8). $\sin^4 x + \cos^{15} x = 1$

$\sin^4 x + \cos^{15} x = \sin^2 x + \cos^2 x \Leftrightarrow \sin^4 x - \sin^2 x = \cos^2 x - \cos^{15} x$

$\Leftrightarrow \sin^2 x\left(\sin^2 x - 1\right) = \cos^2 x\left(1 - \cos^{13} x\right)(1)$

Vì $\begin{cases} \sin^2 x \ge 0 \\ \sin^2 x - 1 \le 0 \end{cases} \Rightarrow \sin^2 x\left(\sin^2 x - 1\right) \le 0, \forall x$,

và $\begin{cases} \cos^2 x \ge 0 \\ 1 - \cos^{13} x \ge 0 \end{cases} \Rightarrow \cos^2 x\left(1 - \cos^{13} x\right) \ge 0, \forall x$

Từ đó $(1) \Leftrightarrow \begin{cases} \sin^2 x\left(\sin^2 x - 1\right) = 0 \\ \cos^2 x\left(1 - \cos^{13} x\right) = 0 \end{cases}$

$$\Leftrightarrow \begin{cases} \left[\begin{array}{l} \sin^2 x = 0 \\ \sin^2 x = 1 \end{array}\right. \\ \left[\begin{array}{l} \cos^2 x = 0 \\ \cos^{13} x = 1 \end{array}\right. \end{cases} \Leftrightarrow \begin{cases} \left[\begin{array}{l} \sin x = 0 \\ \sin x = \pm 1 \end{array}\right. \\ \left[\begin{array}{l} \cos x = 0 \\ \cos x = 1 \end{array}\right. \end{cases} \Leftrightarrow \begin{cases} \left[\begin{array}{l} x = m\pi \\ x = \dfrac{\pi}{2} + m\pi \end{array}\right. \\ \left[\begin{array}{l} x = \dfrac{\pi}{2} + n\pi \\ x = n2\pi \end{array}\right. \end{cases}, \left(m, n \in ¢\right).$$

Vậy nghiệm của phương trình là: $x = \dfrac{\pi}{2} + k\pi, x = k2\pi, \left(k \in ¢\right)$.

9). $2\cos^2\left(\dfrac{\pi}{2}\cos^2 x\right) = 1 + \cos\left(\pi \sin 2x\right)$

$\Leftrightarrow 2\cos^2\left(\dfrac{\pi}{2}\cos^2 x\right) - 1 = \cos\left(\pi \sin 2x\right)$

$\Leftrightarrow \cos\left(\pi\cos^2 x\right) = \cos\left(\pi \sin 2x\right) \Leftrightarrow \left[\begin{array}{l} \pi\cos^2 x = \pi\sin 2x + k2\pi \\ \pi\cos^2 x = -\pi\sin 2x + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \cos^2 x = \sin 2x + k2 \\ \cos^2 x = -\sin 2x + k2 \end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l} \dfrac{1 + \cos 2x}{2} = \sin 2x + k2 \\ \dfrac{1 + \cos 2x}{2} = -\sin 2x + k2 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \cos 2x - 2\sin 2x = 4k - 1 \quad (1) \\ \cos 2x + 2\sin 2x = 4k - 1 \quad (2) \end{array}\right.$

Điều kiện để phương trình (1) hoặc phương trình (2) có nghiệm là:

$1^2 + 2^2 \ge \left(4k - 1\right)^2 \Leftrightarrow 16k^2 - 8k - 4 \le 0 \Leftrightarrow \dfrac{1 - \sqrt{5}}{4} \le k \le \dfrac{1 + \sqrt{5}}{4}$, vì $k \in ¢$ nên chọn $k = 0$

Thay $k = 0$ vào (1) được $\cos 2x - 2\sin 2x = -1 \Leftrightarrow 1 + \cos 2x - 2\sin 2x = 0 \Leftrightarrow 2\cos^2 x - 4\sin x\cos x = 0$

$$\Leftrightarrow 2\cos x\left(\cos x-2\sin x\right)=0 \Leftrightarrow \left[\begin{array}{l}\cos x=0\\ \cos x=2\sin x\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\cos x=0\\ \tan x=\frac{1}{2}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{2}+k\pi\\ x=\arctan\left(\frac{1}{2}\right)+k\pi\end{array}\right.$$

Thay $k=0$ vào (1) được $\cos 2x+2\sin 2x=-1 \Leftrightarrow 1+\cos 2x+2\sin 2x=0 \Leftrightarrow 2\cos^2 x+4\sin x\cos x=0$

$$\Leftrightarrow 2\cos x\left(\cos x+2\sin x\right)=0 \Leftrightarrow \left[\begin{array}{l}\cos x=0\\ \cos x=-2\sin x\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\cos x=0\\ \tan x=-\frac{1}{2}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{2}+k\pi\\ x=\arctan\left(-\frac{1}{2}\right)+k\pi\end{array}\right.$$