

Câu 5: Tìm các giới hạn sau:

- a).  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 - 4x} - x)$    b).  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 1} - x)$    c).  $\lim_{x \rightarrow -\infty} (\sqrt{x^2 - 3x + 2} + x)$   
d).  $\lim_{x \rightarrow -\infty} (\sqrt{4x^2 - 4x + 1} + 2x + 3)$    e).  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 - 4x + 1} - 2x + 3)$

LỜI GIẢI

$$\text{a). } \lim_{x \rightarrow +\infty} (\sqrt{x^2 - 4x} - x) = \lim_{x \rightarrow +\infty} \frac{x^2 - 4x - x^2}{\sqrt{x^2 - 4x} + x} = \lim_{x \rightarrow +\infty} \frac{-4x}{\sqrt{x^2 \left(1 - \frac{4}{x}\right)} + x} = \lim_{x \rightarrow +\infty} \frac{-4x}{|x| + x} = \lim_{x \rightarrow +\infty} \frac{-4x}{2x} = -2.$$

$$\text{b). } \lim_{x \rightarrow +\infty} (\sqrt{x^2 + 1} - x) = \lim_{x \rightarrow +\infty} \frac{x^2 + 1 - x^2}{\sqrt{x^2 + 1} + x} = \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{x^2 \left(1 + \frac{1}{x^2}\right)} + x} = \lim_{x \rightarrow +\infty} \frac{1}{|x| + x} = \lim_{x \rightarrow +\infty} \frac{1}{2x} = 0.$$

$$\text{c). } \lim_{x \rightarrow -\infty} (\sqrt{x^2 - 3x + 2} + x) = \lim_{x \rightarrow -\infty} \frac{x^2 - 3x + 2 - x^2}{\sqrt{x^2 - 3x + 2} - x} = \lim_{x \rightarrow -\infty} \frac{-3x + 2}{\sqrt{x^2 \left(1 - \frac{3}{x} + \frac{2}{x^2}\right)} - x}$$
$$= \lim_{x \rightarrow -\infty} \frac{-x \left(3 + \frac{2}{x}\right)}{|x| - x} = \lim_{x \rightarrow -\infty} \frac{-3x}{-2x} = \frac{3}{2}.$$

$$\text{d). } \lim_{x \rightarrow -\infty} (\sqrt{4x^2 - 4x + 1} + 2x + 3) = \lim_{x \rightarrow -\infty} \left( \frac{4x^2 - 4x + 1 - 4x^2}{\sqrt{4x^2 - 4x + 1} - 2x} \right) + 3 = \lim_{x \rightarrow -\infty} \frac{-4x + 1}{\sqrt{x^2 \left(4 - \frac{4}{x} + \frac{1}{x^2}\right)} - 2x} + 3$$
$$= \lim_{x \rightarrow -\infty} \frac{-x \left(4 + \frac{1}{x}\right)}{2|x| - 2x} + 3 = \lim_{x \rightarrow -\infty} \frac{-4x}{-2x} + 3 = 2 + 3 = 5.$$

$$\text{e). } \lim_{x \rightarrow +\infty} (\sqrt{x^2 - 4x + 1} - 2x + 3) = \lim_{x \rightarrow +\infty} \frac{x^2 - 4x + 1 - 4x^2}{\sqrt{x^2 - 4x + 1} + 2x} + 3 = \lim_{x \rightarrow +\infty} \frac{-3x^2 - 4x + 1}{\sqrt{x^2 \left(1 - \frac{4}{x} + \frac{1}{x^2}\right)} + 2x} + 3$$
$$= \lim_{x \rightarrow +\infty} \frac{x^2 \left(-3 - \frac{4}{x} + \frac{1}{x^2}\right)}{|x| + 2x} + 3 = \lim_{x \rightarrow +\infty} \frac{-8x^2}{x + 2x} + 3 = \lim_{x \rightarrow +\infty} \frac{-8x^2}{3x} + 3 = \lim_{x \rightarrow +\infty} (-3x) + 3 = -\infty.$$