Câu 3: Tìm các giới hạn sau:

1). $\lim\limits_{x\to\frac{\pi}{4}} \tan 2x.\tan\left(\frac{\pi}{4}-x\right)$ 2). $\lim\limits_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$ 3). $\lim\limits_{x\to 1}\frac{\sqrt{x+3}-2}{\tan(x-1)}$

4). $\lim\limits_{x\to-\frac{\pi}{2}}\frac{\cos x}{x+\frac{\pi}{2}}$ 5). $\lim\limits_{x\to\pi}\frac{1+\cos x}{(x-\pi)^2}$ 6). $\lim\limits_{x\to 1}\frac{\sin(x-1)}{x^2-4x+3}$

7). $\lim\limits_{x\to\frac{\pi}{6}}\frac{2\sin x-1}{4\cos^2 x-3}$ 8). $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\sin x-1}{2\cos^2 x-1}$ 9). $\lim\limits_{x\to\frac{\pi}{6}}\frac{\sin\left(\frac{\pi}{6}-x\right)}{1-2\sin x}$

LỜI GIẢI

1). $L=\lim\limits_{x\to\frac{\pi}{4}}\tan 2x.\tan\left(\frac{\pi}{4}-x\right)$. Đặt $t=x-\frac{\pi}{4}$, vì $x\to\frac{\pi}{4}\Rightarrow t\to 0$

$L=\lim\limits_{t\to 0}\left[\tan\left(2t+\frac{\pi}{2}\right)(-1)\tan t\right]=\lim\limits_{t\to 0}\left(\cot 2t.\tan t\right)$

$=\lim\limits_{t\to 0}\frac{\cos 2t}{\sin 2t}\frac{\sin t}{\cos t}=\lim\limits_{t\to 0}\frac{\cos 2t}{2\sin t\cos t}\frac{\sin t}{\cos t}=\lim\limits_{t\to 0}\frac{\cos 2t}{2\cos^2 t}=\frac{1}{2}$

2). $\lim\limits_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}=\lim\limits_{x\to 0}\frac{\tan x-\sin x}{x^3\underbrace{\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}_{A}}=\lim\limits_{x\to 0}\frac{\sin x(x-\cos x)}{x^3.A.\cos x}$

$=\lim\limits_{x\to 0}\frac{2\sin x\sin^2\frac{x}{2}}{x^3.A.\cos x}=\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right).\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2.\frac{1}{2A.\cos x}=\frac{1}{4}.$

3). $\lim\limits_{x\to 1}\frac{\sqrt{x+3}-2}{\tan(x-1)}=\lim\limits_{x\to 1}\frac{x+3-4}{\left(\sqrt{x+3}+2\right)\tan(x-1)}=\lim\limits_{x\to 1}\frac{x-1}{\tan(x-1)}\frac{1}{\sqrt{x+3}+2}$

(Vì $\lim\limits_{x\to 1}\frac{x-1}{\tan(x-1)}=1$, $\lim\limits_{x\to 1}\frac{1}{\sqrt{x+3}+2}=\frac{1}{4}$)

Vậy $L=\frac{1}{4}$.

4). $L=\lim\limits_{x\to-\frac{\pi}{2}}\frac{\cos x}{x+\frac{\pi}{2}}$. Đặt $t=x+\frac{\pi}{2}$, vì $x\to-\frac{\pi}{2}\Rightarrow t\to 0$

$L=\lim\limits_{t\to 0}\frac{\cos\left(t-\frac{\pi}{2}\right)}{t}=\lim\limits_{t\to 0}\frac{\sin t}{t}=1$.

5). $L=\lim\limits_{x\to\pi}\frac{1+\cos x}{(x-\pi)^2}$. Đặt $t=x-\pi$, vì $x\to\pi\Rightarrow t\to 0$

$$L=\lim_{t\to 0}\frac{1+\cos(t+\pi)}{t^2}=\lim_{t\to 0}\frac{1-\cos t}{t^2}=\lim_{t\to 0}\frac{2\sin^2\frac{t}{2}}{t^2}=\lim_{t\to 0}\frac{1}{2}\left(\frac{\sin\frac{t}{2}}{\frac{t}{2}}\right)^2=\frac{1}{2}.$$

6). $L=\lim_{x\to 1}\frac{\sin(x-1)}{x^2-4x+3}=\lim_{x\to 1}\frac{\sin(x-1)}{(x-1)(x-3)}$. Đặt $t=x-1$, vì $x\to 1\Rightarrow t\to 0$

$$L=\lim_{t\to 0}\frac{\sin t}{t.(t-2)}=\lim_{t\to 0}\frac{\sin t}{t}\cdot\frac{1}{t-2}=-\frac{1}{2}.$$

7). $L=\lim_{x\to\frac{\pi}{6}}\frac{2\sin x-1}{4\cos^2 x-3}=\lim_{x\to\frac{\pi}{6}}\frac{2\sin x-1}{4(1-\sin^2 x)-3}=\lim_{x\to\frac{\pi}{6}}\frac{2\sin x-1}{1-4\sin^2 x}$

$$=\lim_{x\to\frac{\pi}{6}}\frac{2\sin x-1}{(1-2\sin x)(1+2\sin x)}=\lim_{x\to\frac{\pi}{6}}\frac{-1}{1+2\sin x}=-\frac{1}{2}$$

8). $L=\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\sin x-1}{2\cos^2 x-1}=\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\sin x-1}{2(1-\sin^2 x)-1}=\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\sin x-1}{1-2\sin^2 x}$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\sin x-1}{(1-\sqrt{2}\sin x)(1+\sqrt{2}\sin x)}=\lim_{x\to\frac{\pi}{4}}\frac{-1}{1+\sqrt{2}\sin x}=-\frac{1}{2}.$$

9). $=\lim_{x\to\frac{\pi}{6}}\frac{\sin\left(\frac{\pi}{6}-x\right)}{1-2\sin x}=\lim_{x\to\frac{\pi}{6}}\frac{\sin\left(\frac{\pi}{6}-x\right)}{-2\left(\sin x-\frac{1}{2}\right)}=\lim_{x\to\frac{\pi}{6}}\frac{\sin\left(x-\frac{\pi}{6}\right)}{2\left(\sin x-\sin\frac{\pi}{6}\right)}$

$$=\lim_{x\to\frac{\pi}{6}}\frac{2\sin\left(\frac{x}{2}-\frac{\pi}{12}\right)\cos\left(\frac{x}{2}-\frac{\pi}{12}\right)}{4\cos\left(\frac{x}{2}+\frac{\pi}{12}\right)\sin\left(\frac{x}{2}-\frac{\pi}{12}\right)}=\lim_{x\to\frac{\pi}{6}}\frac{1}{2}\frac{\cos\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\cos\left(\frac{x}{2}+\frac{\pi}{12}\right)}=\frac{\sqrt{3}}{3}$$