Giải các phương trình sau:

1). $\sqrt{2}\cos\left(\frac{\pi}{4}-x\right).\frac{1+\cos 2x}{\sin x}=1+\cot x$

2). $2\sin x+\cos 3x+\sin 2x=1+\sin 4x$

3). $\cos x+\tan x=1+\tan x.\sin x$

4). $\sin 3x+\cot^2 x=\frac{3\sin^2 x-7\sin^3 x+2\sin^4 x+1}{\sin^2 x}$

5). $(\tan x+1).\sin^2 x+\cos 2x+2=3(\cos x+\sin x).\sin x$

6). $3\tan^3 x-\tan x+\frac{3(1+\sin x)}{\cos^2 x}-8\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)=0$.

7). $3\cot^2 x+2\sqrt{2}\sin^2 x=(2+3\sqrt{2})\cos x$

8). $\cos x.\cos\frac{x}{2}.\cos\frac{3x}{2}-\sin x.\sin\frac{x}{2}.\sin\frac{3x}{2}=\frac{1}{2}$

LỜI GIẢI

1). $\sqrt{2}\cos\left(\frac{\pi}{4}-x\right).\frac{1+\cos 2x}{\sin x}=1+\cot x$

Điều kiện $\sin x\neq 0 \Leftrightarrow x\neq k\pi$

$\Leftrightarrow (\cos x+\sin x).\frac{2\cos^2 x}{\sin x}=\frac{\sin x+\cos x}{\sin x}$

$\Leftrightarrow (\sin x+\cos x)(2\cos^2 x-1)=0$

$\Leftrightarrow (\sin x+\cos x)\cos 2x=0$

$$\Leftrightarrow \left[\begin{array}{l}\sin x+\cos x=0\\ \cos 2x=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=0\\ 2x=\frac{\pi}{2}+k\pi\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=-\frac{\pi}{4}+k\pi\\ x=\frac{\pi}{4}+\frac{k\pi}{2}\end{array}\right. (k\in ¢)$$

So với điều kiện nghiệm của phương trình $x=-\frac{\pi}{4}+k\pi$, $x=\frac{\pi}{4}+\frac{k\pi}{2}$ $(k\in ¢)$

2). $2\sin x+\cos 3x+\sin 2x=1+\sin 4x$

$\Leftrightarrow 2\sin x+\cos 3x=1+\sin 4x-\sin 2x \quad \Leftrightarrow 2\sin x+\cos 3x=1+2\cos 3x.\sin x$

$\Leftrightarrow\ (2\sin x-1)+\cos 3x-2\cos 3x.\sin x=0\ \ \Leftrightarrow\ (2\sin x-1)-\cos 3x(2\sin x-1)=0$

$\Leftrightarrow\ (2\sin x-1)(1-\cos 3x)=0\qquad \Leftrightarrow\ 2\sin x=1$ hoặc $1=\cos 3x$

Với $2\sin x=1\Leftrightarrow \sin x=\dfrac{1}{2}\Leftrightarrow x=\dfrac{\pi}{6}+k2\pi$ hoặc $x=\dfrac{5\pi}{6}+k2\pi,(k\in\mathbb{Z})$

Với $\cos 3x=1\ \Leftrightarrow\ 3x=k2\pi\ \Leftrightarrow\ x=\dfrac{k2\pi}{3}\ \ (k\in\mathbb{Z})$

Kết luận nghiệm của phương trình: $x=\dfrac{\pi}{6}+k2\pi\ ,\ x=\dfrac{5\pi}{6}+k2\pi\ ,\ x=\dfrac{k2\pi}{3}\ \ (k\in\mathbb{Z})$

3). $\cos x+\tan x=1+\tan x.\sin x$

Điều kiện $\cos x\neq 0\Leftrightarrow x\neq\dfrac{\pi}{2}+k\pi,(k\in\mathbb{Z})$

$\Leftrightarrow\ \cos x+\dfrac{\sin x}{\cos x}=1+\dfrac{\sin^2 x}{\cos x}$

$\Leftrightarrow\ \cos^2 x+\sin x=\cos x+\sin^2 x$

$\Leftrightarrow\ \cos^2 x-\sin^2 x+\sin x-\cos x=0$

$\Leftrightarrow\ (\cos x-\sin x)(\cos x+\sin x)-(\cos x-\sin x)=0$

$\Leftrightarrow\ (\cos x-\sin x)(\cos x+\sin x-1)=0\Leftrightarrow \cos x-\sin x=0$ hoặc $\cos x+\sin x=1$

Với $\cos x-\sin x=0\ \Leftrightarrow\ \sqrt{2}\cos\left(x+\dfrac{\pi}{4}\right)=0\ \Leftrightarrow\ \ x=\dfrac{\pi}{4}+k\pi\ \ (k\in\mathbb{Z})$

Với $\cos x+\sin x=1\Leftrightarrow \cos\left(x-\dfrac{\pi}{4}\right)=\dfrac{1}{\sqrt{2}}\Leftrightarrow x=\dfrac{\pi}{2}+k2\pi$ hoặc $x=k2\pi,(k\in\mathbb{Z})$

So với điều kiện nghiệm của phương trình : $x=\dfrac{\pi}{4}+k\pi\ ,\ x=k2\pi\ \ (k\in\mathbb{Z})$

4). $\sin 3x+\cot^2 x=\dfrac{3\sin^2 x-7\sin^3 x+2\sin^4 x+1}{\sin^2 x}\qquad (1)$

Điều kiện $\sin x\neq 0$

$(1)\ \Leftrightarrow\ \sin 3x+\cot^2 x=3-7\sin x+2\sin^2 x+\dfrac{1}{\sin^2 x}$

$\Leftrightarrow\ 3\sin x-4\sin^3 x+\cot^2 x=3-7\sin x+2\sin^2 x+1+\cot^2 x$

$\Leftrightarrow\ 4\sin^3 x+2\sin^2 x-10\sin x+4=0\Leftrightarrow \sin x=-2\vee \sin x=1\vee \sin x=\dfrac{1}{2}$

Với $\sin x=\frac{1}{2}\Leftrightarrow x=\frac{\pi}{6}+k2\pi$ hoặc $x=\frac{5\pi}{6}+k2\pi,(k\in ¢)$

Với $\sin x=1 \Leftrightarrow x=\frac{\pi}{2}+k2\pi\ (k\in ¢)$

Với $\sin x=-2$ (loại).

Vậy nghiệm của phương trình: $x=\frac{\pi}{6}+k2\pi$, $x=\frac{5\pi}{6}+k2\pi$, $x=\frac{\pi}{2}+k2\pi\ (k\in ¢)$

5). $(\tan x+1).\sin^2 x+\cos 2x+2=3(\cos x+\sin x).\sin x$ (1)

Điều kiện : $\cos x\neq 0$

Chia hai vế của (1) cho $\cos^2 x$ ta được :

$$\frac{(\tan x+1).\sin^2 x}{\cos^2 x}+\frac{2\cos^2 x}{\cos^2 x}+\frac{1}{\cos^2 x}=\frac{3(\cos x+\sin x).\sin x}{\cos^2 x}$$

$$(\tan x+1).\tan^2 x+2+1+\tan^2 x=3\frac{\cos x+\sin x}{\cos x}\tan x$$

$$\Leftrightarrow \tan^3 x+\tan^2 x+2+1+\tan^2 x=3(1+\tan x).\tan x$$

$$\Leftrightarrow \tan^3 x-\tan^2 x-3\tan x+3=0\Leftrightarrow \tan x=1 \text{ hoặc } \tan x=\pm\sqrt{3}$$

$$\Leftrightarrow x=\frac{\pi}{4}+k\pi \text{ hoặc } x=\pm\frac{\pi}{3}+k\pi\ (k\in ¢)$$

Kết luận nghiệm của phương trình : $x=\frac{\pi}{4}+k\pi$, $x=\pm\frac{\pi}{3}+k\pi\ (k\in ¢)$.

6). $3\tan^3 x-\tan x+\frac{3(1+\sin x)}{\cos^2 x}-8\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)=0$.

Điều kiện : $\cos x\neq 0\,(*)$ Phương trình :

$$\Leftrightarrow 3\tan^3 x-\tan x+\frac{3(1+\sin x)}{1-\sin^2 x}-4\left(1+\cos\left(\frac{\pi}{2}-x\right)\right)=0$$

$$\Leftrightarrow 3\tan^3 x-\tan x+\frac{3}{(1-\sin x)}-4(1+\sin x)=0$$

$$\Leftrightarrow 3\tan^3 x-\tan x+\frac{3}{1-\sin x}-4(1+\sin x)=0$$

$$\Leftrightarrow \tan x(3\tan^2 x-1)+\frac{3-4(1+\sin x)(1-\sin x)}{1-\sin x}=0$$

$$\Leftrightarrow \tan x\left(3.\frac{\sin^2 x}{\cos^2 x}-1\right)+\frac{3-4\cos^2 x}{1-\sin x}=0$$

$$\Leftrightarrow \tan x\left(\frac{3-4\cos^2 x}{\cos^2 x}\right)+\frac{3-4\cos^2 x}{1-\sin x}=0$$

$$\Leftrightarrow \left(3-4\cos^2 x\right)\left[\frac{\tan x}{\cos^2 x}+\frac{1}{1-\sin x}\right]=0$$

$$\Leftrightarrow 3-4\cos^2 x=0 \vee \frac{\tan x}{\cos^2 x}+\frac{1}{1-\sin x}=0$$

Với $3-4\cos^2 x=0 \Leftrightarrow \cos 2x=\frac{1}{2} \Leftrightarrow 2x=\pm\frac{\pi}{3}+k2\pi,\left(k\in ¢\right)$.

Với $\frac{\tan x}{\cos^2 x}+\frac{1}{1-\sin x}=0 \Leftrightarrow \frac{\sin x}{\cos x\left(1-\sin x\right)\left(1+\sin x\right)}+\frac{1}{1-\sin x}=0$

$$\Leftrightarrow \frac{\sin x+\cos x\left(1+\sin x\right)}{\cos x\left(1-\sin x\right)\left(1+\sin x\right)}=0 \Leftrightarrow \sin x+\cos x+\sin x\cos x=0 \ (1)$$

Đặt $t=\sin x+\cos x \Rightarrow \sin x\cos x=\frac{t^2-1}{2}$, điều kiện $|t|\le\sqrt{2}$

$\left(1\right)\Leftrightarrow t^2+2t-1=0 \Leftrightarrow t=-1+\sqrt{2} \vee t=-1-\sqrt{2}$, so với điều kiện nhận $t=-1+\sqrt{2}$

$$\Leftrightarrow \sin x+\cos x=-1+\sqrt{2} \Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=\frac{2-\sqrt{2}}{2}$$

$$\Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{4}=\arcsin\frac{2-\sqrt{2}}{2}+k2\pi \\ x+\frac{\pi}{4}=\pi-\arcsin\frac{2-\sqrt{2}}{2}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\frac{\pi}{4}+\arcsin\frac{2-\sqrt{2}}{2}+k2\pi \\ x=\frac{3\pi}{4}-\arcsin\frac{2-\sqrt{2}}{2}+k2\pi \end{array}\right.,\left(k\in ¢\right)$$

7). $3\cot^2 x+2\sqrt{2}\sin^2 x=(2+3\sqrt{2})\cos x$

$$\Leftrightarrow \left(3\cot^2 x-3\sqrt{2}\cos x\right)+\left(2\sqrt{2}\sin^2 x-2\cos x\right)=0$$

$$\Leftrightarrow 3\left(\frac{\cos^2 x}{\sin^2 x}-\sqrt{2}\cos x\right)-2\left(\sqrt{2}\sin^2 x-\cos x\right)=0$$

$$\Leftrightarrow 3\cos x\left(\frac{\cos x-\sqrt{2}\sin^2 x}{\sin^2 x}\right)+2\left(\cos x-\sqrt{2}\sin^2 x\right)=0$$

$$\Leftrightarrow \left(\cos x-\sqrt{2}\sin^2 x\right)\left(\frac{3\cos x}{\sin^2 x}+2\right)=0$$

$\Leftrightarrow \cos x-\sqrt{2}\sin^2 x=0 \vee \frac{3\cos x}{\sin^2 x}+2=0$

Với $\cos x-\sqrt{2}\sin^2 x=0 \Leftrightarrow \cos x-\sqrt{2}\left(1-\cos^2 x\right)=1 \Leftrightarrow \sqrt{2}\cos^2 x+\cos x-\sqrt{2}=0$

$\Leftrightarrow \cos x=\frac{\sqrt{2}}{2} \vee \cos x=-\sqrt{2}$ (loại).

Với $\cos x=\frac{\sqrt{2}}{2} \Leftrightarrow x=\pm\frac{\pi}{4}+k2\pi$.

Với $\frac{3\cos x}{\sin^2 x}+2=0 \Leftrightarrow 3\cos x+2\sin^2 x=0 \Leftrightarrow 3\cos x+2\left(1-\cos^2 x\right)=0$

$\Leftrightarrow -2\cos^2 x+3\cos x+2=0 \Leftrightarrow \cos x=-\frac{1}{2} \vee \cos x=-2$.

Với $\cos x=-\frac{1}{2} \Leftrightarrow x=\pm\frac{2\pi}{3}+k2\pi,\left(k\in ¢\right)$

8). $\cos x.\cos\frac{x}{2}.\cos\frac{3x}{2}-\sin x.\sin\frac{x}{2}.\sin\frac{3x}{2}=\frac{1}{2}$

$\Leftrightarrow \frac{1}{2}\left(\cos x+\cos 2x\right)\cos x-\frac{1}{2}\left(\cos x-\cos 2x\right)\sin x=\frac{1}{2}$

$\Leftrightarrow \cos^2 x+\cos 2x\cos x-\sin x\cos x+\cos 2x.\sin x=1$

$\Leftrightarrow \cos 2x\left(\sin x+\cos x\right)-\sin x\cos x-\left(1-\cos^2 x\right)=0$

$\Leftrightarrow \cos 2x\left(\sin x+\cos x\right)-\sin x\cos x-\sin^2 x=0$

$\Leftrightarrow \cos 2x\left(\sin x+\cos x\right)-\sin x\left(\cos x+\sin x\right)=0$

$\Leftrightarrow \left(\sin x+\cos x\right)\left(\cos 2x-\sin x\right)=0 \Leftrightarrow \left[\begin{array}{l}\sin x+\cos x=0\\ \cos 2x-\sin x=0\end{array}\right.$

Với $\sin x+\cos x=0 \Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=0 \Leftrightarrow x+\frac{\pi}{4}=k\pi \Leftrightarrow x=-\frac{\pi}{4}+k\pi,\left(k\in ¢\right)$

Với $\cos 2x-\sin x=0 \Leftrightarrow 2\sin^2 x+\sin x-1=0 \Leftrightarrow \sin x=-1$ hoặc $\sin x=\frac{1}{2}$

$\Leftrightarrow x=-\frac{\pi}{2}+k2\pi$ hoặc $x=\frac{\pi}{6}+k2\pi$ hoặc $x=\frac{5\pi}{6}+k2\pi,\left(k\in ¢\right)$