Câu 17: Tìm các giới hạn sau:

a). $\lim\left(-2n^3+3n+5\right)$ b). $\lim\sqrt{2n^4+5n^3-7n}$ c). $\lim\sqrt[3]{1+2n-n^3}$

d). $\lim\left(3n+\cos n\right)$ e). $\lim\left(\frac{2}{3}n^2-3\sin n^3+5\right)$ f). $\lim\left(2n^2\cos n^2-4n^3\right)$

LỜI GIẢI

a). Ta có $L=\lim\left(-2n^3+3n+5\right)=\lim n^3\left(\frac{-2n^3+3n+5}{n^3}\right)=\lim n^3\left(-2+\frac{3}{n^2}+\frac{5}{n^3}\right)$.

Do $\lim\frac{3}{n^2}=0$ và $\lim\frac{5}{n^3}=0$ nên $\lim\left(-2+\frac{3}{n^2}+\frac{5}{n^3}\right)=-2$ (1), ngoài ra $\lim n^3=+\infty$ (2). Từ (1) và (2) có $L=-\infty$.

b). Ta có $L=\lim\sqrt{2n^4+5n^3-7n}=\lim\sqrt{n^4\left(\frac{2n^4+5n^3-7n}{n^4}\right)}=\lim n^2\sqrt{2+\frac{5}{n}-\frac{7}{n^3}}$

Do $\lim\frac{5}{n}=0$, $\lim\frac{7}{n^3}=0$ nên $\lim\sqrt{2+\frac{5}{n}-\frac{7}{n^3}}=2$ (1) và $\lim n^2=+\infty$ (2). Từ (1) và (2) suy ra $L=+\infty$.

c). Ta có $L=\lim\sqrt[3]{1+2n-n^3}=\lim\sqrt[3]{n^3\left(\frac{1+2n-n^3}{n^3}\right)}=\lim n.\sqrt[3]{\frac{1}{n^3}+\frac{2}{n^2}-1}$. Ta có $\lim\frac{1}{n^3}=0$, $\lim\frac{2}{n^2}=0$ nên $\lim\left(\sqrt[3]{\frac{1}{n^3}+\frac{2}{n^2}-1}\right)=-1$ (1) và $\lim n=+\infty$ (2). Từ (1) và (2) suy ra $L=-\infty$.

d). $L=\lim\left(3n+\cos n\right)=\lim\left[n\left(\frac{3n+\cos n}{n}\right)\right]=\lim\left[n\left(3+\frac{\cos n}{n}\right)\right]$.

Có $\left|\cos n\right|\le 1$ nên $\left|\frac{\cos n}{n}\right|\le\left|\frac{1}{n}\right|=\frac{1}{n}$ mà $\lim\frac{1}{n}=0$ nên $\lim\frac{\cos n}{n}=0$ (1) và $\lim n=+\infty$ (2). Từ (1) va (2) suy ra $L=+\infty$.

e). $L=\lim n^2\left(\frac{\frac{2}{3}n^2-3\sin n^3+5}{n^2}\right)=\lim n^2\left(\frac{2}{3}-3.\frac{\sin n^3}{n^2}+\frac{5}{n^2}\right)$. Có $\lim\frac{5}{n^2}=0$, có $\left|\frac{\sin n^3}{n^2}\right|\le\frac{1}{n^2}$ mà $\lim\frac{1}{n^2}=0$ nên $\lim\frac{\sin n^3}{n^2}=0$, do đó $\lim\left(\frac{2}{3}-3.\frac{\sin n^3}{n^2}+\frac{5}{n^2}\right)=\frac{2}{3}$ (1) ngoài ra $\lim n^2=+\infty$ (2). Từ (1) và (2) có $L=+\infty$.

f). $L=\lim\left(2n^2\cos n^2-4n^3\right)=\lim n^3\left(\frac{2n^2\cos n^2-4n^3}{n^3}\right)=\lim n^3\left(2.\frac{\cos n^2}{n}-4\right)$. Ta có $\left|\frac{\cos n^2}{n}\right|\le\left|\frac{1}{n}\right|=\frac{1}{n}$ mà $\lim\frac{1}{n}=0\Rightarrow\lim\frac{\cos n^2}{n}=0$ do đó $\lim\left(2.\frac{\cos n^2}{n}-4\right)=-4$ (1), ngoài ra $\lim n^3=+\infty$ (2). Từ (1) và (2) có $L=-\infty$.

Câu 18: Tìm các giới hạn sau:

a). $\lim\frac{n^5+n^4-3n-2}{4n^3+6n^2+9}$	b). $\lim\frac{-2n^3+3n-2}{4n+5}$	c). $\lim\frac{\sqrt[3]{n^6-7n^3-5n+8}}{n+2}$
d). $\lim\frac{n\sqrt{2n^2-1}}{\sqrt[3]{n^2+2n}}$	e). $\lim\left(\sqrt{n^2+n+1}-\sqrt{n+3}\right)$	f). $\lim\left(2^{n+3}-3^{n-2}\right)$.

LỜI GIẢI.

a). $L=\lim\frac{n^5+n^4-3n-2}{4n^3+6n^2+9}=\lim\frac{\frac{n^5+n^4-3n-2}{n^5}}{\frac{4n^3+6n^2+9}{n^5}}=\lim\left(n.\frac{1+\frac{1}{n}-\frac{3}{n^2}-\frac{2}{n^5}}{4+\frac{6}{n^3}+\frac{9}{n^5}}\right)$

Ta có $\lim\left(\frac{1+\frac{1}{n}-\frac{3}{n^2}-\frac{2}{n^5}}{4+\frac{6}{n^3}+\frac{9}{n^5}}\right)=\frac{1}{4}$ và $\lim n=+\infty$. Do đó $L=+\infty$.

b). Tương tự câu a).

c). $L=\lim\frac{\sqrt[3]{n^6-7n^3-5n+8}}{n+2}=\lim\frac{\sqrt[3]{n^6.\frac{n^6-7n^3-5n+8}{n^6}}}{n+2}=\lim\frac{n^3.\sqrt[3]{1-\frac{7}{n^3}-\frac{5}{n^5}+\frac{8}{n^6}}}{n+2}$

$=\lim\left(n^2.\frac{\sqrt[3]{1-\frac{7}{n^3}-\frac{5}{n^5}+\frac{8}{n^6}}}{1+\frac{2}{n}}\right).$

Ta có $\lim\left(\frac{\sqrt[3]{1-\frac{7}{n^3}-\frac{5}{n^5}+\frac{8}{n^6}}}{1+\frac{2}{n}}\right)=1$ và $\lim n^2=+\infty$, từ đó suy ra $L=+\infty$.

d). $L=\lim\frac{n\sqrt{2n^2-1}}{\sqrt[3]{n^2+2n}}$

$=\lim\frac{n^2.\sqrt{2-\frac{1}{n^2}}}{\sqrt[3]{n^2\left(\frac{n^2+2n}{n^2}\right)}}=\lim\frac{\left(\sqrt[3]{n^2}\right)^3.\sqrt{2-\frac{1}{n^2}}}{\sqrt[3]{n^2}.\sqrt[3]{1+\frac{2}{n^2}}}=\lim\left[\left(\sqrt[3]{n^2}\right)^2.\frac{\sqrt{2-\frac{1}{n^2}}}{\sqrt[3]{1+\frac{2}{n^2}}}\right].$

Do $\lim\left[\frac{\sqrt{2-\frac{1}{n^2}}}{\sqrt[3]{1+\frac{2}{n^2}}}\right]=\sqrt{2}$ và $\lim\left(\sqrt[3]{n^2}\right)^2=+\infty$ nên $L=+\infty$.

e). $L=\lim\left(\sqrt{n^2+n+1}-\sqrt{n+3}\right)=\lim\frac{n^2-2}{\sqrt{n^2+n+1}+\sqrt{n+3}}=\lim\left(n.\frac{1-\frac{2}{n^2}}{\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+\sqrt{\frac{1}{n}+\frac{3}{n^2}}}\right)$

Do $\lim \dfrac{1-\dfrac{2}{n^2}}{\sqrt{1+\dfrac{1}{n}+\dfrac{1}{n^2}}+\sqrt{\dfrac{1}{n}+\dfrac{3}{n^2}}}=1$ và $\lim n=+\infty$ nên $L=+\infty$.

f). $L=\lim\left(2^{n+3}-3^{n-2}\right)=\lim\left(8.2^n-\dfrac{3^n}{9}\right)=\lim 3^n\left(8.\left(\dfrac{2}{3}\right)^n-\dfrac{1}{9}\right)$. Do $\lim\left(\dfrac{2}{3}\right)^n=0$ nên

$\lim\left(8.\left(\dfrac{2}{3}\right)^n-\dfrac{1}{9}\right)=-\dfrac{1}{9}$ ngoài ra $\lim 3^n=+\infty$. Vậy $L=-\infty$.