

**Câu 4: Tìm các giới hạn sau:**

$$\begin{array}{lll}
 1). \lim_{x \rightarrow \frac{\pi}{6}} \frac{1-2 \sin x}{\frac{\pi}{6}-x} & 2). \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin\left(\frac{\pi}{4}-x\right)}{1-\sqrt{2} \sin x} & 3). \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2}-2 \cos x}{\sin\left(x-\frac{\pi}{4}\right)} \\
 4). \lim_{x \rightarrow 0} \frac{\sqrt{x^2+1}-\cos 2x}{x^2} & 5). \lim_{x \rightarrow 0} \frac{\sqrt{1+2x}-\cos x-x}{x^2} & 6). \lim_{x \rightarrow 0} \frac{\sqrt[3]{2x+1}-\sqrt{1-x}}{\sin 2x} \\
 7). \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin x-\sqrt{3} \cos x}{\sin 3x} & 8). \lim_{x \rightarrow 0} \frac{1-\cos x \sqrt{\cos 2x}}{x^2} & 9). \lim_{x \rightarrow 0} \frac{1-\sqrt[3]{\cos x}}{\tan^2 x}
 \end{array}$$

**LỜI GIẢI**

$$\begin{aligned}
 1). \lim_{x \rightarrow \frac{\pi}{6}} \frac{1-2 \sin x}{\frac{\pi}{6}-x} &= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin x-1}{x-\frac{\pi}{6}} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{2\left(\sin x-\frac{1}{2}\right)}{x-\frac{\pi}{6}} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{2\left(\sin x-\sin \frac{\pi}{6}\right)}{x-\frac{\pi}{6}} \\
 &= \lim_{x \rightarrow \frac{\pi}{6}} \frac{4 \cos \left(\frac{x+\frac{\pi}{12}}{2}\right) \sin \left(\frac{x-\frac{\pi}{12}}{2}\right)}{2\left(\frac{x-\frac{\pi}{12}}{2}\right)} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin \left(\frac{x-\frac{\pi}{12}}{2}\right)}{\left(\frac{x-\frac{\pi}{12}}{2}\right)} 2 \cos \left(\frac{x+\frac{\pi}{12}}{2}\right) = \sqrt{3} \\
 2). \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin\left(\frac{\pi}{4}-x\right)}{1-\sqrt{2} \sin x} &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin\left(x-\frac{\pi}{4}\right)}{\sqrt{2} \sin x-1} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin\left(x-\frac{\pi}{4}\right)}{\sqrt{2}\left(\sin x-\frac{\sqrt{2}}{2}\right)} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin\left(x-\frac{\pi}{4}\right)}{\sqrt{2}\left(\sin x-\sin \frac{\pi}{4}\right)} \\
 &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 \sin\left(\frac{x-\frac{\pi}{8}}{2}\right) \cos\left(\frac{x-\frac{\pi}{8}}{2}\right)}{\sqrt{2} \cos\left(\frac{x+\frac{\pi}{8}}{2}\right) \sin\left(\frac{x-\frac{\pi}{8}}{2}\right)} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \cos\left(\frac{x-\frac{\pi}{8}}{2}\right)}{\cos\left(\frac{x+\frac{\pi}{8}}{2}\right)} = 2 \\
 3). \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2}-2 \cos x}{\sin\left(x-\frac{\pi}{4}\right)} &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{-2\left(\cos x-\frac{\sqrt{2}}{2}\right)}{\sin\left(x-\frac{\pi}{4}\right)} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{-2\left(\cos x-\cos \frac{\pi}{4}\right)}{\sin\left(x-\frac{\pi}{4}\right)} \\
 &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{4 \sin\left(\frac{x+\frac{\pi}{8}}{2}\right) \sin\left(\frac{x-\frac{\pi}{8}}{2}\right)}{2 \sin\left(\frac{x-\frac{\pi}{8}}{2}\right) \cos\left(\frac{x-\frac{\pi}{8}}{2}\right)} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 \sin\left(\frac{x+\frac{\pi}{8}}{2}\right)}{\cos\left(\frac{x-\frac{\pi}{8}}{2}\right)} = \sqrt{2} \\
 4). \lim_{x \rightarrow 0} \frac{\sqrt{x^2+1}-\cos 2x}{x^2} &. \text{Đặt } f(x) = \frac{\sqrt{x^2+1}-\cos 2x}{x^2} \\
 \lim_{x \rightarrow 0} \frac{\sqrt{x^2+1}-1+1-\cos 2x}{x^2} &= \lim_{x \rightarrow 0} \frac{\sqrt{x^2+1}-1}{x^2} + \lim_{x \rightarrow 0} \frac{1-\cos 2x}{x^2} \\
 \bullet \text{ Tính } \lim_{x \rightarrow 0} \frac{\sqrt{x^2+1}-1}{x^2} &= \lim_{x \rightarrow 0} \frac{x^2+1-1}{x^2\left(\sqrt{x^2+1}+1\right)} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{x^2+1}+1} = \frac{1}{2}
 \end{aligned}$$

• Tính  $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} = \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^2 = 2$

Vậy  $\lim_{x \rightarrow 0} f(x) = \frac{1}{2} + 2 = \frac{5}{2}$

5).  $L = \lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \cos x - x}{x^2}$ . Đặt  $f(x) = \frac{\sqrt{1+2x} - \cos x - x}{x^2}$

$L = \lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - (1+x) + 1 - \cos x}{x^2} = \lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - (1+x)}{x^2} + \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

• Tính  $\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - (1+x)}{x^2} = \lim_{x \rightarrow 0} \frac{1+2x - (1+x)^2}{x^2 (\sqrt{1+2x} - (1+x))}$

$= \lim_{x \rightarrow 0} \frac{-x^2}{x^2 (\sqrt{1+2x} - (1+x))} = \lim_{x \rightarrow 0} \frac{-1}{(\sqrt{1+2x} + (1+x))} = -\frac{1}{2}$

• Tính  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{x^2} = \lim_{x \rightarrow 0} \frac{1}{2} \left( \frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 = \frac{1}{2}$

Vậy  $\lim_{x \rightarrow 0} f(x) = -\frac{1}{2} + \frac{1}{2} = 0$ .

6).  $L = \lim_{x \rightarrow 0} \frac{\sqrt[3]{2x+1} - \sqrt{1-x}}{\sin 2x}$ . Đặt  $f(x) = \frac{\sqrt[3]{2x+1} - \sqrt{1-x}}{\sin 2x}$

$L = \lim_{x \rightarrow 0} \frac{\sqrt[3]{2x+1} - 1 + 1 - \sqrt{1-x}}{\sin 2x} = \lim_{x \rightarrow 0} \frac{\sqrt[3]{2x+1} - 1}{\sin 2x} + \lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{\sin 2x}$

• Tính  $\lim_{x \rightarrow 0} \frac{\sqrt[3]{2x+1} - 1}{\sin 2x} = \lim_{x \rightarrow 0} \frac{2x+1-1}{\sin 2x \left[ (\sqrt[3]{2x+1})^2 + \sqrt[3]{2x+1} + 1 \right]}$

$= \lim_{x \rightarrow 0} \frac{2x}{2 \sin x \cos x \left[ (\sqrt[3]{2x+1})^2 + \sqrt[3]{2x+1} + 1 \right]} = \lim_{x \rightarrow 0} \frac{x}{\sin x} \cdot \frac{1}{2 \cos x \left[ (\sqrt[3]{2x+1})^2 + \sqrt[3]{2x+1} + 1 \right]} = \frac{1}{3}$

• Tính  $\lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{\sin 2x} = \lim_{x \rightarrow 0} \frac{x}{2 \sin x \cos x (1 + \sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{x}{\sin x} \cdot \frac{1}{2 \cos x (1 + \sqrt{1-x})} = \frac{1}{4}$

Vậy  $\lim_{x \rightarrow 0} f(x) = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

7).  $\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin x - \sqrt{3} \cos x}{\sin 3x} = \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin x - \sqrt{3} \cos x}{3 \sin x - 4 \sin^3 x} = \frac{\sin^2 x - 3 \cos^2 x}{\sin x (3 - 4 \sin^2 x) (\sin x + \sqrt{3} \cos x)}$

$= \frac{4 \sin^2 x - 3}{\sin x (3 - 4 \sin^2 x) (\sin x + \sqrt{3} \cos x)} = \frac{-1}{\sin x (\sin x + \sqrt{3} \cos x)} = \frac{-2}{3}$

8).  $L = \lim_{x \rightarrow 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2}$

$L = \lim_{x \rightarrow 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2} = \lim_{x \rightarrow 0} \frac{1 - \cos^2 x \cos 2x}{x^2 (1 + \cos x \sqrt{\cos 2x})}$

$$= \lim_{x \rightarrow 0} \frac{\cos^2 x + \sin^2 x - \cos^2 x \cos 2x}{x^2 (1 + \cos x \sqrt{\cos 2x})} = \lim_{x \rightarrow 0} \frac{\cos^2 x (1 - \cos 2x) + \sin^2 x}{x^2 (1 + \cos x \sqrt{\cos 2x})}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin^2 x \cos^2 x + \sin^2 x}{x^2 (1 + \cos x \sqrt{\cos 2x})} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \cdot \frac{2 \cos^2 x + 1}{1 + \cos x \sqrt{\cos 2x}} = \frac{3}{2}$$

9).  $L = \lim_{x \rightarrow 0} \frac{1 - \sqrt[3]{\cos x}}{\tan^2 x}$

$$L = \lim_{x \rightarrow 0} \frac{1 - \cos x}{\tan^2 x \left[ 1 + \sqrt[3]{\cos x} + \left( \sqrt[3]{\cos x} \right)^2 \right]} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2} \cos^2 x}{4 \sin^2 \frac{x}{2} \cos^2 \frac{x}{2} \left[ 1 + \sqrt[3]{\cos x} + \left( \sqrt[3]{\cos x} \right)^2 \right]}$$

$$= \lim_{x \rightarrow 0} \frac{\cos^2 x}{2 \cos^2 \frac{x}{2} \left[ 1 + \sqrt[3]{\cos x} + \left( \sqrt[3]{\cos x} \right)^2 \right]} = \frac{1}{6}.$$

hoc360.net

**Câu 5: Tìm các giới hạn sau:**

$$1). \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt[3]{\tan x} - 1}{2 \sin^2 x - 1} \quad 2). \lim_{x \rightarrow 0} \frac{1 - \cos x \cos 2x}{x^2} \quad 3). \lim_{x \rightarrow 0} \left( \frac{2}{\sin 2x} - \cot x \right)$$

$$4). \lim_{x \rightarrow 0} \frac{1 - \sqrt{2x+1} + \sin x}{\sqrt{3x+4} - 2 - x} \quad 5). \lim_{x \rightarrow 0} \frac{\sqrt{x^2+1} - \cos x}{x^2} \quad 6). L = \lim_{x \rightarrow 0} \frac{1 - \sin 2x - \cos 2x}{1 + \sin 2x - \cos 2x}$$

$$7). \lim_{x \rightarrow \frac{\pi}{3}} \frac{\cos 3x + 2 \cos 2x + 2}{\sin 3x} \quad 8). \lim_{x \rightarrow 0} \frac{\cos\left(\frac{\pi}{2} \cos x\right)}{\sin^2 \frac{x}{2}} \quad 9). \lim_{x \rightarrow 0} \frac{1 - \cos x}{(1 - \sqrt{1-x})^2}$$

**LỜI GIẢI**

$$1). L = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt[3]{\tan x} - 1}{2 \sin^2 x - 1}$$

$$L = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{(\sin^2 x - \cos^2 x) \left[ (\sqrt[3]{\tan x})^2 + \sqrt[3]{\tan x} + 1 \right]}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\cos x (\sin x - \cos x) (\sin x + \cos x) \left[ (\sqrt[3]{\tan x})^2 + \sqrt[3]{\tan x} + 1 \right]}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{1}{\cos x (\sin x + \cos x) \left[ (\sqrt[3]{\tan x})^2 + \sqrt[3]{\tan x} + 1 \right]} = \frac{1}{3}$$

$$2). \lim_{x \rightarrow 0} \frac{1 - \cos x \cos 2x}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{\sin^2 x + \cos^2 x - \cos x \cos 2x}{x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} + \lim_{x \rightarrow 0} \frac{\cos x (\cos x - \cos 2x)}{x^2}$$

$$= 1 + \lim_{x \rightarrow 0} \frac{2 \cos x \sin \frac{3x}{2} \sin \frac{x}{2}}{x^2} = 1 + \lim_{x \rightarrow 0} \left( \frac{\sin \frac{3x}{2}}{\frac{3x}{2}} \right) \left( \frac{\sin \frac{x}{2}}{\frac{x}{2}} \right) \cdot \frac{9}{8} \cdot \cos x = 1 + \frac{9}{8} = \frac{17}{8}$$

$$3). L = \lim_{x \rightarrow 0} \left( \frac{2}{\sin 2x} - \cot x \right)$$

$$L = \lim_{x \rightarrow 0} \left( \frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x} \right) = \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{\sin x \cos x} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sin x \cos x} = \lim_{x \rightarrow 0} \tan x = 0$$

$$4). L = \lim_{x \rightarrow 0} \frac{1 - \sqrt{2x+1} + \sin x}{\sqrt{3x+4} - 2 - x}$$

$$L = \lim_{x \rightarrow 0} \frac{1 - \sqrt{2x+1} + \sin x}{\sqrt{3x+4} - 2 - x} = \lim_{x \rightarrow 0} \frac{1 - \sqrt{2x+1}}{\sqrt{3x+4} - 2 - x} + \lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{3x+4} - 2 - x}$$

$$= \lim_{x \rightarrow 0} \frac{-2x(\sqrt{3x+4} + 2 + x)}{(-x^2 - x)(1 + \sqrt{2x+1})} + \lim_{x \rightarrow 0} \frac{(\sqrt{3x+4} + 2 + x) \sin x}{-x^2 - x}$$

$$= \lim_{x \rightarrow 0} \frac{2(\sqrt{3x+4} + 2 + x)}{(x+1)(1+\sqrt{2x+1})} + \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{\sqrt{3x+4} + 2 + x}{-x-1} = 4 - 4 = 0$$

$$5). \lim_{x \rightarrow 0} \frac{\sqrt{x^2+1} - \cos x}{x^2}$$

$$I = \lim_{x \rightarrow 0} \frac{(\sqrt{1+x^2} - 1) + (1 - \cos x)}{x^2} = \lim_{x \rightarrow 0} \left[ \frac{(\sqrt{1+x^2} - 1)}{x^2} + \frac{1 - \cos x}{x^2} \right]$$

$$= \lim_{x \rightarrow 0} \left( \frac{x^2}{x^2(\sqrt{1+x^2} + 1)} + \frac{2 \sin^2 \frac{x}{2}}{x^2} \right) = \lim_{x \rightarrow 0} \left( \frac{1}{\sqrt{1+x^2} + 1} + \frac{\sin^2 \frac{x}{2}}{2 \frac{x^2}{4}} \right) = 1$$

$$6). L = \lim_{x \rightarrow 0} \frac{1 - \sin 2x - \cos 2x}{1 + \sin 2x - \cos 2x}$$

$$L = \lim_{x \rightarrow 0} \frac{1 - \sin 2x - \cos 2x}{1 + \sin 2x - \cos 2x} = \lim_{x \rightarrow 0} \frac{1 - \cos 2x - \sin 2x}{1 - \cos 2x + \sin 2x} = \lim_{x \rightarrow 0} \frac{2 \sin^2 x - 2 \sin x \cos x}{2 \sin^2 x + 2 \sin x \cos x}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin x (\sin x - \cos x)}{2 \sin x (\sin x + \cos x)} = \lim_{x \rightarrow 0} \frac{\sin x - \cos x}{\sin x + \cos x} = -1$$

$$7). \lim_{x \rightarrow \frac{\pi}{3}} \frac{\cos 3x + 2 \cos 2x + 2}{\sin 3x} = \lim_{x \rightarrow \frac{\pi}{3}} \frac{4 \cos^3 x + 4 \cos^2 x - 3 \cos x}{3 \sin x - 4 \sin^3 x}$$

$$= \lim_{x \rightarrow \frac{\pi}{3}} \frac{\cos x (2 \cos x + 3)(2 \cos x - 1)}{\sin x (2 \cos x - 1)(2 \cos x + 1)} =$$

$$8). \lim_{x \rightarrow 0} \frac{\cos\left(\frac{\pi}{2} \cos x\right)}{\sin^2 \frac{x}{2}}$$

$$= \lim_{x \rightarrow 0} \frac{\sin\left(\frac{\pi}{2} - \frac{\pi}{2} \cos x\right)}{\sin^2 \frac{x}{2}} = \lim_{x \rightarrow 0} \frac{\sin\left(\frac{\pi}{2} 2 \sin^2 \frac{x}{2}\right)}{\sin^2 \frac{x}{2}} = \lim_{x \rightarrow 0} \left[ \pi \cdot \frac{\sin\left(\pi \sin^2 \frac{x}{2}\right)}{\pi \sin^2 \frac{x}{2}} \right] = \pi$$

$$9). \lim_{x \rightarrow 0} \frac{1 - \cos x}{(1 - \sqrt{1-x})^2}$$

$$= \lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \sqrt{1-x})^2}{(1 - \sqrt{1-x})^2 (1 + \sqrt{1-x})^2} = \lim_{x \rightarrow 0} \frac{2 \sin^2\left(\frac{x}{2}\right) (1 + \sqrt{1-x})^2}{4 \left(\frac{x}{2}\right)^2} = \lim_{x \rightarrow 0} \frac{(1 + \sqrt{1-x})^2}{2} = 2$$