

Bài 2: Giải các phương trình sau:

- 1).  $\frac{1}{\sin x} + \frac{1}{\cos x} = 2\sqrt{2}$ .                      2).  $\sin x - 2\sin 2x = \frac{1}{2} - \cos x$ .
- 3).  $\sin 2x + \sqrt{2} \sin\left(x - \frac{\pi}{4}\right) = 1$ .                      4).  $2\sin 2x - 3\sqrt{6}|\sin x + \cos x| + 8 = 0$ .
- 5).  $\cot x - 1 = \frac{\cos 2x}{1 + \tan x} + \sin^2 x - \frac{1}{2}\sin 2x$  (1) [ĐH A03]
- 6).  $\frac{1}{\cos x} - \frac{1}{\sin x} = 2\sqrt{2} \cos\left(x + \frac{\pi}{4}\right)$  (1) [Dự bị 2 ĐH B04]
- 7)  $\sin 2x - 2\sqrt{2}(\sin x + \cos x) - 5 = 0$  [Dự bị 2 ĐH D04]

LỜI GIẢI

1).  $\frac{1}{\sin x} + \frac{1}{\cos x} = 2\sqrt{2}$ . Điều kiện:  $\begin{cases} \sin x \neq 0 \\ \cos x \neq 0 \end{cases} \Leftrightarrow \begin{cases} x \neq k\pi \\ x \neq \frac{\pi}{2} + k\pi \end{cases} (k \in \mathbb{Z})$

Phương trình  $\Leftrightarrow \sin x + \cos x = 2\sqrt{2} \sin x \cos x$

Đặt  $t = \sin x + \cos x$  (Đk:  $|t| \leq \sqrt{2}$ )  $\Rightarrow \sin x \cos x = \frac{t^2 - 1}{2}$

Ta được:  $t = \sqrt{2}(t^2 - 1) \Leftrightarrow \sqrt{2}t^2 - t - \sqrt{2} = 0 \Leftrightarrow t = \sqrt{2} \vee t = -\frac{\sqrt{2}}{2}$

Với  $t = \sqrt{2} \Leftrightarrow \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) = \sqrt{2} \Leftrightarrow \sin\left(x + \frac{\pi}{4}\right) = 1$

$\Leftrightarrow x + \frac{\pi}{4} = \frac{\pi}{2} + k2\pi \Leftrightarrow x = \frac{\pi}{4} + k2\pi (k \in \mathbb{Z})$

Với  $t = -\frac{\sqrt{2}}{2} \Leftrightarrow \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \Leftrightarrow \sin\left(x + \frac{\pi}{4}\right) = -\frac{1}{2}$

$\Leftrightarrow \begin{cases} x + \frac{\pi}{4} = -\frac{\pi}{6} + k2\pi \\ x + \frac{\pi}{4} = \pi + \frac{\pi}{6} + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = -\frac{5\pi}{12} + k2\pi \\ x = \frac{11\pi}{12} + k2\pi \end{cases} (k \in \mathbb{Z})$

So với điều kiện phương trình có các nghiệm:  $x = \frac{\pi}{4} + k2\pi; x = -\frac{5\pi}{12} + k2\pi; x = \frac{11\pi}{12} + k2\pi, (k \in \mathbb{Z})$

2).  $\sin x - 2\sin 2x = \frac{1}{2} - \cos x$ .

$\Leftrightarrow 2(\sin x + \cos x) - 4\sin 2x = 1$ .

Đặt  $t = \sin x + \cos x$  (Đk:  $|t| \leq \sqrt{2}$ )  $\Rightarrow \sin 2x = t^2 - 1$

Ta được:  $2t - 4(t^2 - 1) = 1 \Leftrightarrow 4t^2 - 2t - 3 = 0 \Leftrightarrow t = \frac{1 + \sqrt{13}}{4} \vee t = \frac{1 - \sqrt{13}}{4}$

Với  $t = \frac{1 + \sqrt{13}}{4} \Leftrightarrow \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) = \frac{1 + \sqrt{13}}{4} \Leftrightarrow \sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2} + \sqrt{26}}{8}$

$$\Leftrightarrow \begin{cases} x + \frac{\pi}{4} = \arcsin\left(\frac{\sqrt{2} + \sqrt{26}}{8}\right) + k2\pi \\ x + \frac{\pi}{4} = \pi - \arcsin\left(\frac{\sqrt{2} - \sqrt{26}}{8}\right) + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = -\frac{\pi}{4} + \arcsin\left(\frac{\sqrt{2} + \sqrt{26}}{8}\right) + k2\pi \\ x = \frac{3\pi}{4} - \arcsin\left(\frac{\sqrt{2} - \sqrt{26}}{8}\right) + k2\pi \end{cases}$$

$$\text{Với } t = \frac{1 - \sqrt{13}}{4} \Leftrightarrow \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) = \frac{1 - \sqrt{13}}{4} \Leftrightarrow \sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2} - \sqrt{26}}{8}$$

$$\Leftrightarrow \begin{cases} x + \frac{\pi}{4} = \arcsin\left(\frac{\sqrt{2} - \sqrt{26}}{8}\right) + k2\pi \\ x + \frac{\pi}{4} = \pi - \arcsin\left(\frac{\sqrt{2} - \sqrt{26}}{8}\right) + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = -\frac{\pi}{4} + \arcsin\left(\frac{\sqrt{2} - \sqrt{26}}{8}\right) + k2\pi \\ x = \frac{3\pi}{4} - \arcsin\left(\frac{\sqrt{2} - \sqrt{26}}{8}\right) + k2\pi \end{cases}$$

$$\text{Nghiệm của phương trình: } x = -\frac{\pi}{4} + \arcsin\left(\frac{\sqrt{2} + \sqrt{26}}{8}\right) + k2\pi, x = \frac{3\pi}{4} - \arcsin\left(\frac{\sqrt{2} - \sqrt{26}}{8}\right) + k2\pi,$$

$$x = -\frac{\pi}{4} + \arcsin\left(\frac{\sqrt{2} - \sqrt{26}}{8}\right) + k2\pi, x = \frac{3\pi}{4} - \arcsin\left(\frac{\sqrt{2} - \sqrt{26}}{8}\right) + k2\pi, (k \in \mathbb{Z})$$

$$3). \sin 2x + \sqrt{2} \sin\left(x - \frac{\pi}{4}\right) = 1.$$

$$\Leftrightarrow \sin 2x + (\sin x - \cos x) = 1$$

$$\text{Đặt } t = \sin x - \cos x \text{ (Đk: } |t| \leq \sqrt{2} \text{)} \Rightarrow \sin 2x = 1 - t^2$$

$$\text{Ta được: } 1 - t^2 + t = 1 \Leftrightarrow -t^2 + t = 0 \Leftrightarrow t = 1 \vee t = 0$$

$$\text{Với } t = 1 \Leftrightarrow \sqrt{2} \sin\left(x - \frac{\pi}{4}\right) = 1 \Leftrightarrow \sin\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\Leftrightarrow \begin{cases} x - \frac{\pi}{4} = \frac{\pi}{4} + k2\pi \\ x - \frac{\pi}{4} = \pi - \frac{\pi}{4} + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{2} + k2\pi \\ x = \pi + k2\pi \end{cases} (k \in \mathbb{Z})$$

$$\text{Với } t = 0 \Leftrightarrow \sqrt{2} \sin\left(x - \frac{\pi}{4}\right) = 0 \Leftrightarrow x - \frac{\pi}{4} = k\pi \Leftrightarrow x = \frac{\pi}{4} + k\pi.$$

$$\text{Nghiệm của phương trình: } x = \frac{\pi}{2} + k2\pi, x = \pi + k2\pi, x = \frac{\pi}{4} + k\pi, (k \in \mathbb{Z})$$