

Câu 3: Giải các phương trình sau:

1). $2 \cos\left(2x + \frac{\pi}{6}\right) + 4 \sin x \cos x - 1 = 0$

2). $4 \sin\left(x + \frac{\pi}{4}\right) + 2 \cos\left(x - \frac{\pi}{4}\right) - 3\sqrt{2} = 0$

3). $8 \sin x \sin 2x + 6 \sin\left(x + \frac{\pi}{4}\right) \cos\left(\frac{\pi}{4} - 2x\right) = 5 + 7 \cos x$

4). $2\sqrt{3} \sin\left(x - \frac{\pi}{8}\right) \cos\left(x - \frac{\pi}{8}\right) + 2 \cos^2\left(x - \frac{\pi}{8}\right) = \sqrt{3} + 1$

5). $\frac{1 + \cos x + \cos 2x + \cos 3x}{2 \cos^2 x + \cos x - 1} = \frac{2}{3}(3 - \sqrt{3} \sin x)$

6). $8 \sin x = \frac{\sqrt{3}}{\cos x} + \frac{1}{\sin x}$

7). $2 \cos^3 x + 2 \sin^3 x + 2 \sin^2 x \cos x + 2 \cos^2 x \sin x - \sqrt{2} = 0$

8). $5(\cos x + \sin x) + \sin 3x - \cos 3x = 2\sqrt{2}(2 + \sin 2x)$

LỜI GIẢI

1). $2 \cos\left(2x + \frac{\pi}{6}\right) + 4 \sin x \cos x - 1 = 0$

$\Leftrightarrow 2\left(\cos 2x \cos \frac{\pi}{6} - \sin 2x \sin \frac{\pi}{6}\right) + 2 \sin 2x - 1 = 0 \Leftrightarrow \sqrt{3} \cos 2x + \sin 2x = 1$

$\Leftrightarrow \frac{\sqrt{3}}{2} \cos 2x + \frac{1}{2} \sin 2x = \frac{1}{2} \Leftrightarrow \cos 2x \cos \frac{\pi}{6} + \sin 2x \sin \frac{\pi}{6} = \frac{1}{2}$

$\Leftrightarrow \cos\left(2x - \frac{\pi}{6}\right) = \cos \frac{\pi}{3} \Leftrightarrow \begin{cases} 2x - \frac{\pi}{6} = \frac{\pi}{3} + k2\pi \\ 2x - \frac{\pi}{6} = -\frac{\pi}{3} + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{4} + k\pi \\ x = -\frac{\pi}{12} + k\pi \end{cases}, (k \in \mathbb{Z})$

Vậy nghiệm của phương trình: $x = \frac{\pi}{4} + k\pi, x = -\frac{\pi}{12} + k\pi, (k \in \mathbb{Z})$

2). $4 \sin\left(x + \frac{\pi}{4}\right) + 2 \cos\left(x - \frac{\pi}{4}\right) - 3\sqrt{2} = 0$

$\Leftrightarrow 4 \cdot \frac{\sin x + \cos x}{\sqrt{2}} + 2 \cdot \frac{\sin x + \cos x}{\sqrt{2}} = 3\sqrt{2} \Leftrightarrow \sin x + \cos x = 1 \Leftrightarrow \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) = 1$

$\Leftrightarrow \sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \Leftrightarrow \begin{cases} x + \frac{\pi}{4} = \frac{\pi}{4} + k2\pi \\ x + \frac{\pi}{4} = \pi - \frac{\pi}{4} + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = k2\pi \\ x = \frac{\pi}{2} + k2\pi \end{cases}$

Vậy nghiệm của phương trình: $x = k2\pi, x = \frac{\pi}{2} + k2\pi, (k \in \mathbb{Z})$

3). $8 \sin x \sin 2x + 6 \sin\left(x + \frac{\pi}{4}\right) \cos\left(\frac{\pi}{4} - 2x\right) = 5 + 7 \cos x$

$\Leftrightarrow 4(\cos x - \cos 3x) + 3\left(\sin 3x + \sin\left(\frac{\pi}{2} - x\right)\right) = 5 + 7 \cos x$

$$\Leftrightarrow 4(\cos x - \cos 3x) + 3(\sin 3x + \cos x) = 5 + 7 \cos x \Leftrightarrow 3 \sin 3x - 4 \cos 3x = 5$$

$$\Leftrightarrow \frac{3}{5} \sin 3x - \frac{4}{5} \cos 3x = 1 \Leftrightarrow \sin 3x \cdot \cos \alpha - \cos 3x \cdot \sin \alpha = 1 \Leftrightarrow \sin(3x - \alpha) = 1$$

$$\Leftrightarrow 3x - \alpha = \frac{\pi}{2} + k2\pi \Leftrightarrow x = \frac{\alpha}{3} + \frac{\pi}{6} + \frac{k2\pi}{3}. \text{ (Với } \frac{3}{5} = \cos \alpha, \frac{4}{5} = \sin \alpha)$$

Vậy nghiệm của phương trình: $x = \frac{\alpha}{3} + \frac{\pi}{6} + \frac{k2\pi}{3}$

4). $2\sqrt{3} \sin\left(x - \frac{\pi}{8}\right) \cos\left(x - \frac{\pi}{8}\right) + 2 \sin^2\left(x - \frac{\pi}{8}\right) = \sqrt{3} + 1$

Áp dụng công thức nhân đôi: $2 \sin\left(x - \frac{\pi}{8}\right) \cos\left(x - \frac{\pi}{8}\right) = \sin\left(2x - \frac{\pi}{4}\right)$, và hạ bậc

$$2 \sin^2\left(x - \frac{\pi}{8}\right) = 1 - \cos\left(2x - \frac{\pi}{4}\right) \text{ ta được:}$$

$$\Leftrightarrow \sqrt{3} \sin\left(2x - \frac{\pi}{4}\right) + 1 - \cos\left(2x - \frac{\pi}{4}\right) = \sqrt{3} + 1$$

$$\Leftrightarrow \sqrt{3} \sin\left(2x - \frac{\pi}{4}\right) - \cos\left(2x - \frac{\pi}{4}\right) = \sqrt{3}$$

$$\Leftrightarrow \frac{\sqrt{3}}{2} \sin\left(2x - \frac{\pi}{4}\right) - \frac{1}{2} \cos\left(2x - \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$$

$$\Leftrightarrow \sin\left(2x - \frac{\pi}{4}\right) \cos \frac{\pi}{6} - \cos\left(2x - \frac{\pi}{4}\right) \sin \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\Leftrightarrow \sin\left(2x - \frac{\pi}{4} - \frac{\pi}{6}\right) = \sin \frac{\pi}{3} \Leftrightarrow \begin{cases} 2x - \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{3} + k2\pi \\ 2x - \frac{\pi}{4} - \frac{\pi}{6} = \pi - \frac{\pi}{3} + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{3\pi}{8} + k\pi \\ x = \frac{13\pi}{24} + k\pi \end{cases}$$

Vậy nghiệm của phương trình: $x = \frac{3\pi}{8} + k\pi, x = \frac{13\pi}{24} + k\pi, (k \in \mathbb{Z})$

5). $\frac{1 + \cos x + \cos 2x + \cos 3x}{2 \cos^2 x + \cos x - 1} = \frac{2}{3}(3 - \sqrt{3} \sin x)$ (*). Điều kiện $2 \cos^2 x + \cos x - 1 \neq 0$

$$\Leftrightarrow \frac{(1 + \cos 2x) + (\cos 3x + \cos x)}{(2 \cos^2 x - 1) + \cos x} = \frac{2}{3}(3 - \sqrt{3} \sin x)$$

$$\Leftrightarrow \frac{2 \cos^2 x + 2 \cos 2x \cdot \cos x}{\cos 2x + \cos x} = \frac{2}{3}(3 - \sqrt{3} \sin x)$$

$$\Leftrightarrow \frac{2 \cos x (\cos 2x + \cos x)}{\cos 2x + \cos x} = \frac{2}{3}(3 - \sqrt{3} \sin x)$$

$$\Leftrightarrow 2 \cos x = \frac{2}{3}(3 - \sqrt{3} \sin x) \Leftrightarrow \sqrt{3} \cos x + \sin x = \sqrt{3}$$

$$\Leftrightarrow \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = \frac{\sqrt{3}}{2} \Leftrightarrow \cos x \cdot \cos \frac{\pi}{6} + \sin x \sin \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\Leftrightarrow \cos\left(x - \frac{\pi}{6}\right) = \cos \frac{\pi}{6} \Leftrightarrow \begin{cases} x - \frac{\pi}{6} = \frac{\pi}{6} + k2\pi \\ x - \frac{\pi}{6} = -\frac{\pi}{6} + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{3} + k2\pi \\ x = k2\pi \end{cases}$$

Thay hai họ nghiệm của x vào điều kiện ta thấy thỏa.

Vậy nghiệm của phương trình: $x = \frac{\pi}{3} + k2\pi, x = k2\pi$.

$$6). 8 \sin x = \frac{\sqrt{3}}{\cos x} + \frac{1}{\sin x}. \text{ Điều kiện } \begin{cases} \sin x \neq 0 \\ \cos x \neq 0 \end{cases}$$

$$\text{Quy đồng mẫu được: } \Leftrightarrow 8 \sin^2 x \cos x = \sqrt{3} \sin x + \cos x$$

$$\text{Hạ bậc } \sin^2 x \text{ được: } \Leftrightarrow 4(1 - \cos 2x) \cos x = \sqrt{3} \sin x + \cos x$$

$$\Leftrightarrow 4 \cos x - 4 \cos 2x \cos x = \sqrt{3} \sin x + \cos x$$

$$\Leftrightarrow 4 \cos x - 2(\cos x + \cos 3x) = \sqrt{3} \sin x + \cos x$$

$$\Leftrightarrow \cos x - \sqrt{3} \sin x = 2 \cos 3x \quad \Leftrightarrow \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x = \cos 3x$$

$$\Leftrightarrow \cos x \cdot \cos \frac{\pi}{3} - \sin x \cdot \sin \frac{\pi}{3} = \cos 3x$$

$$\Leftrightarrow \cos \left(x + \frac{\pi}{3} \right) = \cos 3x \Leftrightarrow \begin{cases} 3x = x + \frac{\pi}{3} + k2\pi \\ 3x = -\left(x + \frac{\pi}{3} \right) + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{6} + k\pi \\ x = -\frac{\pi}{12} + \frac{k\pi}{2} \end{cases}$$

So với điều kiện nghiệm của phương trình: $x = \frac{\pi}{6} + k\pi, x = -\frac{\pi}{12} + \frac{k\pi}{2}$.

$$7). 2 \cos^3 x + 2 \sin^3 x + 2 \sin^2 x \cos x + 2 \cos^2 x \sin x - \sqrt{2} = 0$$

$$\Leftrightarrow 2(\cos x + \sin x)(\sin^2 x - \sin x \cos x + \cos^2 x) + 2 \sin x \cos x (\sin x + \cos x) - \sqrt{2} = 0$$

$$\Leftrightarrow 2(\cos x + \sin x)(1 - \sin x \cos x) + 2 \sin x \cos x (\sin x + \cos x) - \sqrt{2} = 0$$

$$\Leftrightarrow 2(\cos x + \sin x) = \sqrt{2} \Leftrightarrow 2\sqrt{2} \cos \left(x - \frac{\pi}{4} \right) = \sqrt{2} \Leftrightarrow \cos \left(x - \frac{\pi}{4} \right) = \frac{1}{2}$$

$$\Leftrightarrow \cos \left(x - \frac{\pi}{4} \right) = \cos \frac{\pi}{3} \Leftrightarrow \begin{cases} x - \frac{\pi}{4} = \frac{\pi}{3} + k2\pi \\ x - \frac{\pi}{4} = -\frac{\pi}{3} + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{7\pi}{12} + k2\pi \\ x = -\frac{\pi}{12} + k2\pi \end{cases}$$

So với điều kiện nghiệm của phương trình: $x = \frac{7\pi}{12} + k2\pi, x = -\frac{\pi}{12} + k2\pi, (k \in \mathbb{Z})$

$$8). 5(\cos x + \sin x) + \sin 3x - \cos 3x = 2\sqrt{2}(2 + \sin 2x)$$

$$\Leftrightarrow 5(\cos x + \sin x) + 3 \sin x - 4 \sin^3 x - (4 \cos^3 x - 3 \cos x) = 2\sqrt{2}(2 + \sin 2x)$$

$$\Leftrightarrow 8(\cos x + \sin x) - 4(\sin^3 x + \cos^3 x) = 2\sqrt{2}(2 + \sin 2x)$$

$$\Leftrightarrow 8(\cos x + \sin x) - 4(\sin x + \cos x)(1 - \sin x \cos x) = 2\sqrt{2}(2 + \sin 2x)$$

$$\Leftrightarrow 4(\sin x + \cos x)(2 - 1 + \sin x \cos x) = 2\sqrt{2}(2 + \sin 2x)$$

$$\Leftrightarrow 4(\sin x + \cos x) \left(1 + \frac{1}{2} \sin 2x \right) = 2\sqrt{2}(2 + \sin 2x)$$

$$\Leftrightarrow 2(\sin x + \cos x)(2 + \sin 2x) = 2\sqrt{2}(2 + \sin 2x)$$

$$\Leftrightarrow \sin x + \cos x = \sqrt{2} \quad (\text{vì } 2 + \sin 2x > 0).$$

$$\Leftrightarrow \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) = \sqrt{2} \Leftrightarrow \sin\left(x + \frac{\pi}{4}\right) = 1 \Leftrightarrow x + \frac{\pi}{4} = \frac{\pi}{2} + k2\pi \Leftrightarrow x = \frac{\pi}{4} + k2\pi$$

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