Bài 6:

a). $y=\sin\sqrt{x}$ b). $y=\cos^2 x$ c). $y=\cos\sqrt{2x+1}$ d). $y=\sin 3x.\cos 5x$

e). $y=\dfrac{\sin x+\cos x}{\sin x-\cos x}$ f). $y=\sqrt{\cos 2x}$ g). $y=\dfrac{\sin x}{x}+\dfrac{x}{\sin x}$ h). $y=\sin(\cos x)+\cos(\sin x)$ i). $y=\dfrac{x+\sin x}{x-\sin x}$ k). $y=\left(\dfrac{1+\cos 2x}{1-\cos 2x}\right)^2$

l). $y=\sin^4 x+\cos^4 x$ m). $y=\cos\left(2x-\dfrac{\pi}{4}\right)^2$ n). $y=\dfrac{\sin x-x\cos x}{\cos x+x\sin x}$

LỜI GIẢI

a). $y=\sin\sqrt{x}$. Áp dụng $(\sin u)'$, với $u=\sqrt{x}$

$$y'=\left(\sin\sqrt{x}\right)'=\cos\sqrt{x}.\left(\sqrt{x}\right)'=\frac{1}{2\sqrt{x}}.\cos\sqrt{x}.$$

b). $y=\cos^2 x$. Áp dụng công thức $(u^\alpha)'$, với $u=\cos x$

$$y'=\left(\cos^2 x\right)'=2.\cos(\cos x)'=2\cos x.(-\sin x)=-\sin 2x.$$

c). $y=\cos\sqrt{2x+1}$. Áp dụng $(\cos u)'$, với $u=\sqrt{2x+1}$

$$y'=\left(\cos\sqrt{2x+1}\right)'=-\sin\sqrt{2x+1}\left(\sqrt{2x+1}\right)'=-\sin\sqrt{2x+1}.\frac{(2x+1)'}{2\sqrt{2x+1}}$$

$$=-\sin\sqrt{2x+1}.\frac{2}{2\sqrt{2x+1}}=-\frac{1}{\sqrt{2x+1}}.\sin\sqrt{2x+1}.$$

d). $y=\sin 3x.\cos 5x=\dfrac{1}{2}(\sin(-2x)+\sin 8x)=\dfrac{1}{2}(-\sin 2x+\sin 8x)$

$$y'=\frac{1}{2}(\sin 8x-\sin 2x)'=\frac{1}{2}(\sin 8x)'-\frac{1}{2}(\sin 2x)'=\frac{1}{2}\cos 8x(8x)'-\frac{1}{2}\cos 2x.(2x)'$$

$$=4\cos 8x-\cos 2x$$

e). $y=\dfrac{\sin x+\cos x}{\sin x-\cos x}$. Áp dụng $\left(\dfrac{u}{v}\right)'$

$$y'=\frac{(\sin x+\cos x)'(\sin x-\cos x)-(\sin x-\cos x)'.(\sin x+\cos x)}{(\sin x-\cos x)^2}$$

$$y'=\frac{(\cos x-\sin x)(\sin x-\cos x)-(\cos x+\sin x)(\sin x+\cos x)}{(\sin x-\cos x)^2}$$

$$y'=\frac{-(\sin x-\cos x)^2-(\sin x+\cos x)^2}{(\sin x-\cos x)^2}=\frac{2\sin 2x}{(\sin x-\cos x)^2}.$$

f). $y=\sqrt{\cos 2x}$. Áp dụng $\left(\sqrt{u}\right)'$, với $u=\cos 2x$

$$y'=\frac{(\cos 2x)'}{2\sqrt{\cos 2x}}=\frac{-\sin 2x.(2x)'}{2\sqrt{\cos 2x}}=\frac{-\sin 2x}{\sqrt{\cos 2x}}.$$

g) $y=\frac{\sin x}{x}+\frac{x}{\sin x}\Rightarrow y'=\left(\frac{\sin x}{x}\right)'+\left(\frac{x}{\sin x}\right)'$

$=\frac{(\sin x)'.x-x'.\sin x}{x^2}+\frac{x'.\sin x-(\sin x)'.x}{\sin^2 x}=\frac{x\cos x-\sin x}{x^2}+\frac{\sin x-x\cos x}{\sin^2 x}$. h) h). $y=\sin(\cos x)+\cos(\sin x)$

Bước đầu tiên sử dụng đạo hàm tổng, sau đó sử dụng $(\sin u)'$, $(\cos u)'$.

$y'=(\sin(\cos x))'+(\cos(\sin x))'=\cos(\cos x).(\cos x)'-\sin(\sin x).(\sin x)'$

$=-\sin x.\cos(\cos x)-\cos x.\sin(\sin x)=-(\sin x.\cos(\cos x)+\cos x.\sin(\sin x))$

$=-\sin(x+\cos x)$

i). $y=\frac{x+\sin x}{x-\sin x}$. Sử dụng $\left(\frac{u}{v}\right)'$

$$y'=\frac{(x+\sin x)'.(x-\sin x)-(x-\sin x)'.(x+\sin x)}{(x-\sin x)^2}$$

$$=\frac{(1+\cos x)(x-\sin x)-(1-\cos x)(x+\sin x)}{(x-\sin x)^2}=\frac{-2\sin x+2x\cos x}{(x-\sin x)^2}.$$

k). $y=\left(\frac{1+\cos 2x}{1-\cos 2x}\right)^2$. Sử dụng $(u^\alpha)'$ với $u=\frac{1+\cos 2x}{1-\cos 2x}$

$$y'=2\left(\frac{1+\cos 2x}{1-\cos 2x}\right).\left(\frac{1+\cos 2x}{1-\cos 2x}\right)'$$

$$=2\left(\frac{1+\cos 2x}{1-\cos 2x}\right).\left(\frac{(1+\cos 2x)'(1-\cos 2x)-(1-\cos 2x)'(1+\cos 2x)}{(1-\cos 2x)^2}\right)$$

$$=2\left(\frac{1+\cos 2x}{1-\cos 2x}\right).\left(\frac{-2\sin 2x(1-\cos 2x)-2\sin 2x(1+\cos 2x)}{(1-\cos 2x)^2}\right)$$

$$=2\left(\frac{1+\cos 2x}{1-\cos 2x}\right).\left(\frac{-4\sin 2x}{(1-\cos 2x)^2}\right).$$

l). $y=\sin^4 x+\cos^4 x\ =1-\frac{1}{2}\sin^2 2x=\frac{3}{4}+\frac{1}{4}\cos 4x.$

$$y'=\left(\frac{3}{4}+\frac{1}{4}\cos 4x\right)'=\frac{1}{4}(\cos 4x)'=\frac{1}{4}(-\sin 4x).(4x)'=-\sin 4x.$$

m). $y=\cos\left(2x-\frac{\pi}{4}\right)^2$. Áp dụng $(\cos u)'$ với $u=\left(2x-\frac{\pi}{4}\right)^2$

$$y'=-\sin\left(2x-\frac{\pi}{4}\right)^2.\left[\left(2x-\frac{\pi}{4}\right)^2\right]'=-\sin\left(2x-\frac{\pi}{4}\right)^2.2\left(2x-\frac{\pi}{4}\right).\left(2x-\frac{\pi}{4}\right)'$$

$$=-4\left(2x-\frac{\pi}{4}\right).\sin\left(2x-\frac{\pi}{4}\right)^2.$$

n). $y=\dfrac{\sin x-x\cos x}{\cos x+x\sin x}$

$y'=\dfrac{(\sin x-x\cos x)'(\cos x+x\sin x)-(\cos x-x\sin x)'(\sin x-x\cos x)}{(\cos x+x\sin x)^2}$

Tính $(\sin x-x\cos x)'=\cos x-(x\cos x)'=\cos x-\left(x'.\cos x+x.(\cos x)'\right)$

$=\cos x-(\cos x-x\sin x)=x\sin x$

Tính $(\cos x+x\sin x)'=-\sin x+\left(x'.\sin x+x.(\sin x)'\right)$

$=-\sin x+(\sin x+x\cos x)=x\cos x$

$\Rightarrow y'=\dfrac{x\sin x(\cos x+x\sin x)-x\cos x(\sin x-x\cos x)}{(\cos x+x\sin x)^2}=\dfrac{x^2}{(\cos x+x\sin x)^2}.$

Bài 7:

a). $y=\tan\dfrac{x+1}{2}$ b). $y=\tan^3 x+\cot 2x$ c). $y=\cot\sqrt{x^2+1}$ d). $y=\tan 3x-\cot 3x$

e). $y=x\cot 2x$ f). $y=\dfrac{1+\tan^2 3x}{1-\tan^2 3x}$ j). $y=\dfrac{1}{2}\left(1+\tan^2 x\right)^2$. h) $y=\sqrt{\cot^3 2x}$

LỜI GIẢI

a). $y=\tan\dfrac{x+1}{2}$. Áp dụng $(\tan x)'$ với $u=\dfrac{x+1}{2}$

$y'=\left(1+\tan^2\left(\dfrac{x+1}{2}\right)\right).\left(\dfrac{x+1}{2}\right)'=\dfrac{1}{2}\left(1+\tan^2\left(\dfrac{x+1}{2}\right)\right).$

b). $y=\tan^3 x+\cot 2x$. Đầu tiên áp dụng $(u+v)'$

$y'=(\tan^3 x)'+(\cot 2x)'$

$(\tan^3 x)'$ áp dụng $(u^\alpha)'$ với $u=\tan x$

Vậy $(\tan^3 x)'=3\tan^2 x(\tan x)'=3\tan^2 x\left(1+\tan^2 x\right)$

$(\cot 2x)'$ áp dụng $(\cot u)'$ với $u=2x$

Vậy $(\cot 2x)'=-\left(1+\cot^2 2x\right).(2x)'=-2\left(1+\cot^2 2x\right)$

$\Rightarrow y'=3\tan^2 x\left(1+\tan^2 x\right)-2\left(1+\cot^2 2x\right).$

c). $y=\cot\sqrt{x^2+1}$. Áp dụng $(\cot u)'$ với $u=\sqrt{x^2+1}$

$\Rightarrow y'=-\left(1+\cot^2\left(\sqrt{x^2+1}\right)\right).\left(\sqrt{x^2+1}\right)'=-\left(1+\cot^2\left(\sqrt{x^2+1}\right)\right).\dfrac{(x^2+1)'}{2\sqrt{x^2+1}}$

$=\dfrac{-x}{\sqrt{x^2+1}}\left(1+\cot^2\sqrt{x^2+1}\right).$

d). $y=\tan 3x-\cot 3x$

$y'=(\tan 3x)'-(\cot 3x)'=\left(1+\tan^2 3x\right)(3x)'+\left(1+\cot^2 3x\right)(3x)'$

$=3\left(1+\tan^2 3x\right)+3\left(1+\cot^2 3x\right)=3\left(2+\tan^2 3x+\cot^2 3x\right).$

e). $y = x\cot 2x$

$y' = x'.\cot 2x + x(\cot 2x)' = \cot 2x - x(1+\cot^2 2x).(2x)' = \cot 2x - 2x(1+\cot^2 2x)$.

f). $y = \dfrac{1+\tan^2 3x}{1-\tan^2 3x}$. Bước đầu tiên biến đổi lượng giác, rút gọn biểu thức

$y = \dfrac{1+\dfrac{\sin^2 3x}{\cos^2 3x}}{1-\dfrac{\sin^2 3x}{\cos^2 3x}} = \dfrac{1}{\cos 6x}$. Sau đó áp dụng $\left(\dfrac{1}{u}\right)'$ với $u = \cos 6x$

$y' = \dfrac{-(\cos 6x)'}{\cos^2 6x} = \dfrac{6\sin 6x}{\cos^2 6x}$.

j). $y = \dfrac{1}{2}(1+\tan^2 x)^2$. Áp dụng $(u^\alpha)'$ với $u = 1+\tan^2 x$

$y' = (1+\tan^2 x)(1+\tan^2 x)' = (1+\tan^2 x).2\tan x.(\tan x)'$

$= (1+\tan^2 x).2\tan x.(1+\tan^2 x) = 2\tan x.(1+\tan^2 x)^2$.

h) $y = \sqrt{\cot^3 2x}$. Áp dụng $(\sqrt{u})'$ với $u = \cot 2x$

$y' = \dfrac{1}{2\sqrt{\cot^3 2x}}.3.\cot^2 2x.(\cot 2x)' = \dfrac{1}{2\sqrt{\cot^3 2x}}.3.\cot^2 2x.(-1).(1+\cot^2 2x)$.