0}\dfrac{1-\sqrt{1-2x}}{x\left(1+\sqrt[4]{1-2x}\right)}}{\lim\limits_{x\to 0}(x+1)}$$

$$=\frac{0+\lim\limits_{x\to 0}\dfrac{2}{\left(1+\sqrt[4]{1-2x}\right)\left(1+\sqrt{1-2x}\right)}}{1}=\frac{1}{2}.$$

8). $\lim\limits_{x\to 0}\dfrac{\sqrt{4x+4}+\sqrt{9-6x}-5}{x^2}$

$$=\lim_{x\to 0}\frac{\sqrt{4x+4}-(x+2)+(x-3)+\sqrt{9-6x}}{x^2}$$

$$=\lim_{x\to 0}\frac{\sqrt{4x+4}-(x+2)}{x^2}+\lim_{x\to 0}\frac{(x-3)+\sqrt{9-6x}}{x^2}$$

$$=\lim_{x\to 0}\frac{\left[\sqrt{4x+4}-(x+2)\right]\left[\sqrt{4x+4}+(x+2)\right]}{x^2\left[\sqrt{4x+4}+(x+2)\right]}+\lim_{x\to 0}\frac{\left[(x-3)+\sqrt{9-6x}\right]\left[(x-3)-\sqrt{9-6x}\right]}{x^2\left[(x-3)-\sqrt{9-6x}\right]}$$

$$=\lim_{x\to 0}\frac{4x+4-(x+2)^2}{x^2\left[\sqrt{4x+4}+(x+2)\right]}+\lim_{x\to 0}\frac{(x+3)^2-(9-6x)}{x^2\left[(x-3)-\sqrt{9-6x}\right]}$$

$$=\lim_{x\to 0}\frac{-x^2}{x^2\left[\sqrt{4x+4}+(x+2)\right]}+\lim_{x\to 0}\frac{x^2}{x^2\left[(x-3)-\sqrt{9-6x}\right]}$$

$$=\lim_{x\to 0}\frac{-1}{\sqrt{4x+4}+(x+2)}+\lim_{x\to 0}\frac{1}{(x-3)-\sqrt{9-6x}}=-\frac{1}{4}-\frac{1}{6}=-\frac{5}{12}.$$