Câu 6: Tìm các giới hạn sau:

a). $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-4x+3}-\sqrt{x^2-3x+2}\right)$ b). $\lim\limits_{x\to-\infty}\left(\sqrt{4x^2-9x-21}-\sqrt{4x^2-7x+13}\right)$

c). $\lim\limits_{x\to-\infty}\left(\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2}\right)$

LỜI GIẢI

a). $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-4x+3}-\sqrt{x^2-3x+2}\right)=\lim\limits_{x\to-\infty}\dfrac{x^2-4x+3-x^2+3x-2}{\sqrt{x^2-4x+3}+\sqrt{x^2-3x+2}}$

$=\lim\limits_{x\to-\infty}\dfrac{-x+1}{\sqrt{x^2\left(1-\dfrac{4}{x}+\dfrac{3}{x^2}\right)}+\sqrt{x^2\left(1-\dfrac{4}{x}+\dfrac{3}{x^2}\right)}}=\lim\limits_{x\to-\infty}\dfrac{-x+1}{\sqrt{x^2}\left(\sqrt{1-\dfrac{1}{x}+\dfrac{3}{x^2}}+\sqrt{1-\dfrac{4}{x}+\dfrac{3}{x^2}}\right)}$

$=\lim\limits_{x\to-\infty}\dfrac{-x\left(1-\dfrac{1}{x}\right)}{|x|\left(\sqrt{1-\dfrac{1}{x}+\dfrac{3}{x^2}}+\sqrt{1-\dfrac{4}{x}+\dfrac{3}{x^2}}\right)}=\lim\limits_{x\to-\infty}\dfrac{-x\left(1-\dfrac{1}{x}\right)}{-x\left(\sqrt{1-\dfrac{1}{x}+\dfrac{3}{x^2}}+\sqrt{1-\dfrac{4}{x}+\dfrac{3}{x^2}}\right)}=\dfrac{1}{2}$

b). $\lim\limits_{x\to-\infty}\left(\sqrt{4x^2-9x-21}-\sqrt{4x^2-7x+13}\right)=\lim\limits_{x\to-\infty}\dfrac{4x^2-9x-21-4x^2+7x-13}{\sqrt{4x^2-9x-21}+\sqrt{4x^2-7x+13}}$

$=\lim\limits_{x\to-\infty}\dfrac{-2x-34}{\sqrt{x^2\left(4-\dfrac{9}{x}-\dfrac{21}{x^2}\right)}+\sqrt{x^2\left(4-\dfrac{7}{x}+\dfrac{13}{x^2}\right)}}=\lim\limits_{x\to-\infty}\dfrac{-x\left(2+\dfrac{34}{x}\right)}{\sqrt{x^2}\left(\sqrt{4-\dfrac{9}{x}-\dfrac{21}{x^2}}+\sqrt{4-\dfrac{7}{x}+\dfrac{13}{x^2}}\right)}$

$=\lim\limits_{x\to-\infty}\dfrac{-x\left(2+\dfrac{34}{x}\right)}{|x|\left(\sqrt{4-\dfrac{9}{x}-\dfrac{21}{x^2}}+\sqrt{4-\dfrac{7}{x}+\dfrac{13}{x^2}}\right)}=\lim\limits_{x\to-\infty}\dfrac{-x\left(2+\dfrac{34}{x}\right)}{-x\left(\sqrt{4-\dfrac{9}{x}-\dfrac{21}{x^2}}+\sqrt{4-\dfrac{7}{x}+\dfrac{13}{x^2}}\right)}=\dfrac{1}{2}$

c). $L=\lim\limits_{x\to-\infty}\left(\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2}\right)$

$=\lim\limits_{x\to-\infty}\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2}=\lim\limits_{x\to-\infty}\dfrac{x^3(3x+2)-x^2(3x^2-4)}{(3x^2-4)(3x+2)}=\lim\limits_{x\to-\infty}\dfrac{2x^3+4x^2}{(3x^2-4)(3x+2)}$

$=\lim\limits_{x\to-\infty}\dfrac{x^3\left(\dfrac{2x^3+4x^2}{x^3}\right)}{x^2\left(\dfrac{3x^2-4}{x^2}\right)x\left(\dfrac{3x+2}{x}\right)}=\lim\limits_{x\to-\infty}\dfrac{2+\dfrac{4}{x}}{\left(3-\dfrac{4}{x^2}\right)\left(3+\dfrac{2}{x}\right)}.$

Do $\lim\limits_{x\to-\infty}\dfrac{4}{x}=\lim\limits_{x\to-\infty}\dfrac{4}{x^2}=\lim\limits_{x\to-\infty}\dfrac{2}{x^2}=0$ nên $L=\dfrac{2}{3.3}=\dfrac{2}{9}$.