Câu 10: Tìm các giới hạn sau:

1). $\lim\limits_{x\to 1}\dfrac{2x^4-5x^3+3x^2+x-1}{3x^4-8x^3+6x^2-1}$

2). $\lim\limits_{x\to 0}\dfrac{(1+x)(1+2x)(1+3x)-1}{x}$

3). $\lim\limits_{x\to 0}\dfrac{\sqrt[n]{a+x}-\sqrt[n]{a}}{x}$

4). $\lim\limits_{x\to 0}\dfrac{\sqrt[n]{(1+2x)(1+3x)(1+4x)}-1}{x}$

5). $\lim\limits_{x\to 2}\dfrac{\sqrt{x+2}-\sqrt{2x}}{\sqrt{x-1}-\sqrt{3-x}}$

6). $\lim\limits_{x\to 1}\dfrac{x-1}{\sqrt{x^2+3}+x^3-3x}$

7). $\lim\limits_{x\to 2}\dfrac{\sqrt{x-1}+x^4-3x^3+x^2+3}{\sqrt{2x}-2}$

8). $\lim\limits_{x\to 1}\dfrac{\sqrt{2x-1}+x^2-3x+1}{\sqrt[3]{x-2}+x^2-x+1}$

9). $\lim\limits_{x\to 0}\dfrac{\sqrt[5]{1+5x}-1}{x}$

20). $\lim\limits_{x\to 1}\dfrac{\sqrt[4]{4x-3}-1}{x-1}$

LỜI GIẢI

1). $L=\lim\limits_{x\to 1}\dfrac{2x^4-5x^3+3x^2+x-1}{3x^4-8x^3+6x^2-1}$

Phân tích $2x^4-5x^3+3x^2+x-1=(x-1)(2x^3-3x^2+1)$, bằng sơ đồ Hoocne sau:

	2	-5	3	1	-1
1	2	-3	0	1	0

Phân tích $3x^4-8x^3+6x^2-1=(x-1)(3x^3-5x^2+x+1)$, bằng sơ đồ Hoocne sau:

	3	-8	6	0	-1
1	3	-5	1	1	0

$L=\lim\limits_{x\to 1}\dfrac{(x-1)(2x^3-3x^2+1)}{(x-1)(3x^3-5x^2+x+1)}=\lim\limits_{x\to 1}\dfrac{2x^3-3x^2+1}{3x^3-5x^2+x+1}$ (thay $x=0$ vào tử và mẫu vẫn còn dạng vô định $\dfrac{0}{0}$, nên tiếp tục phân tích đa thức thành nhân tử, cả tử và mẫu).

Phân tích $2x^3-3x^2+1=(x-1)(2x^2-x-1)$, bằng sơ đồ Hoocne sau:

	2	-3	0	1
1	2	-1	-1	0

Phân tích $3x^3-5x^2+x+1=(x-1)(3x^2-2x-1)$, bằng sơ đồ Hoocne sau:

	3	-5	1	1
1	3	-2	-1	0

$L=\lim\limits_{x\to 1}\dfrac{(x-1)(2x^2-x-1)}{(x-1)(3x^2-2x-1)}=\lim\limits_{x\to 1}\dfrac{2x^2-x-1}{3x^2-2x-1}$ (thay $x=0$ vào tử và mẫu vẫn còn dạng vô định $\dfrac{0}{0}$, nên tiếp tục phân tích đa thức thành nhân tử, cả tử và mẫu).

$L=\lim\limits_{x\to 1}\dfrac{(x-1)(2x+1)}{(x-1)(3x+1)}=\lim\limits_{x\to 1}\dfrac{2x+1}{3x+1}=\dfrac{3}{4}$

2). $L=\lim\limits_{x\to 0}\dfrac{(1+x)(1+2x)(1+3x)-1}{x}=\lim\limits_{x\to 0}\dfrac{x(1+2x)(1+3x)}{x}+\lim\limits_{x\to 0}\dfrac{2x(1+3x)}{x}+\lim\limits_{x\to 0}\dfrac{3x}{x}$

$=\lim\limits_{x\to 0}(1+2x)(1+3x)+\lim\limits_{x\to 0}2(1+3x)+3=1+2+3=6$

Tương tự: Tìm $L=\lim\limits_{x\to 0}\frac{(1+x)(1+2x)(1+3x)(1+4x)-1}{x}$

3). $L=\lim\limits_{x\to 0}\frac{\sqrt[n]{a+x}-\sqrt[n]{a}}{x}=\lim\limits_{x\to 0}\frac{x}{x\left(\sqrt[n]{(a+x)^{n-1}}+\sqrt[n]{(a+x)^{n-2}}.\sqrt[n]{a}+\cdots+\sqrt[n]{a^{n-1}}\right)}$

$=\lim\limits_{x\to 0}\frac{1}{\sqrt[n]{(a+x)^{n-1}}+\sqrt[n]{(a+x)^{n-2}}.\sqrt[n]{a}+\cdots+\sqrt[n]{a^{n-1}}}=\frac{1}{n\sqrt[n]{a^{n-1}}}$

4). $L=\lim\limits_{x\to 0}\frac{\sqrt[n]{(1+2x)(1+3x)(1+4x)}-1}{x}$.

Đặt $t=\sqrt[n]{(1+2x)(1+3x)(1+4x)}$. Ta có $x\to 0\Rightarrow t\to 1$

Và $\lim\limits_{x\to 0}\frac{t^n-1}{x}=\lim\limits_{x\to 0}\frac{(1+2x)(1+3x)(1+4x)-1}{x}=9$

Vậy $L=\lim\limits_{x\to 0}\frac{t-1}{x}=\lim\limits_{x\to 0}\frac{t^n-1}{x\left(t^{n-1}+t^{n-2}+\cdots+t+1\right)}=\frac{9}{n}$

5). $L=\lim\limits_{x\to 2}\frac{\sqrt{x+2}-\sqrt{2x}}{\sqrt{x-1}-\sqrt{3-x}}=\lim\limits_{x\to 2}\frac{x+2-2x}{\sqrt{x+2}+\sqrt{2x}}\cdot\frac{\sqrt{x-1}+\sqrt{3-x}}{x-1-(3-x)}$

$=\lim\limits_{x\to 2}\frac{-(x-2)\left(\sqrt{x-1}+\sqrt{3-x}\right)}{2(x-2)\left(\sqrt{x+2}+\sqrt{2x}\right)}=\lim\limits_{x\to 2}\frac{-\left(\sqrt{x-1}+\sqrt{3-x}\right)}{2\left(\sqrt{x+2}+\sqrt{2x}\right)}=-\frac{1}{4}$

6). $\lim\limits_{x\to 1}\frac{x-1}{\sqrt{x^2+3}+x^3-3x}=\lim\limits_{x\to 1}\frac{x-1}{\sqrt{x^2+3}-2+x^3-3x+2}$

$=\lim\limits_{x\to 1}\frac{\frac{x-1}{x-1}}{\frac{\sqrt{x^2+3}-2}{x-1}+\frac{x^3-3x+2}{x-1}}=\lim\limits_{x\to 1}\frac{1}{\frac{x+1}{\sqrt{x^2+3}+2}+x^2+x-2}=2$

7). $L=\lim\limits_{x\to 2}\frac{\sqrt{x-1}+x^4-3x^3+x^2+3}{\sqrt{2x}-2}$

$=\lim\limits_{x\to 2}\frac{\sqrt{x-1}-1+x^4-3x^3+x^2+4}{\sqrt{2x}-2}=\lim\limits_{x\to 2}\frac{\sqrt{x-1}-1}{\sqrt{2x}-2}+\lim\limits_{x\to 2}\frac{x^4-3x^3+x^2+4}{\sqrt{2x}-2}=M+N$

Tính M: $\lim\limits_{x\to 2}\frac{\sqrt{x-1}-1}{\sqrt{2x}-2}=\lim\limits_{x\to 2}\frac{x-1-1}{\sqrt{x-1}+1}\cdot\frac{\sqrt{2x}+2}{2x-4}=\lim\limits_{x\to 2}\frac{\sqrt{2x}+2}{2\left(\sqrt{x-1}+1\right)}=1$

Tính N: $\lim\limits_{x\to 2}\frac{x^4-3x^3+x^2+4}{\sqrt{2x}-2}$

$=\lim\limits_{x\to 2}\frac{(x-1)\left(x^3-x^2-x-2\right)\left(\sqrt{2x}+2\right)}{2x-4}=\lim\limits_{x\to 2}\frac{\left(x^3-x^2-x-2\right)\left(\sqrt{2x}+2\right)}{2}=0$

Vậy $L=1+0=1$

Tương tự: Tìm $\lim\limits_{x\to -1}\frac{\sqrt[3]{x}+x^2+x+1}{x+1}$, $\lim\limits_{x\to 1}\frac{\sqrt[3]{2x-1}-\sqrt[3]{x}}{\sqrt{x}-1}$

8). $L=\lim\limits_{x\to 1}\frac{\sqrt{2x-1}+x^2-3x+1}{\sqrt[3]{x-2}+x^2-x+1}=\lim\limits_{x\to 1}\frac{\sqrt{2x-1}-1+x^2-3x+2}{\sqrt[3]{x-2}+1+x^2-x}$

$$=\lim_{x\to 1}\frac{\frac{\sqrt{2x-1}-1}{x-1}+\frac{x^2-3x+2}{x-1}}{\frac{\sqrt[3]{x-2}+1}{x-1}+\frac{x^2-x}{x-1}}=\lim_{x\to 1}\frac{\frac{2}{\sqrt{2x-1}+1}+x-2}{\frac{1}{\left(\sqrt[3]{x-2}\right)^2-\sqrt[3]{x-2}+1}+x}=0.$$ Vậy $L=0$

9). $L=\lim_{x\to 0}\frac{\sqrt[5]{1+5x}-1}{x}$

Đặt $t=\sqrt[5]{1+5x}\Rightarrow t^5=1+5x\Rightarrow x=\frac{t^5-1}{5}$. Ta có khi $x\to 0$ thì $t\to 1$

Vậy $L=\lim_{t\to 1}\frac{5(t-1)}{t^5-1}=\lim_{t\to 1}\frac{5(t-1)}{(t-1)(t^4+t^3+t^2+t+1)}=\lim_{t\to 1}\frac{5}{(t^4+t^3+t^2+t+1)}=1$

10). $\lim_{x\to 1}\frac{\sqrt[4]{4x-3}-1}{x-1}=\lim_{x\to 1}\frac{\sqrt{4x-3}-1}{(x-1)\left(\sqrt[4]{4x-3}+1\right)}=\lim_{x\to 1}\frac{4(x-1)}{(x-1)\left(\sqrt[4]{4x-3}+1\right)\left(\sqrt{4x-3}+1\right)}$

$$=\lim_{x\to 1}\frac{4}{\left(\sqrt[4]{4x-3}+1\right)\left(\sqrt{4x-3}+1\right)}=\frac{4}{4}=1$$

Câu 10: Tìm các giới hạn sau:

1). $L=\lim_{x\to 1}\frac{\sqrt[7]{2-x}-1}{x-1}$ 2). $\lim_{x\to 0}\frac{\left(x^2+2004\right)\sqrt[7]{1-2x}-2004}{x}$

3). $L=\lim_{x\to 0}\frac{\sqrt[3]{1+x^2}-\sqrt[4]{1-2x}}{x+x^2}$ 4). $L=\lim_{x\to 1}\frac{\sqrt[4]{2x-1}+\sqrt[5]{x-2}}{x-1}$

5). $L=\lim_{x\to 0}\frac{\sqrt{1+4x}-\sqrt[3]{1+6x}}{x^2}$ 6). $\lim_{x\to 1}\frac{x\sqrt{2x-1}+\sqrt[3]{3x-2}-2}{x^2-1}$

7). $\lim_{x\to 0}\frac{\sqrt[3]{1+x^2}-\sqrt[4]{1-2x}}{x^2+x}$ 8). $\lim_{x\to 0}\frac{\sqrt{4x+4}+\sqrt{9-6x}-5}{x^2}$

LỜI GIẢI

1). $L=\lim_{x\to 1}\frac{\sqrt[7]{2-x}-1}{x-1}$. Đặt $t=\sqrt[7]{2-x}\Rightarrow t^7=2-x\Rightarrow x=2-t^7$

Ta có $x\to 1\Rightarrow t\to 1$

Vậy $L=\lim_{t\to 1}\frac{t-1}{1-t^7}=\lim_{t\to 1}\frac{t-1}{(1-t)\left(1+t+t^2+t^3+\cdots+t^7\right)}=\lim_{t\to 1}\frac{-1}{1+t+t^2+t^3+\cdots+t^7}=-\frac{1}{8}$

2). $\lim_{x\to 0}\frac{\left(x^2+2004\right)\sqrt[7]{1-2x}-2004}{x}$

$$=\lim_{x\to 0}\frac{x^2.\sqrt[7]{1-2x}+2004\left(\sqrt[7]{1-2x}-1\right)}{x}=\lim_{x\to 0}\frac{x^2.\sqrt[7]{1-2x}}{x}+\lim_{x\to 0}\frac{2004\left(\sqrt[7]{1-2x}-1\right)}{x}$$

$$=\lim_{x\to 0}x.\sqrt[7]{1-2x}+\lim_{x\to 0}\frac{2004\left(\sqrt[7]{1-2x}-1\right)}{x}=\lim_{x\to 0}\frac{2004\left(\sqrt[7]{1-2x}-1\right)}{x}$$

Đặt $t=\sqrt[7]{1-2x}\Rightarrow t^7=1-2x\Rightarrow x=\frac{1-t^7}{2}$

Ta có khi $x\to 0$ thì $t\to 1$

Vậy $\lim_{x\to 0}\frac{2004\left(\sqrt[7]{1-2x}-1\right)}{x}=\lim_{t\to 1}\frac{2.2004\left(t-1\right)}{1-t^7}=\lim_{t\to 1}\frac{4008\left(t-1\right)}{(1-t)(1+t+t^2+\cdots+t^7)}$

$$=\lim_{t\to 1}\frac{-4008}{1+t+t^2+\cdots+t^7}=\frac{-4008}{8}=-501$$

Tương tự: $\lim\limits_{x\to 0}\dfrac{\left(x^2+2001\right)\sqrt[9]{1-5x}-2001}{x}$

3). $L=\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x^2}-\sqrt[4]{1-2x}}{x+x^2}$

$$L=\lim_{x\to 0}\frac{\sqrt[3]{1+x^2}-1+1-\sqrt[4]{1-2x}}{x+x^2}=\lim_{x\to 0}\frac{\sqrt[3]{1+x^2}-1}{x+x^2}+\lim_{x\to 0}\frac{1-\sqrt[4]{1-2x}}{x+x^2}$$

- Tính $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x^2}-1}{x+x^2}$

$$=\lim_{x\to 0}\frac{x^2}{x\left(1+x\right)\left[\left(\sqrt[3]{1+x^2}\right)^2+\sqrt[3]{1+x^2}+1\right]}=\lim_{x\to 0}\frac{x}{\left(1+x\right)\left[\left(\sqrt[3]{1+x^2}\right)^2+\sqrt[3]{1+x^2}+1\right]}=0$$

- Tính $\lim\limits_{x\to 0}\dfrac{1-\sqrt[4]{1-2x}}{x+x^2}$

$$=\lim_{x\to 0}\frac{1-\sqrt{1-2x}}{\left(x+x^2\right)\left(1+\sqrt[4]{1-2x}\right)}=\lim_{x\to 0}\frac{2x}{x\left(1+x\right)\left(1+\sqrt[4]{1-2x}\right)\left(1+\sqrt{1-2x}\right)}$$

$$=\lim_{x\to 0}\frac{2}{\left(1+x\right)\left(1+\sqrt[4]{1-2x}\right)\left(1+\sqrt{1-2x}\right)}=\frac{1}{2}$$

Vậy $L=0+\dfrac{1}{2}=\dfrac{1}{2}$

4). $L=\lim\limits_{x\to 1}\dfrac{\sqrt[4]{2x-1}+\sqrt[5]{x-2}}{x-1}$

$$L=\lim_{x\to 1}\frac{\sqrt[4]{2x-1}-1+1+\sqrt[5]{x-2}}{x-1}=\lim_{x\to 1}\frac{\sqrt[4]{2x-1}-1}{x-1}+\lim_{x\to 1}\frac{1+\sqrt[5]{x-2}}{x-1}=M+N$$

Tính M: $\lim\limits_{x\to 1}\dfrac{\sqrt[4]{2x-1}-1}{x-1}=\lim\limits_{x\to 1}\dfrac{\sqrt{2x-1}-1}{\left(x-1\right)\left(\sqrt[4]{2x-1}+1\right)}=\lim\limits_{x\to 1}\dfrac{2(x-1)}{\left(x-1\right)\left(\sqrt[4]{2x-1}+1\right)\left(\sqrt{2x-1}+1\right)}$

$$=\lim_{x\to 1}\frac{2}{\left(\sqrt[4]{2x-1}+1\right)\left(\sqrt{2x-1}+1\right)}=\frac{1}{2}$$

Tính N: $\lim\limits_{x\to 1}\dfrac{1+\sqrt[5]{x-2}}{x-1}$

Đặt $t=\sqrt[5]{x-2}\Rightarrow t^5=x-2\Rightarrow x=t^5+2$

Ta có $x\to 1\Rightarrow t\to -1$

$$N=\lim_{t\to -1}\frac{1+t}{t^5+1}=\lim_{t\to -1}\frac{t+1}{(t+1)(t^4-t^3+t^2-t+1)}=\lim_{t\to -1}\frac{1}{t^4-t^3+t^2-t+1}=\frac{1}{5}$$

Vậy $L=\dfrac{1}{2}+\dfrac{1}{5}=\dfrac{7}{10}$

Tương tự tính: $\lim\limits_{x\to 0}\dfrac{\sqrt{x+1}+\sqrt[3]{x-1}}{x}$, $\lim\limits_{x\to 1}\dfrac{x^6-6x+5}{\left(x-1\right)^2}$

5). $L=\lim\limits_{x\to 0}\dfrac{\sqrt{1+4x}-\sqrt[3]{1+6x}}{x^2}=\lim\limits_{x\to 0}\dfrac{\sqrt{1+4x}-(1+2x)+(1+2x)-\sqrt[3]{1+6x}}{x^2}$

$$= \lim_{x\to 0}\frac{\sqrt{1+4x}-(1+2x)}{x^2}+\lim_{x\to 0}\frac{(1+2x)-\sqrt[3]{1+6x}}{x^2}$$

- Tính $\lim_{x\to 0}\frac{\sqrt{1+4x}-(1+2x)}{x^2}=\lim_{x\to 0}\frac{1+4x-\left(1+4x+4x^2\right)}{x^2\left(\sqrt{1+4x}+1+2x\right)}=\lim_{x\to 0}\frac{-4}{\sqrt{1+4x}+1+2x}=-2$

- Tính $\lim_{x\to 0}\frac{(1+2x)-\sqrt[3]{1+6x}}{x^2}$

$$=\lim_{x\to 0}\frac{\left(1+2x\right)^3-\left(1+6x\right)}{x^2\left[(1+2x)^2+(1+2x)\sqrt[3]{1+6x}+\left(\sqrt[3]{1+6x}\right)^2\right]}=\lim_{x\to 0}\frac{8x^3+12x^2}{x^2\left[(1+2x)^2+(1+2x)\sqrt[3]{1+6x}+\left(\sqrt[3]{1+6x}\right)^2\right]}$$

$$=\lim_{x\to 0}\frac{8x+12}{(1+2x)^2+(1+2x)\sqrt[3]{1+6x}+\left(\sqrt[3]{1+6x}\right)^2}=4$$

Vậy $L=-2+4=2$

6). Ta có $\lim_{x\to 1}\frac{x\sqrt{2x-1}+\sqrt[3]{3x-2}-2}{x^2-1}=\lim_{x\to 1}\frac{x\sqrt{2x-1}-1+\sqrt[3]{3x-2}-1}{x^2-1}$

$$=\lim_{x\to 1}\left[\frac{x\sqrt{2x-1}-1}{x^2-1}+\frac{\sqrt[3]{3x-2}-1}{x^2-1}\right]$$

$$=\lim_{x\to 1}\left[\frac{2x^3-x^2-1}{\left(x^2-1\right)\left(x\sqrt{2x-1}+1\right)}+\frac{3x-3}{\left(x^2-1\right)\left[\sqrt[3]{\left(3x-2\right)^2}+\sqrt[3]{3x-2}+1\right]}\right]$$

$$=\lim_{x\to 1}\left[\frac{2x^2+x+1}{\left(x+1\right)\left(x\sqrt{2x-1}+1\right)}+\frac{3}{\left(x+1\right)\left[\sqrt[3]{\left(3x-2\right)^2}+\sqrt[3]{3x-2}+1\right]}\right]$$

$$=\frac{4}{4}+\frac{3}{6}=\frac{3}{2}.$$

7). $\lim_{x\to 0}\frac{\sqrt[3]{1+x^2}-\sqrt[4]{1-2x}}{x^2+x}$

$$=\lim_{x\to 0}\frac{\sqrt[3]{1+x^2}-1+1-\sqrt[4]{1-2x}}{x^2+x}=\lim_{x\to 0}\frac{\frac{\sqrt[3]{1+x^2}-1}{x}+\frac{1-\sqrt[4]{1-2x}}{x}}{\frac{x^2+x}{x}}$$

$$=\frac{\lim_{x\to 0}\frac{x}{\left(\sqrt[3]{1+x^2}\right)^2+\sqrt[3]{1+x^2}+1}+\lim_{x\to 0}\frac{1-\sqrt{1-2x}}{x\left(1+\sqrt[4]{1-2x}\right)}}{\lim_{x\to 0}\left(x+1\right)}$$

$$=\frac{0+\lim_{x\to 0}\frac{2}{\left(1+\sqrt[4]{1-2x}\right)\left(1+\sqrt{1-2x}\right)}}{1}=\frac{1}{2}.$$

8). $\lim\limits_{x\to 0}\dfrac{\sqrt{4x+4}+\sqrt{9-6x}-5}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{\sqrt{4x+4}-\left(x+2\right)+\left(x-3\right)+\sqrt{9-6x}}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{\sqrt{4x+4}-\left(x+2\right)}{x^2}+\lim\limits_{x\to 0}\dfrac{\left(x-3\right)+\sqrt{9-6x}}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{\left[\sqrt{4x+4}-\left(x+2\right)\right]\left[\sqrt{4x+4}+\left(x+2\right)\right]}{x^2\left[\sqrt{4x+4}+\left(x+2\right)\right]}+\lim\limits_{x\to 0}\dfrac{\left[\left(x-3\right)+\sqrt{9-6x}\right]\left[\left(x-3\right)-\sqrt{9-6x}\right]}{x^2\left[\left(x-3\right)-\sqrt{9-6x}\right]}$

$=\lim\limits_{x\to 0}\dfrac{4x+4-(x+2)^2}{x^2\left[\sqrt{4x+4}+\left(x+2\right)\right]}+\lim\limits_{x\to 0}\dfrac{(x+3)^2-(9-6x)}{x^2\left[\left(x-3\right)-\sqrt{9-6x}\right]}$

$=\lim\limits_{x\to 0}\dfrac{-x^2}{x^2\left[\sqrt{4x+4}+\left(x+2\right)\right]}+\lim\limits_{x\to 0}\dfrac{x^2}{x^2\left[\left(x-3\right)-\sqrt{9-6x}\right]}$

$=\lim\limits_{x\to 0}\dfrac{-1}{\sqrt{4x+4}+\left(x+2\right)}+\lim\limits_{x\to 0}\dfrac{1}{\left(x-3\right)-\sqrt{9-6x}}=-\dfrac{1}{4}-\dfrac{1}{6}=-\dfrac{5}{12}$.