4). $2\sin 2x - 3\sqrt{6}\left|\sin x + \cos x\right| + 8 = 0.$

Đặt $t = \left|\sin x + \cos x\right|$ (Đk : $0 \le t \le \sqrt{2}$) $\Rightarrow$ $t^2 = (\sin x + \cos x)^2$ $\Rightarrow \sin 2x = t^2 - 1$

Ta được : $2(t^2 - 1) - 3\sqrt{6}t + 8 = 0 \Leftrightarrow 2t^2 - 3\sqrt{6} + 8 = 0 \Leftrightarrow t = \dfrac{\sqrt{6}}{2} \vee t = \sqrt{6}$ (loại).

Với $t = \dfrac{\sqrt{6}}{2}$ $\Leftrightarrow$ $\left|\sqrt{2}\sin\left(x + \dfrac{\pi}{4}\right)\right| = \dfrac{\sqrt{6}}{2}$ $\Leftrightarrow$ $\left|\sin\left(x + \dfrac{\pi}{4}\right)\right| = \dfrac{\sqrt{3}}{2}$ $\Leftrightarrow$ $\sin\left(x + \dfrac{\pi}{4}\right) = \pm\dfrac{\sqrt{3}}{2}$

Với $\sin\left(x + \dfrac{\pi}{4}\right) = \dfrac{\sqrt{3}}{2}$ $\Leftrightarrow$ $\left[\begin{array}{l} x + \dfrac{\pi}{4} = \dfrac{\pi}{3} + k2\pi \\ x + \dfrac{\pi}{4} = \pi - \dfrac{\pi}{3} + k2\pi \end{array}\right.$ $\Leftrightarrow$ $\left[\begin{array}{l} x = \dfrac{\pi}{12} + k2\pi \\ x = \dfrac{5\pi}{12} + k2\pi \end{array}\right.$ $,(k \in \mathbb{Z})$

Với $\sin\left(x + \dfrac{\pi}{4}\right) = -\dfrac{\sqrt{3}}{2}$ $\Leftrightarrow$ $\left[\begin{array}{l} x + \dfrac{\pi}{4} = -\dfrac{\pi}{3} + k2\pi \\ x + \dfrac{\pi}{4} = \pi + \dfrac{\pi}{3} + k2\pi \end{array}\right.$ $\Leftrightarrow$ $\left[\begin{array}{l} x = -\dfrac{7\pi}{12} + k2\pi \\ x = \dfrac{13\pi}{12} + k2\pi \end{array}\right.$ $,(k \in \mathbb{Z})$

Nghiệm của phương trình: $x = \dfrac{\pi}{12} + k2\pi, x = \dfrac{5\pi}{12} + k2\pi$, $x = -\dfrac{7\pi}{12} + k2\pi$, $x = \dfrac{13\pi}{12} + k2\pi, (k \in \mathbb{Z})$

5). $\cot x - 1 = \dfrac{\cos 2x}{1 + \tan x} + \sin^2 x - \dfrac{1}{2}\sin 2x$ (1) [ĐH A03]

LỜI GIẢI

Điều kiện : $\left\{\begin{array}{l} \sin 2x \ne 0 \\ \tan x \ne -1 \end{array}\right.$

$(1) \Leftrightarrow \dfrac{\cos x}{\sin x} - 1 = \dfrac{\cos^2 x - \sin^2 x}{1 + \dfrac{\sin x}{\cos x}} + \sin x(\sin x - \cos x)$

$\Leftrightarrow \dfrac{\cos x - \sin x}{\sin x} = \dfrac{\cos x(\cos^2 x - \sin^2 x)}{\sin x + \cos x} + \sin x(\sin x - \cos x)$

$\Leftrightarrow \dfrac{\cos x - \sin x}{\sin x} = \cos x(\cos x - \sin x) + \sin x(\sin x - \cos x)$

$\Leftrightarrow (\cos x - \sin x)\left(\sin^2 x - \sin x\cos x + 1\right) = 0$

$\Leftrightarrow \left[\begin{array}{l} \cos x - \sin x = 0 \\ \sin^2 x - \sin x\cos x + 1 = 0 \end{array}\right.$

* $\cos x - \sin x = 0 \Leftrightarrow \sqrt{2}\cos\left(x + \dfrac{\pi}{4}\right) = 0$ $\Leftrightarrow \cos\left(x + \dfrac{\pi}{4}\right) = 0 \Leftrightarrow x + \dfrac{\pi}{4} = \dfrac{\pi}{2} + k\pi \Leftrightarrow x = \dfrac{\pi}{4} + k\pi; k \in \mathbb{Z}$

* $\sin^2 x - \sin x\cos x + 1 = 0 \Leftrightarrow \dfrac{1 - \cos 2x}{2} - \dfrac{\sin 2x}{2} + 1 = 0$

$\Leftrightarrow \sin 2x + \cos 2x - 3 = 0$ (vô nghiệm)

6). $\dfrac{1}{\cos x} - \dfrac{1}{\sin x} = 2\sqrt{2}\cos\left(x + \dfrac{\pi}{4}\right)$ (1) [Dự bị 2 ĐH B04]

LỜI GIẢI

Điều kiện : $\sin 2x \ne 0 \Leftrightarrow 2x \ne k\pi \Leftrightarrow x \ne \dfrac{k\pi}{2}, (k \in \mathbb{Z})$

$(1) \Leftrightarrow \dfrac{\sin x-\cos x}{\sin x\cos x}=2\sqrt{2}\cos\left(x+\dfrac{\pi}{4}\right)$

$\Leftrightarrow -\sqrt{2}\cos\left(x+\dfrac{\pi}{4}\right)=2\sqrt{2}\sin x\cos x\cos\left(x+\dfrac{\pi}{4}\right)$

$\Leftrightarrow \cos\left(x+\dfrac{\pi}{4}\right)(1+\sin 2x)=0 \Leftrightarrow \left[\begin{array}{l}\cos\left(x+\dfrac{\pi}{4}\right)=0\\ \sin 2x=-1\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x+\dfrac{\pi}{4}=\dfrac{\pi}{2}+k\pi\\ 2x=-\dfrac{\pi}{2}+k2\pi\end{array}\right.,(k\in\mathbb{Z})$

$\Leftrightarrow \left[\begin{array}{l}x=\dfrac{\pi}{4}+k\pi\\ x=-\dfrac{\pi}{4}+k\pi\end{array}\right.,(k\in\mathbb{Z}) \Leftrightarrow x=\dfrac{\pi}{4}+\dfrac{k\pi}{2},(k\in\mathbb{Z})$

Nghiệm của phương trình: $x=\dfrac{\pi}{4}+\dfrac{k\pi}{2},(k\in\mathbb{Z})$

7) $\sin 2x-2\sqrt{2}(\sin x+\cos x)-5=0$ (1) [Dự bị 2 ĐH D04]

LỜI GIẢI

Đặt $t=\sin x+\cos x$ với $-\sqrt{2}\le t\le\sqrt{2}$ $\Rightarrow \sin 2x=t^2-1$.

(1) $\Leftrightarrow t^2-2\sqrt{2}t-6=0 \Leftrightarrow t=-\sqrt{2} \vee t=3\sqrt{2}$ (loại).

Với $t=-\sqrt{2} \Leftrightarrow \sin x+\cos x=-\sqrt{2} \Leftrightarrow \sqrt{2}\cos\left(x-\dfrac{\pi}{4}\right)=-\sqrt{2}$

$\Leftrightarrow \cos\left(x-\dfrac{\pi}{4}\right)=-1 \Leftrightarrow x-\dfrac{\pi}{4}=\pi+k2\pi \Leftrightarrow x=\dfrac{5\pi}{4}+k2\pi,(k\in\mathbb{Z})$