

GIỚI HẠN KHI $x \rightarrow \pm\infty$ TỐI VÔ CỰC

Câu 1: Tìm các giới hạn sau:

$$\begin{array}{lll} \text{a). } \lim_{x \rightarrow -\infty} \frac{3x^2 - x + 7}{2x^3 - 1} & \text{b). } \lim_{x \rightarrow +\infty} \frac{(4x^2 + 1)(7x - 1)}{(2x^3 - 1)(x + 3)} & \text{c). } \lim_{x \rightarrow +\infty} \frac{x\sqrt{x+3}}{x^2 - x + 2} \\ \text{d). } \lim_{x \rightarrow +\infty} \left[(x+1) \sqrt{\frac{x}{2x^4 + x^2 + 1}} \right] & \text{e). } \lim_{x \rightarrow -\infty} x \sqrt{\frac{2x^3 + x}{x^5 - x^2 + 3}}. \end{array}$$

LỜI GIẢI

$$\text{a). } \lim_{x \rightarrow -\infty} \frac{3x^2 - x + 7}{2x^3 - 1} = \lim_{x \rightarrow -\infty} \frac{x^2 \left(3 - \frac{1}{x} + \frac{7}{x^2} \right)}{x^3 \left(2 - \frac{1}{x^3} \right)} = \lim_{x \rightarrow -\infty} \frac{3 - \frac{1}{x} + \frac{7}{x^2}}{x \left(2 - \frac{1}{x^3} \right)} = \lim_{x \rightarrow -\infty} \frac{3}{2x} = 0$$

$$\text{b). } \lim_{x \rightarrow +\infty} \frac{(4x^2 + 1)(7x - 1)}{(2x^3 - 1)(x + 3)} = \lim_{x \rightarrow +\infty} \frac{x^2 \left(4 + \frac{1}{x^2} \right) x \left(7 - \frac{1}{x} \right)}{x^3 \left(2 - \frac{1}{x^3} \right) x \left(1 + \frac{3}{x} \right)} = \lim_{x \rightarrow +\infty} \frac{\left(4 + \frac{1}{x^2} \right) \left(7 - \frac{1}{x} \right)}{x \left(2 - \frac{1}{x^3} \right) \left(1 + \frac{3}{x} \right)} = \lim_{x \rightarrow +\infty} \frac{28}{2x} = 0$$

$$\text{c). } \lim_{x \rightarrow +\infty} \frac{x\sqrt{x+3}}{x^2 - x + 2} = \lim_{x \rightarrow +\infty} \frac{x\sqrt{x} \left(1 + \frac{3}{x\sqrt{x}} \right)}{x^2 \left(1 - \frac{1}{x} + \frac{2}{x^2} \right)} = \lim_{x \rightarrow +\infty} \frac{1 + \frac{3}{x\sqrt{x}}}{\sqrt{x} \left(1 - \frac{1}{x} + \frac{2}{x^2} \right)} = \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{x}} = 0.$$

$$\begin{aligned} \text{d). } \lim_{x \rightarrow +\infty} \left[(x+1) \sqrt{\frac{x}{2x^4 + x^2 + 1}} \right] &= \lim_{x \rightarrow +\infty} \left[(x+1) \sqrt{\frac{x}{x^4 \left(2 + \frac{1}{x^2} + \frac{1}{x^4} \right)}} \right] \\ &= \lim_{x \rightarrow +\infty} \left[(x+1) \sqrt{\frac{1}{2x^3}} \right] = \lim_{x \rightarrow +\infty} \left[(x+1) \frac{1}{x\sqrt{x}\sqrt{2}} \right] = \lim_{x \rightarrow +\infty} \left[x \left(1 + \frac{1}{x} \right) \frac{1}{x\sqrt{x}\sqrt{2}} \right] = 0 \end{aligned}$$

$$\text{e). } \lim_{x \rightarrow -\infty} x \sqrt{\frac{2x^3 + x}{x^5 - x^2 + 3}} = \lim_{x \rightarrow -\infty} x \sqrt{\frac{x^3 \left(x + \frac{1}{x^2} \right)}{x^5 \left(1 - \frac{1}{x^3} + \frac{3}{x^5} \right)}} = \lim_{x \rightarrow -\infty} x \frac{\sqrt{2}}{\sqrt{x^2}} = \lim_{x \rightarrow -\infty} x \frac{\sqrt{2}}{|x|} = \lim_{x \rightarrow -\infty} x \frac{\sqrt{2}}{-x} = -\sqrt{2}$$

Câu 2: Tìm các giới hạn sau:

$$\begin{array}{lll} \text{a). } \lim_{x \rightarrow -\infty} \left(\sqrt{2x^2 + 1} + x \right) & \text{b). } \lim_{x \rightarrow +\infty} \frac{3x^2 - x + 3}{x - 4} & \text{c). } \lim_{x \rightarrow +\infty} \frac{x^4 - x^3 + 11}{2x - 7} \\ \text{d). } \lim_{x \rightarrow +\infty} \frac{\sqrt{2x^4 + x^2 - 1}}{1 - 2x} & \text{e). } \lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 - x}}{1 - 2x} & \text{f). } \lim_{x \rightarrow -\infty} \frac{2x^4 + 7x^3 - 15}{x^4 + 1}. \end{array}$$

LỜI GIẢI

$$\begin{aligned} \text{a). } \lim_{x \rightarrow -\infty} \left(\sqrt{2x^2 + 1} + x \right) &= \lim_{x \rightarrow -\infty} \left(\sqrt{x^2 \left(2 + \frac{1}{x^2} \right)} + x \right) = \lim_{x \rightarrow -\infty} \left(\sqrt{2x^2} + x \right) \\ &= \lim_{x \rightarrow -\infty} (|x|\sqrt{2} + x) = \lim_{x \rightarrow -\infty} (-\sqrt{2}x + x) = \lim_{x \rightarrow -\infty} x(-\sqrt{2} + 1) = +\infty \end{aligned}$$

$$b). \lim_{x \rightarrow +\infty} \frac{3x^2 - x + 3}{x - 4} = \lim_{x \rightarrow +\infty} \frac{x^2 \left(3 - \frac{1}{x} + \frac{3}{x^2}\right)}{x \left(1 - \frac{1}{x}\right)} = \lim_{x \rightarrow +\infty} 3x = +\infty$$

$$c). \lim_{x \rightarrow +\infty} \frac{x^4 - x^3 + 11}{2x - 7} = \lim_{x \rightarrow +\infty} \frac{x^4 \left(1 - \frac{1}{x} + \frac{11}{x^3}\right)}{x \left(2 - \frac{7}{x}\right)} = \lim_{x \rightarrow +\infty} \frac{1}{2} x^3 = +\infty$$

$$d). \lim_{x \rightarrow +\infty} \frac{\sqrt{2x^4 + x^2 - 1}}{1 - 2x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 \left(2 + \frac{1}{x^2} - \frac{1}{x^4}\right)}}{x \left(\frac{1}{x} - 2\right)} = \lim_{x \rightarrow +\infty} \frac{x \sqrt{2 + \frac{1}{x^2} - \frac{1}{x^4}}}{\frac{1}{x} - 2} = \lim_{x \rightarrow +\infty} \frac{\sqrt{2x}}{-2} = -\infty$$

$$e). \lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 - x}}{1 - 2x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 \left(1 - \frac{1}{x^3}\right)}}{x \left(\frac{1}{x} - 2\right)} = \lim_{x \rightarrow +\infty} \frac{x \sqrt{1 - \frac{1}{x^3}}}{\frac{1}{x} - 2} = +\infty$$

$$f). \lim_{x \rightarrow -\infty} \frac{2x^4 + 7x^3 - 15}{x^4 + 1} = \lim_{x \rightarrow -\infty} \frac{x^4 \left(2 + \frac{7}{x} - \frac{15}{x^3}\right)}{x^4 \left(1 + \frac{1}{x^4}\right)} = 2$$

Câu 3: Tìm các giới hạn sau:

$$a). \lim_{x \rightarrow -\infty} \frac{(x-1)^2(5x+2)^2}{(3x+1)^4} \quad b). \lim_{x \rightarrow -\infty} \frac{2|x|+3}{\sqrt{x^2+x+5}} \quad c). \lim_{x \rightarrow -\infty} \frac{2|x|+3}{\sqrt{x^2+x+5}}$$

$$d). \lim_{x \rightarrow +\infty} \sqrt{\frac{2x^5 + x^3 - 1}{(2x^2 - 1)(x^3 + x)}} \quad e). \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+x} + 2x}{2x+3} \quad f). \lim_{x \rightarrow -\infty} \frac{|x| - \sqrt{x^2+x}}{x+10}$$

LỜI GIẢI

$$a). \lim_{x \rightarrow -\infty} \frac{(x-1)^2(5x+2)^2}{(3x+1)^4} = \lim_{x \rightarrow -\infty} \frac{x^2 \left(1 - \frac{1}{x}\right)^2 x^2 \left(5 + \frac{2}{x}\right)^2}{x^4 \left(3 + \frac{1}{x}\right)^4} = \lim_{x \rightarrow -\infty} \frac{\left(1 - \frac{1}{x}\right)^2 \left(5 + \frac{2}{x}\right)^2}{\left(3 + \frac{1}{x}\right)^4} = \frac{25}{81}$$

$$b). \lim_{x \rightarrow +\infty} \frac{\sqrt{x^6 + 2}}{3x^3 - 1} = \lim_{x \rightarrow +\infty} \sqrt{\frac{x^6 \left(1 + \frac{2}{x^6}\right)}{x^3 \left(3 - \frac{1}{x^3}\right)}} = \lim_{x \rightarrow +\infty} \frac{x^3 \sqrt{1 + \frac{2}{x^3}}}{x^3 \left(3 - \frac{1}{x^3}\right)} = \lim_{x \rightarrow +\infty} \frac{\sqrt{1 + \frac{2}{x^3}}}{3 - \frac{1}{x^3}} = \frac{1}{3}$$

$$c). \lim_{x \rightarrow -\infty} \frac{2|x|+3}{\sqrt{x^2+x+5}} = \lim_{x \rightarrow -\infty} \frac{-2x+3}{\sqrt{x^2 \left(1 + \frac{1}{x} + \frac{5}{x^2}\right)}} = \lim_{x \rightarrow -\infty} \frac{x \left(-2 + \frac{3}{x}\right)}{x \sqrt{1 + \frac{1}{x} + \frac{5}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{-2 + \frac{3}{x}}{1 + \frac{1}{x} + \frac{5}{x^2}} = -2$$

$$d). \lim_{x \rightarrow +\infty} \sqrt{\frac{2x^5 + x^3 - 1}{(2x^2 - 1)(x^3 + x)}} = \lim_{x \rightarrow +\infty} \frac{x^5 \left(2 + \frac{1}{x^2} - \frac{1}{x^5}\right)}{x^2 \left(2 - \frac{1}{x^2}\right)x^3 \left(1 + \frac{1}{x^2}\right)} = \lim_{x \rightarrow +\infty} \frac{2 + \frac{1}{x^2} - \frac{1}{x^3}}{\left(2 - \frac{1}{x^2}\right)\left(1 + \frac{1}{x^2}\right)} = 1$$

$$e). \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + x} + 2x}{2x + 3} = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 \left(1 + \frac{1}{x}\right)} + 2x}{x \left(2 + \frac{3}{x}\right)} = \lim_{x \rightarrow -\infty} \frac{x \left(\sqrt{1 + \frac{1}{x}} + 2\right)}{x \left(2 + \frac{3}{x}\right)} = \lim_{x \rightarrow -\infty} \frac{\sqrt{1 + \frac{1}{x}} + 2}{2 + \frac{3}{x}} = 1$$

$$f). \lim_{x \rightarrow -\infty} \frac{|x| - \sqrt{x^2 + x}}{x + 10} = \lim_{x \rightarrow -\infty} \frac{-x - \sqrt{x^2 \left(1 + \frac{1}{x}\right)}}{x \left(1 + \frac{10}{x}\right)} = \lim_{x \rightarrow -\infty} \frac{-x \left(1 + \sqrt{1 + \frac{1}{x}}\right)}{x \left(1 + \frac{10}{x}\right)} = \lim_{x \rightarrow -\infty} \frac{1 + \sqrt{1 + \frac{1}{x}}}{1 + \frac{10}{x}} = 1$$

Câu 4: Tìm các giới hạn sau:

$$a). \lim_{x \rightarrow -\infty} x \sqrt{\frac{2x^3 + x}{x^5 - x^2 + 3}} \quad b). \lim_{x \rightarrow +\infty} \left[(x+2) \sqrt{\frac{x-1}{x^3 + x}} \right] \quad c). \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 2x + 3} + 4x + 1}{\sqrt{4x^2 + 1} - x + 2}$$

$$d). \lim_{x \rightarrow +\infty} \left(\sqrt{x^2 - 4x} - x \right) \quad e). \lim_{x \rightarrow -\infty} \left(\sqrt[3]{8x^3 + 1} - 2x + 1 \right)$$

LỜI GIẢI

$$a). \lim_{x \rightarrow -\infty} x \sqrt{\frac{2x^3 + x}{x^5 - x^2 + 3}} = \lim_{x \rightarrow -\infty} x \sqrt{\frac{x^3 \left(2 + \frac{1}{x^2}\right)}{x^5 \left(1 - \frac{1}{x^3} + \frac{3}{x^5}\right)}} = \lim_{x \rightarrow -\infty} x \frac{\sqrt{2 + \frac{1}{x^2}}}{\sqrt{x^2} \sqrt{1 - \frac{1}{x^3} + \frac{3}{x^5}}} \\ = \lim_{x \rightarrow -\infty} \frac{\sqrt{2 + \frac{1}{x^2}}}{\sqrt{1 - \frac{1}{x^3} + \frac{3}{x^5}}} = \sqrt{2}$$

$$b). \lim_{x \rightarrow +\infty} \left[(x+2) \sqrt{\frac{x-1}{x^3 + x}} \right] = \lim_{x \rightarrow +\infty} \left[x \left(1 + \frac{2}{x}\right) \sqrt{\frac{x \left(1 - \frac{1}{x}\right)}{x^3 \left(1 + \frac{1}{x^2}\right)}} \right] = \lim_{x \rightarrow +\infty} \left[x \left(1 + \frac{2}{x}\right) \frac{\sqrt{1 - \frac{1}{x}}}{x \sqrt{1 + \frac{1}{x^2}}} \right] \\ = \lim_{x \rightarrow +\infty} \left[\left(1 + \frac{2}{x}\right) \frac{\sqrt{1 - \frac{1}{x}}}{\sqrt{1 + \frac{1}{x^2}}} \right] = 1.$$

$$c). \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 2x + 3} + 4x + 1}{\sqrt{4x^2 + 1} - x + 2} = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 \left(1 + \frac{2}{x} + \frac{3}{x^2}\right)} + 4x + 1}{\sqrt{x^2 \left(4 + \frac{1}{x^2}\right)} - x + 2} = \lim_{x \rightarrow -\infty} \frac{|x| \sqrt{\left(1 + \frac{2}{x} + \frac{3}{x^2}\right)} + 4x + 1}{|x| \sqrt{4 + \frac{1}{x^2}} - x + 2}$$

$$= \lim_{x \rightarrow -\infty} \frac{-x \sqrt{1 + \frac{2}{x} + \frac{3}{x^2}} + 4x + 1}{-x \sqrt{4 + \frac{1}{x^2}} - x + 2} = \lim_{x \rightarrow -\infty} \frac{-x + 4x + 1}{-4x - x + 2} = \lim_{x \rightarrow -\infty} \frac{-x \left(1 - 4 - \frac{1}{x}\right)}{-x \left(4 + 1 - \frac{2}{x}\right)} = \lim_{x \rightarrow -\infty} \frac{-3 - \frac{1}{x}}{5 - \frac{2}{x}} = \frac{-3}{5}$$

$$\text{d). } \lim_{x \rightarrow +\infty} (\sqrt{x^2 - 4x} - x) = \lim_{x \rightarrow +\infty} \frac{x^2 - 4x - x}{\sqrt{x^2 - 4x} + x} = \lim_{x \rightarrow +\infty} \frac{-4x}{\sqrt{x^2 \left(1 - \frac{1}{x}\right)} + x} = \lim_{x \rightarrow +\infty} \frac{-4x}{x + x} = \lim_{x \rightarrow +\infty} \frac{-4x}{2x} = -2$$

$$\begin{aligned} \text{e). } \lim_{x \rightarrow -\infty} (\sqrt[3]{8x^3 + 1} - 2x + 1) &= \lim_{x \rightarrow -\infty} \frac{8x^3 + 1 - 8x^3}{(\sqrt[3]{8x^3 + 1})^2 + \sqrt[3]{8x^3 + 1}.2x + 4x^2} + 1 \\ &= \lim_{x \rightarrow -\infty} \frac{1}{\sqrt[3]{x^3 \left(8 + \frac{1}{x^3}\right)^2} + \sqrt[3]{x^3 \left(8 + \frac{1}{x^3}\right)}.2x + 4x^2} + 1 \\ &= \lim_{x \rightarrow -\infty} \frac{1}{4 \left(\sqrt[3]{x^3}\right)^2 + 2 \sqrt[3]{x^3}.2x + 4x^2} + 1 = \lim_{x \rightarrow -\infty} \frac{1}{4x^2 + 4x^2 + 4x^2} + 1 = \lim_{x \rightarrow -\infty} \frac{1}{12x^2} + 1 = 1 \end{aligned}$$

Câu 5: Tìm các giới hạn sau:

$$\begin{aligned} \text{a). } \lim_{x \rightarrow +\infty} (\sqrt{x^2 - 4x} - x) &\quad \text{b). } \lim_{x \rightarrow +\infty} (\sqrt{x^2 + 1} - x) \quad \text{c). } \lim_{x \rightarrow -\infty} (\sqrt{x^2 - 3x + 2} + x) \\ \text{d). } \lim_{x \rightarrow -\infty} (\sqrt{4x^2 - 4x + 1} + 2x + 3) &\quad \text{e). } \lim_{x \rightarrow +\infty} (\sqrt{x^2 - 4x + 1} - 2x + 3) \end{aligned}$$

LỜI GIẢI

$$\text{a). } \lim_{x \rightarrow +\infty} (\sqrt{x^2 - 4x} - x) = \lim_{x \rightarrow +\infty} \frac{x^2 - 4x - x^2}{\sqrt{x^2 - 4x} + x} = \lim_{x \rightarrow +\infty} \frac{-4x}{\sqrt{x^2 \left(1 - \frac{4}{x}\right)} + x} = \lim_{x \rightarrow +\infty} \frac{-4x}{|x| + x} = \lim_{x \rightarrow +\infty} \frac{-4x}{2x} = -2.$$

$$\text{b). } \lim_{x \rightarrow +\infty} (\sqrt{x^2 + 1} - x) = \lim_{x \rightarrow +\infty} \frac{x^2 + 1 - x^2}{\sqrt{x^2 + 1} + x} = \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{x^2 \left(1 + \frac{1}{x^2}\right)} + x} = \lim_{x \rightarrow +\infty} \frac{1}{|x| + x} = \lim_{x \rightarrow +\infty} \frac{1}{2x} = 0.$$

$$\begin{aligned} \text{c). } \lim_{x \rightarrow -\infty} (\sqrt{x^2 - 3x + 2} + x) &= \lim_{x \rightarrow -\infty} \frac{x^2 - 3x + 2 - x^2}{\sqrt{x^2 - 3x + 2} + x} = \lim_{x \rightarrow -\infty} \frac{-3x + 2}{\sqrt{x^2 \left(1 - \frac{3}{x} + \frac{2}{x^2}\right)} - x} \\ &= \lim_{x \rightarrow -\infty} \frac{-x \left(3 + \frac{2}{x}\right)}{|x| - x} = \lim_{x \rightarrow -\infty} \frac{-3x}{-2x} = \frac{3}{2}. \end{aligned}$$

$$\text{d). } \lim_{x \rightarrow -\infty} (\sqrt{4x^2 - 4x + 1} + 2x + 3) = \lim_{x \rightarrow -\infty} \left(\frac{4x^2 - 4x + 1 - 4x^2}{\sqrt{4x^2 - 4x + 1} - 2x} \right) + 3 = \lim_{x \rightarrow -\infty} \frac{-4x + 1}{\sqrt{x^2 \left(4 - \frac{4}{x} + \frac{1}{x^2}\right)} - 2x} + 3$$

$$= \lim_{x \rightarrow -\infty} \frac{-x \left(4 + \frac{1}{x}\right)}{2|x| - 2x} + 3 = \lim_{x \rightarrow -\infty} \frac{-4x}{-2x} + 3 = 2 + 3 = 5.$$

$$\begin{aligned}
 e). \lim_{x \rightarrow +\infty} \left(\sqrt{x^2 - 4x + 1} - 2x + 3 \right) &= \lim_{x \rightarrow +\infty} \frac{x^2 - 4x + 1 - 4x^2}{\sqrt{x^2 - 4x + 1} + 2x} + 3 = \lim_{x \rightarrow +\infty} \frac{-3x^2 - 4x + 1}{\sqrt{x^2 \left(1 - \frac{4}{x} + \frac{1}{x^2} \right)} + 2x} + 3 \\
 &= \lim_{x \rightarrow +\infty} \frac{x^2 \left(-3 - \frac{4}{x} + \frac{1}{x^2} \right)}{|x| + 2x} + 3 = \lim_{x \rightarrow +\infty} \frac{-8x^2}{x + 2x} + 3 = \lim_{x \rightarrow +\infty} \frac{-8x^2}{3x} + 3 = \lim_{x \rightarrow +\infty} (-3x) + 3 = -\infty.
 \end{aligned}$$

Câu 6: Tìm các giới hạn sau:

$$\begin{aligned}
 a). \lim_{x \rightarrow -\infty} \left(\sqrt{x^2 - 4x + 3} - \sqrt{x^2 - 3x + 2} \right) & b). \lim_{x \rightarrow -\infty} \left(\sqrt{4x^2 - 9x - 21} - \sqrt{4x^2 - 7x + 13} \right) \\
 c). \lim_{x \rightarrow -\infty} \left(\frac{x^3}{3x^2 - 4} - \frac{x^2}{3x + 2} \right)
 \end{aligned}$$

LỜI GIẢI

$$\begin{aligned}
 a). \lim_{x \rightarrow -\infty} \left(\sqrt{x^2 - 4x + 3} - \sqrt{x^2 - 3x + 2} \right) &= \lim_{x \rightarrow -\infty} \frac{x^2 - 4x + 3 - x^2 + 3x - 2}{\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 3x + 2}} \\
 &= \lim_{x \rightarrow -\infty} \frac{-x + 1}{\sqrt{x^2 \left(1 - \frac{4}{x} + \frac{3}{x^2} \right)} + \sqrt{x^2 \left(1 - \frac{4}{x} + \frac{3}{x^2} \right)}} = \lim_{x \rightarrow -\infty} \frac{-x + 1}{\sqrt{x^2} \left(\sqrt{1 - \frac{1}{x} + \frac{3}{x^2}} + \sqrt{1 - \frac{4}{x} + \frac{3}{x^2}} \right)} \\
 &= \lim_{x \rightarrow -\infty} \frac{-x \left(1 - \frac{1}{x} \right)}{|x| \left(\sqrt{1 - \frac{1}{x} + \frac{3}{x^2}} + \sqrt{1 - \frac{4}{x} + \frac{3}{x^2}} \right)} = \lim_{x \rightarrow -\infty} \frac{-x \left(1 - \frac{1}{x} \right)}{-x \left(\sqrt{1 - \frac{1}{x} + \frac{3}{x^2}} + \sqrt{1 - \frac{4}{x} + \frac{3}{x^2}} \right)} = \frac{1}{2} \\
 b). \lim_{x \rightarrow -\infty} \left(\sqrt{4x^2 - 9x - 21} - \sqrt{4x^2 - 7x + 13} \right) &= \lim_{x \rightarrow -\infty} \frac{4x^2 - 9x - 21 - 4x^2 + 7x - 13}{\sqrt{4x^2 - 9x - 21} + \sqrt{4x^2 - 7x + 13}} \\
 &= \lim_{x \rightarrow -\infty} \frac{-2x - 34}{\sqrt{x^2 \left(4 - \frac{9}{x} - \frac{21}{x^2} \right)} + \sqrt{x^2 \left(4 - \frac{7}{x} + \frac{13}{x^2} \right)}} = \lim_{x \rightarrow -\infty} \frac{-x \left(2 + \frac{34}{x} \right)}{\sqrt{x^2} \left(\sqrt{4 - \frac{9}{x} - \frac{21}{x^2}} + \sqrt{4 - \frac{7}{x} + \frac{13}{x^2}} \right)} \\
 &= \lim_{x \rightarrow -\infty} \frac{-x \left(2 + \frac{34}{x} \right)}{|x| \left(\sqrt{4 - \frac{9}{x} - \frac{21}{x^2}} + \sqrt{4 - \frac{7}{x} + \frac{13}{x^2}} \right)} = \lim_{x \rightarrow -\infty} \frac{-x \left(2 + \frac{34}{x} \right)}{-x \left(\sqrt{4 - \frac{9}{x} - \frac{21}{x^2}} + \sqrt{4 - \frac{7}{x} + \frac{13}{x^2}} \right)} = \frac{1}{2} \\
 c). L &= \lim_{x \rightarrow -\infty} \left(\frac{x^3}{3x^2 - 4} - \frac{x^2}{3x + 2} \right)
 \end{aligned}$$

$$\begin{aligned}&= \lim_{x \rightarrow -\infty} \frac{x^3}{3x^2 - 4} - \frac{x^2}{3x + 2} = \lim_{x \rightarrow -\infty} \frac{x^3(3x+2) - x^2(3x^2 - 4)}{(3x^2 - 4)(3x+2)} = \lim_{x \rightarrow -\infty} \frac{2x^3 + 4x^2}{(3x^2 - 4)(3x+2)} \\&= \lim_{x \rightarrow -\infty} \frac{x^3 \left(\frac{2x^3 + 4x^2}{x^3} \right)}{x^2 \left(\frac{3x^2 - 4}{x^2} \right) x \left(\frac{3x+2}{x} \right)} = \lim_{x \rightarrow -\infty} \frac{2 + \frac{4}{x}}{\left(3 - \frac{4}{x^2} \right) \left(3 + \frac{2}{x} \right)}.\end{aligned}$$

Do $\lim_{x \rightarrow -\infty} \frac{4}{x} = \lim_{x \rightarrow -\infty} \frac{4}{x^2} = \lim_{x \rightarrow -\infty} \frac{2}{x^2} = 0$ nên $L = \frac{2}{3 \cdot 3} = \frac{2}{9}$.