Câu 10: Tìm các giới hạn sau:

1). $\lim\limits_{x\to 0}\dfrac{2\sqrt{1-x}-\sqrt[3]{8-x}}{x}$ 2). $\lim\limits_{x\to 1}\dfrac{\sqrt{3x-2}-\sqrt[3]{4x^2-x-2}}{x^2-3x+2}$

3). $\lim\limits_{x\to 1}\dfrac{\sqrt{5-x^3}-\sqrt[3]{x^2+7}}{x^2-1}$ 4). $\lim\limits_{x\to 4}\dfrac{2-\sqrt{x^2-12}}{\left(\sqrt{x^2+x-19}-1\right)\left(\sqrt{x+12}-2\right)}$ 5).

$\lim\limits_{x\to 2}\dfrac{3\sqrt[3]{4x^3-24}+\sqrt{x+2}-8\sqrt{2x-3}}{4-x^2}$ 7). $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2+2x+6}-4x+1}{x^3-2x+1}$

6). $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2+3}+\sqrt{2x^2+4x+19}-\sqrt{3x^2+46}}{x^2-1}$ 8). $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{x+6}-\sqrt[4]{7x+2}}{x-2}$

9). $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{3x+2}-\sqrt{3x-2}}{x-2}$ 10). $\lim\limits_{x\to 1}\dfrac{\sqrt{6x+3}+2x^2-5x}{(x-1)^2}$

LỜI GIẢI

1). $\lim\limits_{x\to 0}\dfrac{2\sqrt{1-x}-\sqrt[3]{8-x}}{x}=\lim\limits_{x\to 0}\dfrac{2\sqrt{1-x}-2+2-\sqrt[3]{8-x}}{x}=\lim\limits_{x\to 0}\dfrac{2\sqrt{1-x}-2}{x}$

$+\lim\limits_{x\to 0}\dfrac{2-\sqrt[3]{8-x}}{x}=\lim\limits_{x\to 0}\dfrac{4(1-x)-4}{x\left(2\sqrt{1-x}+2\right)}+\lim\limits_{x\to 0}\dfrac{8-(8-x)}{x\left[4+2\sqrt[3]{8-x}+\left(\sqrt[3]{8-x}\right)^2\right]}$

$=\lim\limits_{x\to 0}\dfrac{-4}{2\sqrt{1-x}+2}+\lim\limits_{x\to 0}\dfrac{1}{4+2\sqrt[3]{8-x}+\left(\sqrt[3]{8-x}\right)^2}=\dfrac{-4}{4}+\dfrac{1}{12}=-\dfrac{11}{12}$

2). $\lim\limits_{x\to 1}\dfrac{\sqrt{3x-2}-\sqrt[3]{4x^2-x-2}}{x^2-3x+2}$

$\lim\limits_{x\to 1}\dfrac{\sqrt{3x-2}-1+1-\sqrt[3]{4x^2-x-2}}{x^2-3x+2}=\lim\limits_{x\to 1}\dfrac{\sqrt{3x-2}-1}{x^2-3x+2}+\lim\limits_{x\to 1}\dfrac{1-\sqrt[3]{4x^2-x-2}}{x^2-3x+2}$

- Tính $\lim\limits_{x\to 1}\dfrac{\sqrt{3x-2}-1}{x^2-3x+2}=\lim\limits_{x\to 1}\dfrac{3x-2-1}{(x-1)(x-2)\left(\sqrt{3x-2}+1\right)}$

$=\lim\limits_{x\to 1}\dfrac{3(x-1)}{(x-1)(x-2)\left(\sqrt{3x-2}+1\right)}=\lim\limits_{x\to 1}\dfrac{3}{(x-2)\left(\sqrt{3x-2}+1\right)}=\dfrac{3}{-1.2}=-\dfrac{3}{2}$

- Tính $\lim\limits_{x\to 1}\dfrac{1-\sqrt[3]{4x^2-x-2}}{x^2-3x+2}=\lim\limits_{x\to 1}\dfrac{1-(4x^2-x-2)}{\left(x^2-3x+2\right)\left[1+\sqrt[3]{4x^2-x-2}+\left(\sqrt[3]{4x^2-x-2}\right)^2\right]}$

$=\lim\limits_{x\to 1}\dfrac{-(x-1)(4x+3)}{(x-1)(x-2)\left[1+\sqrt[3]{4x^2-x-2}+\left(\sqrt[3]{4x^2-x-2}\right)^2\right]}$

$=\lim\limits_{x\to 1}\dfrac{-(4x+3)}{(x-2)\left[1+\sqrt[3]{4x^2-x-2}+\left(\sqrt[3]{4x^2-x-2}\right)^2\right]}=\dfrac{-7}{-1.3}=\dfrac{7}{3}$

Vậy giới hạn cần tìm: $-\dfrac{3}{2}+\dfrac{7}{3}=\dfrac{5}{6}$

CÁCH 2:

$$\lim_{x\to1}\frac{\sqrt{3x-2}-1+1-\sqrt[3]{4x^2-x-2}}{x^2-3x+2}=\lim_{x\to1}\frac{\frac{\sqrt{3x-2}-1}{x-1}+\frac{1-\sqrt[3]{4x^2-x-2}}{x-1}}{\frac{x^2-3x+2}{x-1}}$$

$$=\lim_{x\to1}\frac{\frac{2}{\sqrt{3x-2}+1}+\frac{-4x-3}{1+\sqrt[3]{4x^2-x-2}+\left(\sqrt[3]{4x^2-x-2}\right)^2}}{x-2}=\frac{5}{6}$$

3) $L=\lim_{x\to1}\frac{\sqrt{5-x^3}-\sqrt[3]{x^2+7}}{x^2-1}$

$$L=\lim_{x\to1}\frac{\sqrt{5-x^3}-2+2-\sqrt[3]{x^2+7}}{x^2-1}=\lim_{x\to1}\frac{\sqrt{5-x^3}-2}{x^2-1}+\lim_{x\to1}\frac{2-\sqrt[3]{x^2+7}}{x^2-1}$$

g Tính $\lim_{x\to1}\frac{\sqrt{5-x^3}-2}{x^2-1}=\lim_{x\to1}\frac{5-x^3-4}{(x^2-1)\left(\sqrt{5-x^3}+2\right)}=\lim_{x\to1}\frac{(1-x)(1+x+x^2)}{(x-1)(x+1)\left(\sqrt{5-x^3}+2\right)}$

$$=\lim_{x\to1}\frac{-(1+x+x^2)}{(x+1)\left(\sqrt{5-x^3}+2\right)}=\frac{-3}{2.4}=-\frac{3}{8}$$

g Tính $\lim_{x\to1}\frac{2-\sqrt[3]{x^2+7}}{x^2-1}=\lim_{x\to1}\frac{8-(x^2+7)}{(x^2-1)\left(4+\sqrt[3]{x^2+7}+\left(\sqrt[3]{x^2+7}\right)^2\right)}$

$$=\lim_{x\to1}\frac{1-x^2}{(x^2-1)\left(4+\sqrt[3]{x^2+7}+\left(\sqrt[3]{x^2+7}\right)^2\right)}=\lim_{x\to1}\frac{-1}{\left(4+\sqrt[3]{x^2+7}+\left(\sqrt[3]{x^2+7}\right)^2\right)}=-\frac{1}{12}$$

Kết luận $L=-\frac{3}{8}-\frac{1}{12}=-\frac{11}{24}$

4). $\lim_{x\to4}\frac{2-\sqrt{x^2-12}}{\left(\sqrt{x^2+x-19}-1\right)\left(\sqrt{x+12}-2\right)}$

Ta có $2-\sqrt{x^2-12}=\frac{4-(x^2-12)}{2+\sqrt{x^2-12}}=\frac{16-x^2}{2+\sqrt{x^2-12}}=\frac{(4-x)(4+x)}{2+\sqrt{x^2-12}}$

Ta có $\frac{1}{\sqrt{x^2+x-19}-1}=\frac{\sqrt{x^2+x-19}+1}{x^2+x-19-1}=\frac{\sqrt{x^2+x-19}+1}{x^2+x-20}=\frac{\sqrt{x^2+x-19}+1}{(x-4)(x+5)}$

$$=\lim_{x\to4}\frac{(4-x)(4+x)}{2+\sqrt{x^2-12}}\cdot\frac{\sqrt{x^2+x-19}+1}{(x-4)(x+5)}\cdot\frac{1}{\sqrt{x+12}-2}$$

$$=\lim_{x\to4}\frac{-(4+x)}{2+\sqrt{x^2-12}}\cdot\frac{\sqrt{x^2+x-19}+1}{x+5}\cdot\frac{1}{\sqrt{x+12}-2}=\frac{-8}{4}\cdot\frac{2}{9}\cdot\frac{1}{2}=-\frac{2}{9}$$

5). $\lim_{x\to2}\frac{3\sqrt[3]{4x^3-24}+\sqrt{x+2}-8\sqrt{2x-3}}{4-x^2}$. Đặt $f(x)=\frac{3\sqrt[3]{4x^3-24}+\sqrt{x+2}-8\sqrt{2x-3}}{4-x^2}$

$$\lim_{x\to2}f(x)=\lim_{x\to2}\frac{3\sqrt[3]{4x^3-24}-6+\sqrt{x+2}-2+8-8\sqrt{2x-3}}{4-x^2}$$

$$= \lim_{x\to 2}\frac{3\sqrt[3]{4x^3-24}-6}{4-x^2}+\lim_{x\to 2}\frac{\sqrt{x+2}-2}{4-x^2}+\lim_{x\to 2}\frac{8-8\sqrt{2x-3}}{4-x^2}$$

• Tính : $\lim_{x\to 2}\frac{3\sqrt[3]{4x^3-24}-6}{4-x^2}=\lim_{x\to 2}3\cdot\frac{\sqrt[3]{4x^3-24}-2}{4-x^2}$

$$=3\lim_{x\to 2}\frac{4x^3-24-8}{(4-x^2)\underbrace{\left[\left(\sqrt[3]{4x^3-24}\right)+2\sqrt[3]{4x^3-24}+2\right]}_{A}}=3\lim_{x\to 2}\frac{4x^3-24-8}{(4-x^2).A}=3\lim_{x\to 2}\frac{4(x^3-8)}{(4-x^2).A}$$

$$=3\lim_{x\to 2}\frac{4(x-2)(x^2+2x+4)}{(2-x)(2+x).A}=3\lim_{x\to 2}\frac{-4(x^2+2x+4)}{(2+x).A}=3\lim_{x\to 2}\frac{-4.12}{4.8}=-\frac{9}{2}$$

• Tính $\lim_{x\to 2}\frac{\sqrt{x+2}-2}{4-x^2}=\lim_{x\to 2}\frac{x+2-4}{(4-x^2)\left(\sqrt{x+2}+2\right)}=\lim_{x\to 2}\frac{x-2}{(2+x)(2-x)\left(\sqrt{x+2}+2\right)}$

$$=\lim_{x\to 2}\frac{-1}{(2+x)\left(\sqrt{x+2}+2\right)}=-\frac{1}{4}.$$

• Tính $\lim_{x\to 2}\frac{8-8\sqrt{2x-3}}{4-x^2}=\lim_{x\to 2}\frac{8\left(1-\sqrt{2x-3}\right)}{4-x^2}=8\lim_{x\to 2}\frac{1-(2x-3)}{(4-x^2)\left(1+\sqrt{2x-3}\right)}$

$$=8\lim_{x\to 2}\frac{2(2-x)}{(2-x)(2+x)\left(1+\sqrt{2x-3}\right)}=8\lim_{x\to 2}\frac{2}{(2+x)\left(1+\sqrt{2x-3}\right)}=8\cdot\frac{2}{4.2}=2$$

Vậy giới hạn cần tìm : $\lim_{x\to 2}f(x)=-\frac{9}{2}-\frac{1}{4}+2=-\frac{11}{4}$

7). $\lim_{x\to 1}\frac{\sqrt{x^2+2x+6}-4x+1}{x^3-2x+1}$

$$\lim_{x\to 1}\frac{\sqrt{x^2+2x+6}-(4x-1)}{x^3-2x+1}=\lim_{x\to 1}\frac{x^2+2x+6-(4x-1)^2}{(x^3-2x+1)\left(\sqrt{x^2+2x+6}+(4x-1)\right)}$$

$$=\lim_{x\to 1}\frac{-15x^2+10x+5}{(x^3-2x+1)\left(\sqrt{x^2+2x+6}+4x-1\right)}=\lim_{x\to 1}\frac{-5(x-1)(3x+1)}{(x-1)(x^2+x-1)\left(\sqrt{x^2+2x+6}+4x-1\right)}$$

Phân tích $x^3-2x+1=(x-1)(x^2+x-1)$, bằng sơ đồ Hoocne sau:

	1	0	-2	1
1	1	1	-1	0

$$=\lim_{x\to 1}\frac{-5(3x+1)}{(x^2+x-1)\left(\sqrt{x^2+2x+6}+4x-1\right)}=\frac{-20}{1.6}=-\frac{10}{3}$$

6). $L=\lim_{x\to 1}\frac{\sqrt{x^2+3}+\sqrt{2x^2+4x+19}-\sqrt{3x^2+46}}{x^2-1}$

$$L=\lim_{x\to 1}\frac{\sqrt{x^2+3}-2+\sqrt{2x^2+4x+19}-5+7-\sqrt{3x^2+46}}{x^2-1}$$

$$=\lim_{x\to 1}\frac{\sqrt{x^2+3}-2}{x^2-1}+\lim_{x\to 1}\frac{\sqrt{2x^2+4x+19}-5}{x^2-1}+\lim_{x\to 1}\frac{7-\sqrt{3x^2+46}}{x^2-1}$$

• Tính $\lim_{x\to1}\frac{\sqrt{x^2+3}-2}{x^2-1}=\lim_{x\to1}\frac{x^2+3-4}{(x^2-1)\left(\sqrt{x^2+3}+2\right)}$

$=\lim_{x\to1}\frac{x^2-1}{(x^2-1)\left(\sqrt{x^2+3}+2\right)}=\lim_{x\to1}\frac{1}{\sqrt{x^2+3}+2}=\frac{1}{4}$

• Tính $\lim_{x\to1}\frac{\sqrt{2x^2+4x+19}-5}{x^2-1}=\lim_{x\to1}\frac{2x^2+4x+19-25}{(x^2-1)\left(\sqrt{2x^2+4x+19}+5\right)}$

$=\lim_{x\to1}\frac{2x^2+4x-6}{(x^2-1)\left(\sqrt{2x^2+4x+19}+5\right)}=\lim_{x\to1}\frac{2(x-1)(x+3)}{(x-1)(x+1)\left(\sqrt{2x^2+4x+19}+5\right)}$

$=\lim_{x\to1}\frac{2(x+3)}{(x+1)\left(\sqrt{2x^2+4x+19}+5\right)}=\frac{2.4}{2.10}=\frac{2}{5}$

• Tính $\lim_{x\to1}\frac{7-\sqrt{3x^2+46}}{x^2-1}=\lim_{x\to1}\frac{49-(3x^2+46)}{(x^2-1)\left(7+\sqrt{3x^2+46}\right)}=\lim_{x\to1}\frac{-3(x^2-1)}{(x^2-1)\left(7+\sqrt{3x^2+46}\right)}$

$=\lim_{x\to1}\frac{-3}{\left(7+\sqrt{3x^2+46}\right)}=-\frac{3}{14}$

Kết luận $L=\frac{1}{4}+\frac{2}{5}-\frac{3}{14}=\frac{61}{140}$

8). $\lim_{x\to2}\frac{\sqrt[3]{x+6}-\sqrt[4]{7x+2}}{x-2}$. Đặt $f(x)=\frac{\sqrt[3]{x+6}-\sqrt[4]{7x+2}}{x-2}$

$\lim_{x\to2}f(x)=\lim_{x\to2}\frac{\sqrt[3]{x+6}-2+2-\sqrt[4]{7x+2}}{x-2}=\lim_{x\to2}\frac{\sqrt[3]{x+6}-2}{x-2}+\lim_{x\to2}\frac{2-\sqrt[4]{7x+2}}{x-2}$

• Tính $\lim_{x\to2}\frac{\sqrt[3]{x+6}-2}{x-2}=\lim_{x\to2}\frac{x+6-8}{(x-2)\left[\left(\sqrt[3]{x+6}\right)^2+2.\sqrt[3]{x+6}+4\right]}$

$=\lim_{x\to2}\frac{1}{\left(\sqrt[3]{x+6}\right)^2+2.\sqrt[3]{x+6}+4}=\frac{1}{12}$

• Tính $\lim_{x\to2}\frac{2-\sqrt[4]{7x+2}}{x-2}=\lim_{x\to2}\frac{4-\sqrt{7x+2}}{(x-2)\left(2+\sqrt[4]{7x+2}\right)}=\lim_{x\to2}\frac{16-(7x+2)}{(x-2)\left(2+\sqrt[4]{7x+2}\right)\left(4+\sqrt{7x+2}\right)}$

$=\lim_{x\to2}\frac{-7(x-2)}{(x-2)\left(2+\sqrt[4]{7x+2}\right)\left(4+\sqrt{7x+2}\right)}=\lim_{x\to2}\frac{-7}{\left(2+\sqrt[4]{7x+2}\right)\left(4+\sqrt{7x+2}\right)}=-\frac{7}{32}$

Vậy $\lim_{x\to2}f(x)=\frac{1}{12}-\frac{7}{32}=-\frac{13}{96}$.

9). $\lim_{x\to2}\frac{\sqrt[3]{3x+2}-\sqrt{3x-2}}{x-2}$. Đặt $f(x)=\frac{\sqrt[3]{3x+2}-\sqrt{3x-2}}{x-2}$

Có $\lim_{x\to2}f(x)=\lim_{x\to2}\frac{\sqrt[3]{3x+2}-2+2-\sqrt{3x-2}}{x-2}=\lim_{x\to2}\frac{\sqrt[3]{3x+2}-2}{x-2}+\lim_{x\to2}\frac{2-\sqrt{3x-2}}{x-2}$

• Tính $\lim_{x\to 2}\frac{\sqrt[3]{3x+2}-2}{x-2}=\lim_{x\to 2}\frac{3x+2-8}{(x-2)\left[\left(\sqrt[3]{3x+2}\right)^2+2.\sqrt[3]{3x+2}+4\right]}$

$=\lim_{x\to 2}\frac{3(x-2)}{(x-2)\left[\left(\sqrt[3]{3x+2}\right)^2+2.\sqrt[3]{3x+2}+4\right]}=\lim_{x\to 2}\frac{3}{\left[\left(\sqrt[3]{3x+2}\right)^2+2.\sqrt[3]{3x+2}+4\right]}=\frac{1}{4}$

• Tính $\lim_{x\to 2}\frac{2-\sqrt{3x-2}}{x-2}=\lim_{x\to 2}\frac{4-(3x-2)}{(x-2)(2+\sqrt{3x-2})}$

$=\lim_{x\to 2}\frac{-3(x-2)}{(x-2)(2+\sqrt{3x-2})}=\lim_{x\to 2}\frac{-3}{2+\sqrt{3x-2}}=-\frac{3}{4}$

Vậy $\lim_{x\to 2}f(x)=\frac{1}{4}-\frac{3}{4}=-\frac{1}{2}$.

Tương tự: Tìm $\lim_{x\to 2}\frac{\sqrt{6-x}-\sqrt[3]{x^2+4}}{x^2-4}$; $\lim_{x\to 7}\frac{\sqrt{x+2}-\sqrt[3]{x+20}}{\sqrt[4]{x+9}-2}$; $\lim_{x\to 0}\frac{\sqrt{1+4x}-\sqrt[3]{1+6x}}{x}$.

10). $\lim_{x\to 1}\frac{\sqrt{6x+3}+2x^2-5x}{(x-1)^2}=\lim_{x\to 1}\frac{\sqrt{6x+3}-(x+2)+2(x^2-2x+1)}{(x-1)^2}$

$=\lim_{x\to 1}\frac{\sqrt{6x+3}-(x+2)}{(x-1)^2}+\lim_{x\to 1}\frac{2(x^2-2x+1)}{(x-1)^2}$

$=\lim_{x\to 1}\frac{6x+3-(x+2)^2}{(x-1)^2}+2=\lim_{x\to 1}\frac{-x^2+2x-1}{(x-1)^2}+2=\lim_{x\to 1}\frac{-(x-1)^2}{(x-1)^2}+2=-1+2=1$.