GIỚI HẠN HÀM SỐ LƯỢNG GIÁC

Dạng 4: $\lim\limits_{x\to 0}\frac{\sin x}{x}=1$

Câu 1: Tìm các giới hạn sau:

1). $\lim\limits_{x\to 0}\frac{\sin 5x}{x}$ 2). $\lim\limits_{x\to 0}\frac{\tan 2x}{3x}$ 3). $\lim\limits_{x\to 0}\frac{1-\cos x}{\sin x}$

4). $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$ 5). $\lim\limits_{x\to 0}\frac{\sin 5x.\sin 3x.\sin x}{45x^3}$ 6). $\lim\limits_{x\to 0}\frac{\sin 7x-\sin 5x}{\sin x}$

7). $\lim\limits_{x\to 0}\frac{1-\cos 5x}{1-\cos 3x}$ 8). $\lim\limits_{x\to 0}\frac{1-\cos^2 2x}{x.\sin x}$ 9). $L=\lim\limits_{x\to 0}\frac{x.\sin ax}{1-\cos ax}$

LỜI GIẢI

1). $\lim\limits_{x\to 0}\frac{\sin 5x}{x}=\lim\limits_{x\to 0}\frac{1}{5}\cdot\frac{\sin 5x}{5x}=\frac{1}{5}$

2). $\lim\limits_{x\to 0}\frac{\tan 2x}{3x}=\lim\limits_{x\to 0}\frac{2}{3}\cdot\frac{\tan 2x}{2x}=\frac{2}{3}$

3). $\lim\limits_{x\to 0}\frac{1-\cos x}{\sin x}=\lim\limits_{x\to 0}\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}=\lim\limits_{x\to 0}\tan\frac{x}{2}=0$

4). $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}=\lim\limits_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}=\lim\limits_{x\to 0}\frac{1}{2}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2=\frac{1}{2}$

5). $\lim\limits_{x\to 0}\frac{\sin 5x.\sin 3x.\sin x}{45x^3}=\lim\limits_{x\to 0}\frac{1}{3}\cdot\frac{\sin 5x}{5x}\cdot\frac{\sin 3x}{3x}\cdot\frac{\sin x}{x}=\frac{1}{3}$

6). $\lim\limits_{x\to 0}\frac{\sin 7x-\sin 5x}{\sin x}=\lim\limits_{x\to 0}\frac{2\cos 6x\sin x}{\sin x}=\lim\limits_{x\to 0}2\cos 6x=2$

7). $\lim\limits_{x\to 0}\frac{1-\cos 5x}{1-\cos 3x}=\lim\limits_{x\to 0}\frac{2\sin^2\frac{5x}{2}}{2\sin^2\frac{3x}{2}}=\lim\limits_{x\to 0}\frac{25}{9}\cdot\left(\frac{\sin\frac{5x}{2}}{\frac{5x}{2}}\right)^2\cdot\left(\frac{\frac{3x}{2}}{\sin\frac{3x}{2}}\right)^2=\frac{25}{9}$

(Vì $\lim\limits_{x\to 0}\frac{\sin\frac{5x}{2}}{\frac{5x}{2}}=1,\ \lim\limits_{x\to 0}\frac{\frac{3x}{2}}{\sin\frac{3x}{2}}=1$)

8). $\lim\limits_{x\to 0}\frac{1-\cos^2 2x}{x.\sin x}=\lim\limits_{x\to 0}\frac{(1-\cos 2x)(1+\cos 2x)}{x.\sin x}$

$=\lim\limits_{x\to 0}\frac{2\sin^2 x(1+\cos 2x)}{x.\sin x}=\lim\limits_{x\to 0}2(1+\cos 2x)\frac{\sin x}{x}=4$

9). $L=\lim\limits_{x\to 0}\frac{x.\sin ax}{1-\cos ax}=\lim\limits_{x\to 0}\frac{x.2\sin\frac{ax}{2}\cos\frac{ax}{2}}{2\sin^2\frac{ax}{2}}=\lim\limits_{x\to 0}\frac{x}{\sin\frac{ax}{2}}\cdot\cos\frac{ax}{2}=\lim\limits_{x\to 0}\frac{\frac{ax}{2}}{\sin\frac{ax}{2}}\cdot\frac{\cos\frac{ax}{2}}{\frac{a}{2}}$

(Vì $\lim\limits_{x\to 0}\dfrac{\frac{ax}{2}}{\sin\frac{ax}{2}}=1$ và $\lim\limits_{x\to 0}\dfrac{\cos\frac{ax}{2}}{\frac{a}{2}}=\dfrac{2}{a}$). Vậy $L=\dfrac{2}{a}$.

Câu 2: Tìm các giới hạn sau:

1). $\lim\limits_{x\to 0}\dfrac{1-\cos ax}{1-\cos bx}$ 2). $\lim\limits_{x\to 0}\dfrac{\sin x.\sin 2x....\sin nx}{n!x^n}$ 3). $\lim\limits_{x\to 0}\dfrac{1-\cos ax}{x^2}\ (a\neq 0)$

4). $\lim\limits_{x\to 0}\dfrac{\sin x-\tan x}{x^3}$ 5). $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{\sin^3 x}$ 6). $\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{x-a}$

7). $\lim\limits_{x\to b}\dfrac{\cos x-\cos b}{x-b}$ 8). $\lim\limits_{x\to 0}\dfrac{1-\sqrt{2x+1}}{\sin 2x}$ 9). $\lim\limits_{x\to 0}\dfrac{\cos(a+x)-\cos(a-x)}{x}$

LỜI GIẢI

1). $L=\lim\limits_{x\to 0}\dfrac{1-\cos ax}{1-\cos bx}=\dfrac{2\sin^2\frac{ax}{2}}{2\sin^2\frac{bx}{2}}=\lim\limits_{x\to 0}\left(\dfrac{a}{b}\cdot\dfrac{\sin\frac{ax}{2}}{\frac{ax}{2}}\cdot\dfrac{\frac{bx}{2}}{\sin\frac{bx}{2}}\right)$

Vì $\lim\limits_{x\to 0}\dfrac{\sin\frac{ax}{2}}{\frac{ax}{2}}=1,\ \lim\limits_{x\to 0}\dfrac{\frac{bx}{2}}{\sin\frac{bx}{2}}=1$. Vậy $L=\dfrac{a}{b}$

2). $L=\lim\limits_{x\to 0}\dfrac{\sin x.\sin 2x....\sin nx}{n!x^n}=\lim\limits_{x\to 0}\dfrac{\sin x.\sin 2x....\sin nx}{1.2.3....nx^n}=\lim\limits_{x\to 0}\dfrac{\sin x}{x}\cdot\dfrac{\sin 2x}{2x}\cdots\dfrac{\sin nx}{nx}$

Vì $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1,\ \lim\limits_{x\to 0}\dfrac{\sin 2x}{2x}=1,\cdots,\ \lim\limits_{x\to 0}\dfrac{\sin nx}{nx}=1$

Vậy $L=1$.

3). $L=\lim\limits_{x\to 0}\dfrac{1-\cos ax}{x^2}=\lim\limits_{x\to 0}\dfrac{2\sin^2\frac{ax}{2}}{x^2}=\lim\limits_{x\to 0}\dfrac{a^2}{4}\left(\dfrac{\sin\frac{ax}{2}}{\frac{ax}{2}}\right)^2$ (vì $\lim\limits_{x\to 0}\dfrac{\sin\frac{ax}{2}}{\frac{ax}{2}}=1$).

Vậy $L=\dfrac{a^2}{4}$.

4). $L=\lim\limits_{x\to 0}\dfrac{\sin x-\tan x}{x^3}=\lim\limits_{x\to 0}\dfrac{\sin x-\frac{\sin x}{\cos x}}{x^3}=\dfrac{\sin x\left(\cos x-1\right)}{x^3\cos x}$

$=\lim\limits_{x\to 0}\dfrac{-2\sin^2\frac{x}{2}\sin x}{x^3\cos x}=\lim\limits_{x\to 0}\left(\dfrac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\cdot\dfrac{\sin x}{x}\cdot\dfrac{-1}{2\cos x}$

Vì $\lim\limits_{x\to 0}\dfrac{\sin\frac{x}{2}}{\frac{x}{2}}=1,\ \lim\limits_{x\to 0}\dfrac{\sin x}{x}=1,\ \lim\limits_{x\to 0}\dfrac{-1}{2\cos x}=-\dfrac{1}{2}$.

Vậy $L=-\dfrac{1}{2}$

5). $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{\sin^3 x}=\lim\limits_{x\to 0}\dfrac{\frac{\sin x}{\cos x}-\sin x}{\sin^3 x}=\lim\limits_{x\to 0}\dfrac{1-\cos x}{\cos x\sin^2 x}=\lim\limits_{x\to 0}\dfrac{2\sin^2\frac{x}{2}}{\cos x\sin^2 x}$

$$=\lim_{x\to 0}\frac{\frac{1}{2}.\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2}{\cos x.\left(\frac{\sin x}{x}\right)^2}=\frac{1}{2}$$

6). $\lim_{x\to a}\frac{\sin x-\sin a}{x-a}=\lim_{x\to a}\frac{2\sin\frac{x+a}{2}\sin\frac{x-a}{2}}{x-a}=\lim_{x\to a}\sin\frac{x+a}{2}\cdot\frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}}=\sin a$

7). $\lim_{x\to b}\frac{\cos x-\cos b}{x-b}=\lim_{x\to b}\frac{-2\sin\frac{x+b}{2}\sin\frac{x-b}{2}}{x-b}=\lim_{x\to b}\left(-\sin\frac{x+b}{2}\right).\frac{\sin\frac{x-b}{2}}{\frac{x-b}{2}}=-\sin b$

8). $\lim_{x\to 0}\frac{1-\sqrt{2x+1}}{\sin 2x}=\lim_{x\to 0}\frac{2x}{\sin 2x}\cdot\frac{-1}{1+\sqrt{2x+1}}=-\frac{1}{2}$

9). $L=\lim_{x\to 0}\frac{\cos(a+x)-\cos(a-x)}{x}=\lim_{x\to 0}\frac{-2\sin a\sin x}{x}=\lim_{x\to 0}\frac{\sin x}{x}.(-2\sin a)$

(Vì $\lim_{x\to 0}\frac{\sin x}{x}=1$). Vậy $L=-2\sin a$

Câu 2: Tìm các giới hạn sau:

1). $\lim_{x\to c}\frac{\tan x-\tan c}{x-c}$ 2). $\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin x}$ 3). $\lim_{x\to a}\frac{\sin^2 x-\sin^2 a}{x^2-a^2}$

4). $\lim_{x\to 0}\frac{\cos\alpha x-\cos\beta x}{x^2}$ 5). $\lim_{x\to 0}\frac{\sin 5x-\sin 3x}{\sin x}$ 6). $\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2}$

7). $\lim_{x\to -2}\frac{x^3+8}{\tan(x+2)}$ 8). $\lim_{x\to 0}\frac{1-\cos x.\cos 2x.\cos 3x}{1-\cos x}$

9). $\lim_{x\to 0}\frac{\sin(a+2x)-2\sin(a+x)+\sin a}{x^2}$ 10) $\lim_{x\to 0}\frac{\tan(a+2x)-2\tan(a+x)+\tan a}{x^2}$

LỜI GIẢI

1). $\lim_{x\to c}\frac{\tan x-\tan c}{x-c}=\lim_{x\to c}\frac{\sin(x-c)}{x-c}\cdot\frac{1}{\cos x\cos c}=\frac{1}{\cos^2 c}$ (vì $\lim_{x\to c}\frac{\sin(x-c)}{x-c}=1$).

2). $\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin x}=\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x+\cos^2 x)}{x\sin x}$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x.2\sin\frac{x}{2}\cos\frac{x}{2}}(1+\cos x+\cos^2 x)=\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}.\frac{1+\cos x+\cos^2 x}{2\cos\frac{x}{2}}=\frac{3}{2}.$$

3). $\lim_{x\to a}\frac{\sin^2 x-\sin^2 a}{x^2-a^2}=\lim_{x\to a}\frac{(\sin x-\sin a)(\sin x+\sin a)}{(x-a)(x+a)}$

$$=\lim_{x\to a}\frac{2\cos\frac{x+a}{2}\sin\frac{x-a}{2}}{2.\frac{x-a}{2}}\cdot\frac{\sin x+\sin a}{x+a}=\lim_{x\to a}\frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}}\cdot\frac{\cos\frac{x+a}{2}(\sin x+\sin a)}{x+a}$$

$$=\frac{2\cos a.\sin a}{2a}=\frac{\sin 2a}{2a}.$$

4). $\lim\limits_{x\to 0}\frac{\cos\alpha x-\cos\beta x}{x^2}=\lim\limits_{x\to 0}\frac{-2\sin\frac{x(\alpha+\beta)}{2}\cdot\sin\frac{x(\alpha-\beta)}{2}}{x^2}$

$$=\lim_{x\to 0}\frac{\sin\frac{x(\alpha+\beta)}{2}}{\frac{x(\alpha+\beta)}{2}}\cdot\lim_{x\to 0}\frac{\sin\frac{x(\alpha-\beta)}{2}}{\frac{x(\alpha-\beta)}{2}}.\lim_{x\to 0}\frac{(\alpha+\beta)}{2}\frac{(\alpha-\beta)}{2}(-2)=\frac{\beta^2-\alpha^2}{2}.$$

5). $\lim\limits_{x\to 0}\frac{\sin 5x-\sin 3x}{\sin x}=\lim\limits_{x\to 0}\frac{2\cos 4x\sin x}{\sin x}=\lim\limits_{x\to 0}(2\cos 4x)=2$

6). $L=\lim\limits_{x\to 1}(1-x)\tan\frac{\pi x}{2}$. Đặt $t=x-1$, vì $x\to 1\Rightarrow t\to 0$

$$L=\lim_{t\to 0}(-t)\tan\frac{\pi}{2}(t+1)=\lim_{t\to 0}(-t)\tan\left(\frac{\pi}{2}+\frac{\pi}{2}t\right)=\lim_{t\to 0}t\cot\frac{\pi}{2}t$$

$$=\lim_{t\to 0}t.\frac{\cos\frac{\pi}{2}t}{\sin\frac{\pi}{2}t}=\lim_{t\to 0}\frac{\frac{\pi}{2}t}{\sin\frac{\pi}{2}t}\cdot\frac{\cos\frac{\pi}{2}t}{\frac{\pi}{2}}=\frac{2}{\pi}$$

7). $\lim\limits_{x\to -2}\frac{x^3+8}{\tan(x+2)}=\lim\limits_{x\to -2}\frac{(x+2)(x^2-2x+4)}{\tan(x+2)}=\lim\limits_{x\to -2}\frac{x+2}{\tan(x+2)}(x^2-2x+4)=12$

(Vì $\lim\limits_{x\to -2}\frac{x+2}{\tan(x+2)}=1$).

8). $\lim\limits_{x\to 0}\frac{1-\cos x.\cos 2x.\cos 3x}{1-\cos x}$

$$\lim_{x\to 0}\frac{(1-\cos x.)\cos 2x.\cos 3x+(1-\cos 2x)\cos 3x+(1-\cos 3x)}{1-\cos x}$$

$$=\lim_{x\to 0}\frac{(1-\cos x.)\cos 2x.\cos 3x}{1-\cos x}+\lim_{x\to 0}\frac{(1-\cos 2x)\cos 3x}{1-\cos x}+\lim_{x\to 0}\frac{1-\cos 3x}{1-\cos x}$$

$$=\lim_{x\to 0}\cos 2x.\cos 3x+\lim_{x\to 0}\frac{2\sin^2 x\cos 3x}{2\sin^2\frac{x}{2}}+\lim_{x\to 0}\frac{2\sin^2\frac{3x}{2}}{2\sin^2\frac{x}{2}}$$

$$=1+\lim_{x\to 0}\frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}\cos 3x}{\sin^2\frac{x}{2}}+\lim_{x\to 0}9.\frac{\left(\sin\frac{3x}{2}\Big/\frac{3x}{2}\right)^2}{\left(\sin\frac{x}{2}\Big/\frac{x}{2}\right)^2}=1+4+9=14$$

9). $\lim\limits_{x\to 0}\frac{\sin(a+2x)-2\sin(a+x)+\sin a}{x^2}$

$$=\lim_{x\to 0}\frac{\sin(a+2x)-\sin(a+x)+\sin a-\sin(a+x)}{x^2}$$

$$=\lim_{x\to 0}\frac{2\cos\left(a+\frac{3x}{2}\right)\sin\frac{x}{2}-2\cos\left(a+\frac{x}{2}\right)\sin\frac{x}{2}}{x^2}$$

$$=\lim_{x\to 0}\frac{2\sin\frac{x}{2}\left[\cos\left(a+\frac{3x}{2}\right)-\cos\left(a+\frac{x}{2}\right)\right]}{x^2}=\lim_{x\to 0}\frac{-4\sin\frac{x}{2}\sin\left(a+x\right)\sin\frac{x}{2}}{x^2}$$

$$=\lim_{x\to 0}\left(-1\right)\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\sin\left(a+x\right)=-\sin a$$

10). $\lim_{x\to 0}\frac{\tan\left(a+2x\right)-2\tan\left(a+x\right)+\tan a}{x^2}$

$$=\lim_{x\to 0}\frac{\tan\left(a+2x\right)-\tan\left(a+x\right)-\left(\tan\left(a+x\right)-\tan a\right)}{x^2}$$

$$=\lim_{x\to 0}\frac{\frac{\sin x}{\cos(a+2x)\cos(a+x)}-\frac{\sin x}{\cos(a+x)\cos a}}{x^2}$$

$$=\lim_{x\to 0}\frac{\sin x}{x^2}\left(\frac{\cos a-\cos(a+2x)}{\cos(a+2x)\cos(a+x)\cos a}\right)=\lim_{x\to 0}\frac{\sin x}{x^2}\left(\frac{2\sin x\sin(a+x)}{\cos(a+2x)\cos(a+x)\cos a}\right)$$

$$=\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2\left(\frac{2\sin(a+x)}{\cos(a+2x)\cos(a+x)\cos a}\right)=\frac{2\sin a}{\cos^3 a}.$$

Câu 3: Tìm các giới hạn sau:

1). $\lim_{x\to 0}\frac{\sin ax+\tan bx}{(a+b)x}\quad(a+b\neq 0)$ 2). $\lim_{x\to 0}\frac{\cos 3x-\cos 5x.\cos 7x}{x^2}$

3). $\lim_{x\to 0}\frac{\cos ax-\cos bx.\cos cx}{x^2}$ 4). $\lim_{x\to 0}\frac{\sin\left(a+x\right)-\sin\left(a-x\right)}{\tan\left(a+x\right)-\tan\left(a-x\right)}$

5). $\lim_{x\to 0}\frac{\sqrt{2x+1}-\sqrt[3]{x^2+1}}{\sin x}$ 6). $\lim_{x\to 0}\frac{\sin^2 2x-\sin x.\sin 4x}{x^4}$

7). $\lim_{x\to 0}\frac{1-\cos 5x.\cos 7x}{\sin^2 11x}$ 8). $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right)$

9). $\lim_{x\to 0}\frac{\sin x-\sin 2x}{x\left(1-2\sin^2\frac{x}{2}\right)}$ 10). $\lim_{x\to 0}\frac{\sqrt{1+x^2}-\cos x}{x^2}$

LỜI GIẢI

1). $\lim_{x\to 0}\frac{\sin ax+\tan bx}{(a+b)x}=\lim_{x\to 0}\frac{\sin ax+\frac{\sin bx}{\cos bx}}{(a+b)x}=\lim_{x\to 0}\frac{\sin ax}{(a+b)x}+\lim_{x\to 0}\frac{\sin bx}{(a+b)x.\cos bx}$

$$=\lim_{x\to 0}\frac{a}{a+b}\cdot\frac{\sin ax}{ax}+\lim_{x\to 0}\frac{b}{(a+b)\cos bx}\cdot\frac{\sin bx}{bx}=\frac{a}{a+b}+\frac{b}{a+b}=1$$

2). $\lim_{x\to 0}\frac{\cos 3x-\cos 5x.\cos 7x}{x^2}=\lim_{x\to 0}\frac{\cos 3x-1+\left(1-\cos 5x\right)\cos 7x+1-\cos 7x}{x^2}$

$$=\lim_{x\to 0}\frac{\cos 3x-1}{x^2}\lim_{x\to 0}\frac{\left(1-\cos 5x\right)\cos 7x}{x^2}+\lim_{x\to 0}\frac{1-\cos 7x}{x^2}$$

$$=\lim_{x\to 0}\frac{-2\sin^2\frac{3x}{2}}{x^2}+\lim_{x\to 0}\frac{2\sin^2\frac{5x}{2}\cos 7x}{x^2}+\lim_{x\to 0}\frac{2\sin^2\frac{7x}{2}}{x^2}$$

$$=\lim_{x\to 0}\frac{-9}{2}.\left(\frac{\sin\frac{3x}{2}}{\frac{3x}{2}}\right)^2+\lim_{x\to 0}\frac{25\cos 7x}{2}\left(\frac{\sin\frac{5x}{2}}{\frac{5x}{2}}\right)^2+\lim_{x\to 0}\frac{49}{2}\left(\frac{\sin\frac{7x}{2}}{\frac{7x}{2}}\right)^2=-\frac{9}{2}+\frac{25}{2}+\frac{49}{2}=\frac{65}{2}$$

3). $$\lim_{x\to 0}\frac{\cos ax-\cos bx.\cos cx}{x^2}=\lim_{x\to 0}\frac{\cos ax-1-\left(\cos bx-1\right)\cos cx+1-\cos cx}{x^2}$$

$$=\lim_{x\to 0}\frac{-2\sin^2\frac{ax}{2}}{x^2}+\lim_{x\to 0}\frac{2\sin^2\frac{bx}{2}\cos cx}{x^2}+\lim_{x\to 0}\frac{2\sin^2\frac{cx}{2}}{x^2}$$

$$=\lim_{x\to 0}\frac{-a^2}{2}\left(\frac{\sin\frac{ax}{2}}{\frac{ax}{2}}\right)^2+\lim_{x\to 0}\frac{b^2\cos cx}{2}\cdot\left(\frac{\sin\frac{bx}{2}}{\frac{bx}{2}}\right)^2+\lim_{x\to 0}\frac{c^2}{2}\cdot\left(\frac{\sin\frac{cx}{2}}{\frac{cx}{2}}\right)^2=\frac{-a^2+b^2+c^2}{2}$$

4). $$\lim_{x\to 0}\frac{\sin\left(a+x\right)-\sin\left(a-x\right)}{\tan\left(a+x\right)-\tan\left(a-x\right)}=\lim_{x\to 0}\frac{2\cos a\sin x}{\frac{\sin 2x}{\cos(a+x)\cos(a-x)}}$$

$$=\lim_{x\to 0}\frac{\cos a\cos(a+x)\cos(a-x)}{\cos x}=\cos^3 a$$

5). $$\lim_{x\to 0}\frac{\sqrt{2x+1}-\sqrt[3]{x^2+1}}{\sin x}$$

$$\lim_{x\to 0}\frac{\sqrt{2x+1}-1+1-\sqrt[3]{x^2+1}}{\sin x}=\lim_{x\to 0}\frac{\sqrt{2x+1}-1}{\sin x}+\lim_{x\to 0}\frac{1-\sqrt[3]{x^2+1}}{\sin x}$$

$$=\lim_{x\to 0}\frac{2x}{\sin x\left(\sqrt{2x+1}+1\right)}+\lim_{x\to 0}\frac{-x^2}{\sin x\left[1+\sqrt[3]{x^2+1}+\left(\sqrt[3]{x^2+1}\right)^2\right]}$$

$$=\lim_{x\to 0}\frac{x}{\sin x}\cdot\frac{2}{\sqrt{2x+1}+1}+\lim_{x\to 0}\frac{x}{\sin x}\cdot\frac{-x}{1+\sqrt[3]{x^2+1}+\left(\sqrt[3]{x^2+1}\right)^2}=\frac{2}{1+1}+0=1$$

6). $$\lim_{x\to 0}\frac{\sin^2 2x-\sin x.\sin 4x}{x^4}=\lim_{x\to 0}\frac{\sin^2 2x-2\sin x\sin 2x\cos 2x}{x^4}$$

$$=\lim_{x\to 0}\frac{\sin 2x\left(2\sin x\cos x-2\sin x\cos 2x\right)}{x^4}$$

$$=\lim_{x\to 0}\frac{2\sin 2x.\sin x\left(\cos x-\cos 2x\right)}{x^4}=\lim_{x\to 0}\frac{4\sin 2x.\sin x.\sin\frac{3x}{2}.\sin\frac{x}{2}}{x^4}$$

$$=\lim_{x\to 0}6\cdot\left(\frac{\sin 2x}{2x}\right)\cdot\left(\frac{\sin x}{x}\right)\cdot\left(\frac{\sin\frac{3x}{2}}{\frac{3x}{2}}\right)\cdot\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)=6$$

7). $\lim\limits_{x\to 0}\dfrac{1-\cos 5x.\cos 7x}{\sin^2 11x}$

$=\lim\limits_{x\to 0}\dfrac{(1-\cos 5x)\cos 7x+1-\cos 7x}{\sin^2 11x}=\lim\limits_{x\to 0}\dfrac{2\sin^2\frac{5x}{2}\cos 7x}{\sin^2 11x}+\lim\limits_{x\to 0}\dfrac{2\sin^2\frac{7x}{2}}{\sin^2 11x}$

$=\lim\limits_{x\to 0}\dfrac{\left(\dfrac{\sin\frac{5x}{2}}{\frac{5x}{2}}\right)^2\cos 7x}{\left(\dfrac{\sin 11x}{11x}\right)^2}\cdot\dfrac{25}{484}+\lim\limits_{x\to 0}\dfrac{\left(\dfrac{\sin\frac{7x}{2}}{\frac{7x}{2}}\right)^2}{\left(\dfrac{\sin 11x}{11x}\right)^2}\cdot\dfrac{49}{484}=\dfrac{25}{484}+\dfrac{49}{484}=\dfrac{37}{242}$

8). $\lim\limits_{x\to 0}\left(\dfrac{1}{\sin x}-\dfrac{1}{\tan x}\right)=\lim\limits_{x\to 0}\left(\dfrac{1}{\sin x}-\dfrac{\cos x}{\sin x}\right)=\lim\limits_{x\to 0}\dfrac{1-\cos x}{\sin x}=\lim\limits_{x\to 0}\dfrac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$=\lim\limits_{x\to 0}\tan\dfrac{x}{2}=0$.

9). $\lim\limits_{x\to 0}\dfrac{\sin x-\sin 2x}{x\left(1-2\sin^2\frac{x}{2}\right)}=\lim\limits_{x\to 0}\dfrac{2\cos\frac{3x}{2}\sin\frac{-x}{2}}{x\cos x}=\lim\limits_{x\to 0}\dfrac{\sin\frac{x}{2}}{\frac{x}{2}}\cdot\dfrac{-\cos\frac{3x}{2}}{\cos x}=-1$

10). $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x^2}-\cos x}{x^2}=\lim\limits_{x\to 0}\dfrac{\sqrt{1+x^2}-1+1-\cos x}{x^2}=\lim\limits_{x\to 0}\dfrac{\sqrt{1+x^2}-1}{x^2}+\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{x^2}{x^2\left(\sqrt{1+x^2}+1\right)}+\lim\limits_{x\to 0}\dfrac{2\sin^2\frac{x}{2}}{x^2}=\lim\limits_{x\to 0}\dfrac{1}{\sqrt{1+x^2}+1}+\lim\limits_{x\to 0}\dfrac{1}{2}\cdot\left(\dfrac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2=\dfrac{1}{2}+\dfrac{1}{2}=1$.

Câu 3: Tìm các giới hạn sau:

1). $\lim\limits_{x\to\frac{\pi}{4}}\tan 2x.\tan\left(\dfrac{\pi}{4}-x\right)$ 2). $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$ 3). $\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{\tan(x-1)}$

4). $\lim\limits_{x\to-\frac{\pi}{2}}\dfrac{\cos x}{x+\frac{\pi}{2}}$ 5). $\lim\limits_{x\to\pi}\dfrac{1+\cos x}{(x-\pi)^2}$ 6). $\lim\limits_{x\to 1}\dfrac{\sin(x-1)}{x^2-4x+3}$

7). $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin x-1}{4\cos^2 x-3}$ 8). $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}\sin x-1}{2\cos^2 x-1}$ 9). $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\sin\left(\frac{\pi}{6}-x\right)}{1-2\sin x}$

LỜI GIẢI

1). $L=\lim\limits_{x\to\frac{\pi}{4}}\tan 2x.\tan\left(\dfrac{\pi}{4}-x\right)$. Đặt $t=x-\dfrac{\pi}{4}$, vì $x\to\dfrac{\pi}{4}\Rightarrow t\to 0$

$L=\lim\limits_{t\to 0}\left[\tan\left(2t+\dfrac{\pi}{2}\right)(-1)\tan t\right]=\lim\limits_{t\to 0}(\cot 2t.\tan t)$

$=\lim\limits_{t\to 0}\dfrac{\cos 2t}{\sin 2t}\dfrac{\sin t}{\cos t}=\lim\limits_{t\to 0}\dfrac{\cos 2t}{2\sin t\cos t}\dfrac{\sin t}{\cos t}=\lim\limits_{t\to 0}\dfrac{\cos 2t}{2\cos^2 t}=\dfrac{1}{2}$

2). $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}=\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{x^3\left(\underbrace{\sqrt{1+\tan x}+\sqrt{1+\sin x}}_{A}\right)}=\lim\limits_{x\to 0}\dfrac{\sin x\left(x-\cos x\right)}{x^3.A.\cos x}$

$=\lim\limits_{x\to 0}\dfrac{2\sin x\sin^2\dfrac{x}{2}}{x^3.A.\cos x}=\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right).\left(\dfrac{\sin\dfrac{x}{2}}{\dfrac{x}{2}}\right)^2.\dfrac{1}{2A.\cos x}=\dfrac{1}{4}$.

3). $\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{\tan(x-1)}=\lim\limits_{x\to 1}\dfrac{x+3-4}{\left(\sqrt{x+3}+2\right)\tan(x-1)}=\lim\limits_{x\to 1}\dfrac{x-1}{\tan(x-1)}\dfrac{1}{\sqrt{x+3}+2}$

(Vì $\lim\limits_{x\to 1}\dfrac{x-1}{\tan(x-1)}=1,\ \lim\limits_{x\to 1}\dfrac{1}{\sqrt{x+3}+2}=\dfrac{1}{4}$)

Vậy $L=\dfrac{1}{4}$.

4). $L=\lim\limits_{x\to-\frac{\pi}{2}}\dfrac{\cos x}{x+\dfrac{\pi}{2}}$. Đặt $t=x+\dfrac{\pi}{2}$, vì $x\to-\dfrac{\pi}{2}\Rightarrow t\to 0$

$L=\lim\limits_{t\to 0}\dfrac{\cos\left(t-\dfrac{\pi}{2}\right)}{t}=\lim\limits_{t\to 0}\dfrac{\sin t}{t}=1$.

5). $L=\lim\limits_{x\to\pi}\dfrac{1+\cos x}{\left(x-\pi\right)^2}$. Đặt $t=x-\pi$, vì $x\to\pi\Rightarrow t\to 0$

$L=\lim\limits_{t\to 0}\dfrac{1+\cos(t+\pi)}{t^2}=\lim\limits_{t\to 0}\dfrac{1-\cos t}{t^2}=\lim\limits_{t\to 0}\dfrac{2\sin^2\dfrac{t}{2}}{t^2}=\lim\limits_{t\to 0}\dfrac{1}{2}\left(\dfrac{\sin\dfrac{t}{2}}{\dfrac{t}{2}}\right)^2=\dfrac{1}{2}$.

6). $L=\lim\limits_{x\to 1}\dfrac{\sin(x-1)}{x^2-4x+3}=\lim\limits_{x\to 1}\dfrac{\sin(x-1)}{(x-1)(x-3)}$. Đặt $t=x-1$, vì $x\to 1\Rightarrow t\to 0$

$L=\lim\limits_{t\to 0}\dfrac{\sin t}{t.(t-2)}=\lim\limits_{t\to 0}\dfrac{\sin t}{t}\cdot\dfrac{1}{t-2}=-\dfrac{1}{2}$.

7). $L=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin x-1}{4\cos^2x-3}=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin x-1}{4\left(1-\sin^2x\right)-3}=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin x-1}{1-4\sin^2x}$

$=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin x-1}{\left(1-2\sin x\right)\left(1+2\sin x\right)}=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{-1}{1+2\sin x}=-\dfrac{1}{2}$

8). $L=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}\sin x-1}{2\cos^2x-1}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}\sin x-1}{2\left(1-\sin^2x\right)-1}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}\sin x-1}{1-2\sin^2x}$

$=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}\sin x-1}{\left(1-\sqrt{2}\sin x\right)\left(1+\sqrt{2}\sin x\right)}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{-1}{1+\sqrt{2}\sin x}=-\dfrac{1}{2}$.

9). $=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\sin\left(\dfrac{\pi}{6}-x\right)}{1-2\sin x}=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\sin\left(\dfrac{\pi}{6}-x\right)}{-2\left(\sin x-\dfrac{1}{2}\right)}=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\sin\left(x-\dfrac{\pi}{6}\right)}{2\left(\sin x-\sin\dfrac{\pi}{6}\right)}$

$$=\lim_{x\to\frac{\pi}{6}}\frac{2\sin\left(\frac{x}{2}-\frac{\pi}{12}\right)\cos\left(\frac{x}{2}-\frac{\pi}{12}\right)}{4\cos\left(\frac{x}{2}+\frac{\pi}{12}\right)\sin\left(\frac{x}{2}-\frac{\pi}{12}\right)}=\lim_{x\to\frac{\pi}{6}}\frac{1}{2}\frac{\cos\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\cos\left(\frac{x}{2}+\frac{\pi}{12}\right)}=\frac{\sqrt{3}}{3}$$

Câu 4: Tìm các giới hạn sau:

1). $\lim_{x\to\frac{\pi}{6}}\frac{1-2\sin x}{\frac{\pi}{6}-x}$ 2). $\lim_{x\to\frac{\pi}{4}}\frac{\sin\left(\frac{\pi}{4}-x\right)}{1-\sqrt{2}\sin x}$ 3). $\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}-2\cos x}{\sin\left(x-\frac{\pi}{4}\right)}$

4). $\lim_{x\to0}\frac{\sqrt{x^2+1}-\cos 2x}{x^2}$ 5). $\lim_{x\to0}\frac{\sqrt{1+2x}-\cos x-x}{x^2}$ 6). $\lim_{x\to0}\frac{\sqrt[3]{2x+1}-\sqrt{1-x}}{\sin 2x}$

7). $\lim_{x\to\frac{\pi}{3}}\frac{\sin x-\sqrt{3}\cos x}{\sin 3x}$ 8). $\lim_{x\to0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}$ 9). $\lim_{x\to0}\frac{1-\sqrt[3]{\cos x}}{\tan^2 x}$

LỜI GIẢI

1). $$\lim_{x\to\frac{\pi}{6}}\frac{1-2\sin x}{\frac{\pi}{6}-x}=\lim_{x\to\frac{\pi}{6}}\frac{2\sin x-1}{x-\frac{\pi}{6}}=\lim_{x\to\frac{\pi}{6}}\frac{2\left(\sin x-\frac{1}{2}\right)}{x-\frac{\pi}{6}}=\lim_{x\to\frac{\pi}{6}}\frac{2\left(\sin x-\sin\frac{\pi}{6}\right)}{x-\frac{\pi}{6}}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{4\cos\left(\frac{x}{2}+\frac{\pi}{12}\right)\sin\left(\frac{x}{2}-\frac{\pi}{12}\right)}{2\left(\frac{x}{2}-\frac{\pi}{12}\right)}=\lim_{x\to\frac{\pi}{6}}\frac{\sin\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\left(\frac{x}{2}-\frac{\pi}{12}\right)}2\cos\left(\frac{x}{2}+\frac{\pi}{12}\right)=\sqrt{3}$$

2). $$\lim_{x\to\frac{\pi}{4}}\frac{\sin\left(\frac{\pi}{4}-x\right)}{1-\sqrt{2}\sin x}=\lim_{x\to\frac{\pi}{4}}\frac{\sin\left(x-\frac{\pi}{4}\right)}{\sqrt{2}\sin x-1}=\lim_{x\to\frac{\pi}{4}}\frac{\sin\left(x-\frac{\pi}{4}\right)}{\sqrt{2}\left(\sin x-\frac{\sqrt{2}}{2}\right)}=\lim_{x\to\frac{\pi}{4}}\frac{\sin\left(x-\frac{\pi}{4}\right)}{\sqrt{2}\left(\sin x-\sin\frac{\pi}{4}\right)}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{2\sin\left(\frac{x}{2}-\frac{\pi}{8}\right)\cos\left(\frac{x}{2}-\frac{\pi}{8}\right)}{\sqrt{2}\cos\left(\frac{x}{2}+\frac{\pi}{8}\right)\sin\left(\frac{x}{2}-\frac{\pi}{8}\right)}=\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\cos\left(\frac{x}{2}-\frac{\pi}{8}\right)}{\cos\left(\frac{x}{2}+\frac{\pi}{8}\right)}=2$$

3). $$\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}-2\cos x}{\sin\left(x-\frac{\pi}{4}\right)}=\lim_{x\to\frac{\pi}{4}}\frac{-2\left(\cos x-\frac{\sqrt{2}}{2}\right)}{\sin\left(x-\frac{\pi}{4}\right)}=\lim_{x\to\frac{\pi}{4}}\frac{-2\left(\cos x-\cos\frac{\pi}{4}\right)}{\sin\left(x-\frac{\pi}{4}\right)}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{4\sin\left(\frac{x}{2}+\frac{\pi}{8}\right)\sin\left(\frac{x}{2}-\frac{\pi}{8}\right)}{2\sin\left(\frac{x}{2}-\frac{\pi}{8}\right)\cos\left(\frac{x}{2}-\frac{\pi}{8}\right)}=\lim_{x\to\frac{\pi}{4}}\frac{2\sin\left(\frac{x}{2}+\frac{\pi}{8}\right)}{\cos\left(\frac{x}{2}-\frac{\pi}{8}\right)}=\sqrt{2}$$

4). $\lim_{x\to0}\frac{\sqrt{x^2+1}-\cos 2x}{x^2}$. Đặt $f(x)=\frac{\sqrt{x^2+1}-\cos 2x}{x^2}$

$$\lim_{x\to0}\frac{\sqrt{x^2+1}-1+1-\cos 2x}{x^2}=\lim_{x\to0}\frac{\sqrt{x^2+1}-1}{x^2}+\lim_{x\to0}\frac{1-\cos 2x}{x^2}$$

• Tính $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+1}-1}{x^2}=\lim\limits_{x\to 0}\dfrac{x^2+1-1}{x^2\left(\sqrt{x^2+1}+1\right)}=\lim\limits_{x\to 0}\dfrac{1}{\sqrt{x^2+1}+1}=\dfrac{1}{2}$

• Tính $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{x^2}=\lim\limits_{x\to 0}\dfrac{2\sin^2 x}{x^2}=2\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)^2=2$

Vậy $\lim\limits_{x\to 0}f(x)=\dfrac{1}{2}+2=\dfrac{5}{2}$

5). $L=\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}-\cos x-x}{x^2}$. Đặt $f(x)=\dfrac{\sqrt{1+2x}-\cos x-x}{x^2}$

$L=\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}-(1+x)+1-\cos x}{x^2}=\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}-(1+x)}{x^2}+\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}$

• Tính $\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}-(1+x)}{x^2}=\lim\limits_{x\to 0}\dfrac{1+2x-(1+x)^2}{x^2\left(\sqrt{1+2x}-(1+x)\right)}$

$=\lim\limits_{x\to 0}\dfrac{-x^2}{x^2\left(\sqrt{1+2x}-(1+x)\right)}=\lim\limits_{x\to 0}\dfrac{-1}{\left(\sqrt{1+2x}+(1+x)\right)}=-\dfrac{1}{2}$

• Tính $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}=\lim\limits_{x\to 0}\dfrac{2\sin^2\frac{x}{2}}{x^2}=\lim\limits_{x\to 0}\dfrac{1}{2}\left(\dfrac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2=\dfrac{1}{2}$

Vậy $\lim\limits_{x\to 0}f(x)=-\dfrac{1}{2}+\dfrac{1}{2}=0$.

6). $L=\lim\limits_{x\to 0}\dfrac{\sqrt[3]{2x+1}-\sqrt{1-x}}{\sin 2x}$. Đặt $f(x)=\dfrac{\sqrt[3]{2x+1}-\sqrt{1-x}}{\sin 2x}$

$L=\lim\limits_{x\to 0}\dfrac{\sqrt[3]{2x+1}-1+1-\sqrt{1-x}}{\sin 2x}=\lim\limits_{x\to 0}\dfrac{\sqrt[3]{2x+1}-1}{\sin 2x}+\lim\limits_{x\to 0}\dfrac{1-\sqrt{1-x}}{\sin 2x}$

• Tính $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{2x+1}-1}{\sin 2x}=\lim\limits_{x\to 0}\dfrac{2x+1-1}{\sin 2x\left[\left(\sqrt[3]{2x+1}\right)^2+\sqrt[3]{2x+1}+1\right]}$

$=\lim\limits_{x\to 0}\dfrac{2x}{2\sin x\cos x\left[\left(\sqrt[3]{2x+1}\right)^2+\sqrt[3]{2x+1}+1\right]}=\lim\limits_{x\to 0}\dfrac{x}{\sin x}\cdot\dfrac{1}{\cos x\left[\left(\sqrt[3]{2x+1}\right)^2+\sqrt[3]{2x+1}+1\right]}=\dfrac{1}{3}$

• Tính $\lim\limits_{x\to 0}\dfrac{1-\sqrt{1-x}}{\sin 2x}=\lim\limits_{x\to 0}\dfrac{x}{2\sin x\cos x\left(1+\sqrt{1-x}\right)}=\lim\limits_{x\to 0}\dfrac{x}{\sin x}\cdot\dfrac{1}{2\cos x\left(1+\sqrt{1-x}\right)}=\dfrac{1}{4}$

Vậy $\lim\limits_{x\to 0}f(x)=\dfrac{1}{3}+\dfrac{1}{4}=\dfrac{7}{12}$

7). $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin x-\sqrt{3}\cos x}{\sin 3x}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin x-\sqrt{3}\cos x}{3\sin x-4\sin^3 x}=\dfrac{\sin^2 x-3\cos^2 x}{\sin x\left(3-4\sin^2 x\right)\left(\sin x+\sqrt{3}\cos x\right)}$

$=\dfrac{4\sin^2 x-3}{\sin x\left(3-4\sin^2 x\right)\left(\sin x+\sqrt{3}\cos x\right)}=\dfrac{-1}{\sin x\left(\sin x+\sqrt{3}\cos x\right)}=\dfrac{-2}{3}$

8). $L=\lim\limits_{x\to 0}\dfrac{1-\cos x\sqrt{\cos 2x}}{x^2}$

$$L=\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}=\lim_{x\to 0}\frac{1-\cos^2 x\cos 2x}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}$$

$$=\lim_{x\to 0}\frac{\cos^2 x+\sin^2 x-\cos^2 x\cos 2x}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}=\lim_{x\to 0}\frac{\cos^2 x\left(1-\cos 2x\right)+\sin^2 x}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}$$

$$=\lim_{x\to 0}\frac{2\sin^2 x\cos^2 x+\sin^2 x}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}=\lim_{x\to 0}\frac{\sin^2 x}{x^2}\cdot\frac{2\cos^2 x+1}{1+\cos x\sqrt{\cos 2x}}=\frac{3}{2}$$

9). $L=\lim\limits_{x\to 0}\dfrac{1-\sqrt[3]{\cos x}}{\tan^2 x}$

$$L=\lim_{x\to 0}\frac{1-\cos x}{\tan^2 x\left[1+\sqrt[3]{\cos x}+\left(\sqrt[3]{\cos x}\right)^2\right]}=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}\cos^2 x}{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}\left[1+\sqrt[3]{\cos x}+\left(\sqrt[3]{\cos x}\right)^2\right]}$$

$$=\lim_{x\to 0}\frac{\cos^2 x}{2\cos^2\frac{x}{2}\left[1+\sqrt[3]{\cos x}+\left(\sqrt[3]{\cos x}\right)^2\right]}=\frac{1}{6}.$$

Câu 5: Tìm các giới hạn sau:

1). $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt[3]{\tan x}-1}{2\sin^2 x-1}$ 2). $\lim\limits_{x\to 0}\dfrac{1-\cos x\cos 2x}{x^2}$ 3). $\lim\limits_{x\to 0}\left(\dfrac{2}{\sin 2x}-\cot x\right)$

4). $\lim\limits_{x\to 0}\dfrac{1-\sqrt{2x+1}+\sin x}{\sqrt{3x+4}-2-x}$ 5). $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+1}-\cos x}{x^2}$ 6). $L=\lim\limits_{x\to 0}\dfrac{1-\sin 2x-\cos 2x}{1+\sin 2x-\cos 2x}$

7). $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\cos 3x+2\cos 2x+2}{\sin 3x}$ 8). $\lim\limits_{x\to 0}\dfrac{\cos\left(\frac{\pi}{2}\cos x\right)}{\sin^2\frac{x}{2}}$ 9). $\lim\limits_{x\to 0}\dfrac{1-\cos x}{\left(1-\sqrt{1-x}\right)^2}$

LỜI GIẢI

1). $L=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt[3]{\tan x}-1}{2\sin^2 x-1}$

$$L=\lim_{x\to\frac{\pi}{4}}\frac{\tan x-1}{\left(\sin^2 x-\cos^2 x\right)\left[\left(\sqrt[3]{\tan x}\right)^2+\sqrt[3]{\tan x}+1\right]}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{\cos x\left(\sin x-\cos x\right)\left(\sin x+\cos x\right)\left[\left(\sqrt[3]{\tan x}\right)^2+\sqrt[3]{\tan x}+1\right]}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{1}{\cos x\left(\sin x+\cos x\right)\left[\left(\sqrt[3]{\tan x}\right)^2+\sqrt[3]{\tan x}+1\right]}=\frac{1}{3}$$

2). $\lim\limits_{x\to 0}\dfrac{1-\cos x\cos 2x}{x^2}$

$$=\lim_{x\to 0}\frac{\sin^2 x+\cos^2 x-\cos x\cos 2x}{x^2}=\lim_{x\to 0}\frac{\sin^2 x}{x^2}+\lim_{x\to 0}\frac{\cos x\left(\cos x-\cos 2x\right)}{x^2}$$

$$=1+\lim_{x\to 0}\frac{2\cos x\sin\frac{3x}{2}\sin\frac{x}{2}}{x^2}=1+\lim_{x\to 0}\left(\frac{\sin\frac{3x}{2}}{\frac{3x}{2}}\right)\cdot\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)\cdot\frac{9}{8}\cdot\cos x=1+\frac{9}{8}=\frac{17}{8}$$

3). $L=\lim_{x\to 0}\left(\frac{2}{\sin 2x}-\cot x\right)$

$$L=\lim_{x\to 0}\left(\frac{1}{\sin x\cos x}-\frac{\cos x}{\sin x}\right)=\lim_{x\to 0}\frac{1-\cos^2 x}{\sin x\cos x}=\lim_{x\to 0}\frac{\sin^2 x}{\sin x\cos x}=\lim_{x\to 0}\tan x=0$$

4). $L=\lim_{x\to 0}\frac{1-\sqrt{2x+1}+\sin x}{\sqrt{3x+4}-2-x}$

$$L=\lim_{x\to 0}\frac{1-\sqrt{2x+1}+\sin x}{\sqrt{3x+4}-2-x}=\lim_{x\to 0}\frac{1-\sqrt{2x+1}}{\sqrt{3x+4}-2-x}+\lim_{x\to 0}\frac{\sin x}{\sqrt{3x+4}-2-x}$$

$$=\lim_{x\to 0}\frac{-2x\left(\sqrt{3x+4}+2+x\right)}{\left(-x^2-x\right)\left(1+\sqrt{2x+1}\right)}+\lim_{x\to 0}\frac{\left(\sqrt{3x+4}+2+x\right)\sin x}{-x^2-x}$$

$$=\lim_{x\to 0}\frac{2\left(\sqrt{3x+4}+2+x\right)}{(x+1)\left(1+\sqrt{2x+1}\right)}+\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\sqrt{3x+4}+2+x}{-x-1}=4-4=0$$

5). $\lim_{x\to 0}\frac{\sqrt{x^2+1}-\cos x}{x^2}$

$$I=\lim_{x\to 0}\frac{\left(\sqrt{1+x^2}-1\right)+(1-\cos x)}{x^2}=\lim_{x\to 0}\left[\frac{\left(\sqrt{1+x^2}-1\right)}{x^2}+\frac{1-\cos x}{x^2}\right]$$

$$=\lim_{x\to 0}\left(\frac{x^2}{x^2(\sqrt{1+x^2}+1)}+\frac{2\sin^2\frac{x}{2}}{x^2}\right)=\lim_{x\to 0}\left(\frac{1}{\sqrt{1+x^2}+1}+\frac{\sin^2\frac{x}{2}}{2\frac{x^2}{4}}\right)=1$$

6). $L=\lim_{x\to 0}\frac{1-\sin 2x-\cos 2x}{1+\sin 2x-\cos 2x}$

$$L=\lim_{x\to 0}\frac{1-\sin 2x-\cos 2x}{1+\sin 2x-\cos 2x}=\lim_{x\to 0}\frac{1-\cos 2x-\sin 2x}{1-\cos 2x+\sin 2x}=\lim_{x\to 0}\frac{2\sin^2 x-2\sin x\cos x}{2\sin^2 x+2\sin x\cos x}$$

$$=\lim_{x\to 0}\frac{2\sin x\left(\sin x-\cos x\right)}{2\sin x\left(\sin x+\cos x\right)}=\lim_{x\to 0}\frac{\sin x-\cos x}{\sin x+\cos x}=-1$$

7). $\lim_{x\to\frac{\pi}{3}}\frac{\cos 3x+2\cos 2x+2}{\sin 3x}=\lim_{x\to\frac{\pi}{3}}\frac{4\cos^3 x+4\cos^2 x-3\cos x}{3\sin x-4\sin^3 x}$

$$=\lim_{x\to\frac{\pi}{3}}\frac{\cos x\left(2\cos x+3\right)\left(2\cos x-1\right)}{\sin x\left(2\cos x-1\right)\left(2\cos x+1\right)}=$$

8). $\lim\limits_{x\to 0}\dfrac{\cos\left(\dfrac{\pi}{2}\cos x\right)}{\sin^2\dfrac{x}{2}}$

$$=\lim\limits_{x\to 0}\frac{\sin\left(\dfrac{\pi}{2}-\dfrac{\pi}{2}\cos x\right)}{\sin^2\dfrac{x}{2}}=\lim\limits_{x\to 0}\frac{\sin\left(\dfrac{\pi}{2}2\sin^2\dfrac{x}{2}\right)}{\sin^2\dfrac{x}{2}}=\lim\limits_{x\to 0}\left[\pi.\frac{\sin\left(\pi\sin^2\dfrac{x}{2}\right)}{\pi\sin^2\dfrac{x}{2}}\right]=\pi$$

9). $\lim\limits_{x\to 0}\dfrac{1-\cos x}{\left(1-\sqrt{1-x}\right)^2}$

$$=\lim\limits_{x\to\infty}\frac{(1-\cos x)\left(1+\sqrt{1-x}\right)^2}{\left(1-\sqrt{1-x}\right)^2\left(1+\sqrt{1-x}\right)^2}=\lim\limits_{x\to 0}\frac{2\sin^2\left(\dfrac{x}{2}\right)\left(1+\sqrt{1-x}\right)^2}{4.\left(\dfrac{x}{2}\right)^2}=\lim\limits_{x\to 0}\frac{\left(1+\sqrt{1-x}\right)^2}{2}=2$$