

- 1). $\sin^2 3x - \cos^2 4x = \sin^2 5x - \cos^2 6x$ (1) [ĐH B02]
- 2). Tìm $x \in [0; 14]$: $\cos 3x - 4 \cos 2x + 3 \cos x - 4 = 0$ [ĐH D02]
- 3). $(2 \cos x - 1)(2 \sin x + \cos x) = \sin 2x - \sin x$ [ĐH D04]
- 4). $\sqrt{1 - \sin x} + \sqrt{1 - \cos x} = 1$ [Dự bị 2 ĐH A04]
- 5). $\sin 4x \sin 7x = \cos 3x \cos 6x$ [Dự bị 1 ĐH D04]
- 6). $\cos^2 3x \cos 2x - \cos^2 x = 0$ [ĐH A05]
- 7). $1 + \sin x + \cos x + \sin 2x + \cos 2x = 0$ [ĐH B05]
- 8). Tìm $x \in (0; \pi)$ $4 \sin^2 \frac{x}{2} - \sqrt{3} \cos 2x = 1 + 2 \cos^2 \left(x - \frac{3\pi}{4} \right)$ [Dự bị 1 ĐH A05]

LỜI GIẢI

1). $\sin^2 3x - \cos^2 4x = \sin^2 5x - \cos^2 6x$ (1)

$$(1) \Leftrightarrow \frac{1 - \cos 6x}{2} - \frac{1 + \cos 8x}{2} = \frac{1 - \cos 10x}{2} - \frac{1 + \cos 12x}{2} \Leftrightarrow \cos 12x + \cos 10x = \cos 8x + \cos 6x$$

$$\Leftrightarrow 2 \cos x (\cos 11x - \cos 7x) = 0 \Leftrightarrow -4 \cos x \cdot \sin 9x \cdot \sin 2x = 0$$

$$\Leftrightarrow \sin 9x = 0 \text{ hoặc } \sin 2x = 0 \Leftrightarrow x = \frac{k\pi}{9} \text{ hoặc } x = \frac{k\pi}{2}, (k \in \mathbb{Z})$$

Nghiệm phương trình: $x = \frac{k\pi}{9}, x = \frac{k\pi}{2}, (k \in \mathbb{Z})$

2). Tìm $x \in [0; 14]$: $\cos 3x - 4 \cos 2x + 3 \cos x - 4 = 0$ (1)

Ta có : $\cos 3x = 4 \cos^3 x - 3 \cos x$

$$(1) \Leftrightarrow 4 \cos^3 x - 3 \cos x - 4(2 \cos^2 x - 1) + 3 \cos x - 4 = 0$$

$$\Leftrightarrow 4 \cos^3 x - 8 \cos^2 x = 0$$

$$\Leftrightarrow 4 \cos^2 x (\cos x - 2) = 0 \Leftrightarrow \cos x = 0 \Leftrightarrow x = \frac{\pi}{2} + k\pi, (k \in \mathbb{Z}) \text{ (vì } \cos x = 2 \text{ vô nghiệm).}$$

$$\text{Ta có } 0 < x < 14 \Leftrightarrow 0 < \frac{\pi}{2} + k\pi < 14 \Leftrightarrow -\frac{\pi}{2} < k\pi < 14 - \frac{\pi}{2} \Leftrightarrow -0,5 < k < 3,96.$$

Từ đó suy ra $k = 0, 1, 2, 3$. Kết luận nghiệm cần tìm $x \in \left\{ \frac{\pi}{2}; \frac{3\pi}{2}; \frac{5\pi}{2}; \frac{7\pi}{2} \right\}$

3). $(2 \cos x - 1)(2 \sin x + \cos x) = \sin 2x - \sin x$ (1)

$$(1) \Leftrightarrow (2 \cos x - 1)(2 \sin x + \cos x) = 2 \sin x \cos x - \sin x$$

$$\Leftrightarrow (2 \cos x - 1)(2 \sin x + \cos x) = \sin x(2 \cos x - 1)$$

$$\Leftrightarrow (2 \cos x - 1)(\sin x + \cos x) = 0 \Leftrightarrow 2 \cos x - 1 = 0 \vee \sin x + \cos x = 0$$

$$\text{Với } 2 \cos x - 1 = 0 \Leftrightarrow \cos x = \frac{1}{2} \Leftrightarrow x = \pm \frac{\pi}{3} + k2\pi, (k \in \mathbb{Z})$$

$$\text{Với: } \sin x + \cos x = 0 \Leftrightarrow \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) = 0 \Leftrightarrow x + \frac{\pi}{4} = k\pi \Leftrightarrow x = -\frac{\pi}{4} + k\pi, (k \in \mathbb{Z})$$

$$\text{Nghiệm phương trình: } x = \pm \frac{\pi}{3} + k2\pi, x = -\frac{\pi}{4} + k\pi, (k \in \mathbb{Z}).$$

$$4). \sqrt{1 - \sin x} + \sqrt{1 - \cos x} = 1$$

$$\sqrt{1 - \sin x} + \sqrt{1 - \cos x} = 1 \quad (1) \quad \text{TXĐ: } D = \mathbb{R}$$

Chú ý: $1 - \sin x \geq 0$; $1 - \cos x \geq 0$ (đúng)

$$(1) \Leftrightarrow 2 - (\sin x + \cos x) + 2\sqrt{(1 - \sin x)(1 - \cos x)} = 1$$

$$\Leftrightarrow 2 - (\sin x + \cos x) + 2\sqrt{1 - (\sin x + \cos x) - \sin x \cos x} = 1 \quad (2)$$

$$\text{Đặt: } t = \sin x + \cos x; |t| \leq \sqrt{2}, \text{ khi đó: } \sin x \cos x = \frac{t^2 - 1}{2}$$

$$(2) \Leftrightarrow 1 - t + 2\sqrt{\frac{t^2 - 2t + 1}{2}} = 0 \Leftrightarrow 1 - t + \sqrt{2}\sqrt{(t-1)^2} = 0 \Leftrightarrow 1 - t + \sqrt{2}|t-1| = 0$$

$$\Leftrightarrow \sqrt{2}|t-1| = t-1 \Leftrightarrow t-1 = 0 \Leftrightarrow t = 1 \Leftrightarrow \sin x + \cos x = 1 \Leftrightarrow \sqrt{2} \cos\left(x - \frac{\pi}{4}\right) = 1$$

$$\Leftrightarrow \cos\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \Leftrightarrow x = k2\pi \text{ hoặc } x = -\frac{\pi}{2} + k2\pi, (k \in \mathbb{Z}).$$

$$5). \sin 4x \sin 7x = \cos 3x \cos 6x \quad (1)$$

$$(1) \Leftrightarrow \frac{1}{2}(\cos 3x - \cos 11x) = \frac{1}{2}(\cos 9x + \cos 3x)$$

$$\Leftrightarrow \cos 3x - \cos 11x = \cos 9x + \cos 3x \Leftrightarrow \cos 11x + \cos 9x = 0$$

$$\Leftrightarrow 2 \cos 10x \cos x = 0 \Leftrightarrow \cos 10x = 0 \text{ hoặc } \cos x = 0$$

$$\Leftrightarrow x = \frac{\pi}{20} + k10\pi \text{ hoặc } x = \frac{\pi}{2} + k\pi, (k \in \mathbb{Z}).$$

Nghiệm phương trình: $x = \frac{\pi}{20} + k10\pi, x = \frac{\pi}{2} + k\pi, (k \in \mathbb{Z})$

$$6). \cos^2 3x \cos 2x - \cos^2 x = 0 \quad (1)$$

$$(1) \Leftrightarrow \frac{(1 + \cos 6x) \cos 2x}{2} - \frac{1 + \cos 2x}{2} = 0$$

$$\Leftrightarrow \cos 2x + \cos 6x \cos 2x - 1 - \cos 2x = 0$$

$$\Leftrightarrow \cos 6x \cos 2x - 1 = 0 \Leftrightarrow \cos 8x + \cos 4x - 2 = 0 \Leftrightarrow 2 \cos^2 4x - 1 + \cos 4x - 2 = 0$$

$$\Leftrightarrow 2 \cos^2 4x + \cos 4x - 3 = 0 \Leftrightarrow \cos 4x = 1 \vee \cos 4x = -\frac{3}{2} \text{ (loại)}.$$

Với $\cos 4x = 1 \Leftrightarrow x = \frac{k\pi}{2}, (k \in \mathbb{Z})$

Nghiệm phương trình: $x = \frac{k\pi}{2}, (k \in \mathbb{Z})$

$$7). 1 + \sin x + \cos x + \sin 2x + \cos 2x = 0$$

$$\Leftrightarrow \sin x + \cos x + 2 \sin x \cos x + 2 \cos^2 x = 0 \Leftrightarrow (\sin x + \cos x) + 2 \cos x (\sin x + \cos x) = 0$$

$$\Leftrightarrow (\sin x + \cos x)(1 + 2 \cos x) = 0 \quad \Leftrightarrow \sin x + \cos x = 0 \vee 1 + 2 \cos x = 0$$

Với $\sin x + \cos x = 0 \Leftrightarrow \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) = 0 \Leftrightarrow x = -\frac{\pi}{4} + k\pi, (k \in \mathbb{Z})$

Với $1 + 2 \cos x = 0 \Leftrightarrow \cos x = -\frac{1}{2} \Leftrightarrow x = \pm \frac{2\pi}{3} + k2\pi, (k \in \mathbb{Z})$

Nghiệm phương trình: $x = -\frac{\pi}{4} + k\pi, x = \pm \frac{2\pi}{3} + k2\pi, (k \in \mathbb{Z})$.

$$8). \text{ Tìm } x \in (0; \pi) \quad 4 \sin^2 \frac{x}{2} - \sqrt{3} \cos 2x = 1 + 2 \cos^2 \left(x - \frac{3\pi}{4}\right) \quad (1)$$

$$(1) \Leftrightarrow 2(1 - \cos x) - \sqrt{3} \cos 2x = 1 + 1 + \cos \left(2x - \frac{3\pi}{2}\right)$$

$$\Leftrightarrow 2 - 2 \cos x - \sqrt{3} \cos 2x = 2 - \sin 2x \Leftrightarrow \sqrt{3} \cos 2x - \sin 2x = -2 \cos x$$

$$\Leftrightarrow \frac{\sqrt{3}}{2} \cos 2x - \frac{1}{2} \sin 2x = -\cos x \Leftrightarrow \cos \left(2x + \frac{\pi}{6}\right) = \cos(\pi - x) \Leftrightarrow \cos \left(2x + \frac{\pi}{6}\right) = \cos(\pi - x) \Leftrightarrow x = \frac{5\pi}{18} + \frac{k_1 2\pi}{3}$$

hoặc $x = -\frac{7\pi}{6} + k_2 2\pi, (k_1; k_2 \in \mathbb{Z})$

$$\forall \begin{cases} k_1 \in \mathcal{C} \\ k_1 \in (0; \pi) \end{cases} \Rightarrow k_1 \in \{0; 1\} \Rightarrow x = \frac{5\pi}{18}; x = \frac{17\pi}{18}$$

$$\forall \begin{cases} k_2 \in \mathcal{C} \\ k_2 \in (0; \pi) \end{cases} \Rightarrow k_2 = 1 \Rightarrow x = \frac{5\pi}{6}$$

Kết luận nghiệm phương trình: $x = \frac{5\pi}{18}; x = \frac{17\pi}{18}; x = \frac{5\pi}{6}$.

Câu : Giải các phương trình sau:

1). $\sin 2x + \cos 2x + 3 \sin x - \cos x - 2 = 0$ [Dự bị 2 ĐH D05]

2). $\cos 3x + \cos 2x - \cos x - 1 = 0$ [ĐH D06]

3). $\cos 3x \cos^3 x - \sin 3x \sin^3 x = \frac{2+3\sqrt{2}}{8}$ [Dự bị 1 ĐH A06]

4). $(2 \sin^2 x - 1) \tan^2 2x + 3(2 \cos^2 x - 1) = 0$ [Dự bị 1 ĐH B06]

5). $\cos 2x + (1 + 2 \cos x)(\sin x - \cos x) = 0$ [Dự bị 2 ĐH B06]

6). $\cos^3 x + \sin^3 x + 2 \sin^2 x = 1$ [Dự bị 1 ĐH D06]

7). $4 \sin^3 x + 4 \sin^2 x + 3 \sin 2x + 6 \cos x = 0$ [Dự bị 2 ĐH D06]

8). $(1 + \sin^2 x) \cos x + (1 + \cos^2 x) \sin x = 1 + \sin 2x$ [ĐH A07]

LỜI GIẢI

1). $\sin 2x + \cos 2x + 3 \sin x - \cos x - 2 = 0$

$$\Leftrightarrow 2 \sin x \cos x + 1 - 2 \sin^2 x + 3 \sin x - \cos x - 2 = 0$$

$$\Leftrightarrow (2 \sin x \cos x - \cos x) + (-2 \sin^2 x + 3 \sin x - 1) = 0$$

$$\Leftrightarrow \cos x(2 \sin x - 1) - (\sin x - 1)(2 \sin x - 1) = 0 \Leftrightarrow (2 \sin x - 1)(\cos x - \sin x + 1) = 0$$

$$\Leftrightarrow 2 \sin x - 1 = 0 \text{ hoặc } \cos x - \sin x + 1 = 0$$

$$\text{Với } 2 \sin x - 1 = 0 \Leftrightarrow \sin x = \frac{1}{2} \Leftrightarrow x = \frac{\pi}{6} + k2\pi \text{ hoặc } x = \frac{5\pi}{6} + k2\pi, (k \in \mathcal{C})$$

$$\text{Với } \sin x - \cos x = 1 \Leftrightarrow \sin\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \Leftrightarrow x = \frac{\pi}{2} + k2\pi \text{ hoặc } x = \pi + k2\pi, (k \in \mathcal{C})$$

$$\text{Nghiệm phương trình: } x = \frac{\pi}{6} + k2\pi, x = \frac{5\pi}{6} + k2\pi, x = \frac{\pi}{2} + k2\pi, x = \pi + k2\pi, (k \in \mathcal{C})$$

$$2). \cos 3x + \cos 2x - \cos x - 1 = 0$$

$$\Leftrightarrow \cos 3x - \cos x + \cos 2x - 1 = 0 \Leftrightarrow 2 \sin 2x \sin x + 2 \sin^2 x = 0$$

$$\Leftrightarrow 2 \sin x (\sin 2x + \sin x) = 0 \Leftrightarrow 2 \sin x (2 \sin x \cos x - \sin x) = 0$$

$$\Leftrightarrow 2 \sin^2 x (2 \cos x - 1) = 0 \Leftrightarrow \sin x = 0 \vee 2 \cos x - 1 = 0$$

$$\text{Với } \sin x = 0 \Leftrightarrow x = k\pi, (k \in \mathbb{Z})$$

$$\text{Với } 2 \cos x - 1 = 0 \Leftrightarrow \cos x = \frac{1}{2} \Leftrightarrow x = \pm \frac{\pi}{3} + k2\pi, (k \in \mathbb{Z})$$

$$\text{Nghiem phương trình: } x = k\pi, x = \pm \frac{\pi}{3} + k2\pi, (k \in \mathbb{Z})$$

$$3). \cos 3x \cos^3 x - \sin 3x \sin^3 x = \frac{2+3\sqrt{2}}{8} \quad (1)$$

LỜI GIẢI

Nhắc lại công thức nhân ba:

$$\cos 3x = 4 \cos^3 x - 3 \cos x \Leftrightarrow \cos^3 x = \frac{1}{4} (\cos 3x + 3 \cos x)$$

$$\sin 3x = 3 \sin x - 4 \sin^3 x \Leftrightarrow \sin^3 x = \frac{1}{4} (3 \sin x - \sin 3x)$$

$$(1) \Leftrightarrow \frac{1}{4} [\cos 3x (\cos 3x + 3 \cos x) - \sin 3x (3 \sin x - \sin 3x)] = \frac{2+3\sqrt{2}}{8}$$

$$\Leftrightarrow \cos 3x (\cos 3x + 3 \cos x) - \sin 3x (3 \sin x - \sin 3x) = \frac{2+3\sqrt{2}}{2}$$

$$\Leftrightarrow \cos^2 x + 3 \cos 3x \cos x - 3 \sin 3x \sin x + \sin^2 3x = 1 + \frac{3\sqrt{2}}{2}$$

$$\Leftrightarrow 1 + 3 (\cos 3x \cos x - \sin 3x \sin x) = 1 + \frac{3\sqrt{2}}{2}$$

$$\Leftrightarrow \cos 4x = \frac{\sqrt{2}}{2} = \sin \frac{\pi}{4} \Leftrightarrow 4x = \pm \frac{\pi}{4} + k2\pi \Leftrightarrow x = \pm \frac{\pi}{16} + \frac{k\pi}{2}; (k \in \mathbb{Z})$$

$$\text{Nghiem phương trình: } x = \pm \frac{\pi}{16} + \frac{k\pi}{2}; (k \in \mathbb{Z})$$

$$4). (2 \sin^2 x - 1) \tan^2 2x + 3(2 \cos^2 x - 1) = 0 \quad (1)$$

LỜI GIẢI

Điều kiện : $\cos 2x \neq 0$

$$(1) \Leftrightarrow -\cos 2x \cdot \tan^2 2x + 3 \cos 2x = 0 \Leftrightarrow \cos 2x (\tan^2 2x - 3) = 0 \Leftrightarrow \tan^2 2x = 3$$

$$\Leftrightarrow \tan 2x = \pm\sqrt{3} \Leftrightarrow x = \pm\frac{\pi}{6} + \frac{k\pi}{2}, (k \in \mathbb{Z})$$

Kết luận nghiệm của phương trình là: $x = \pm\frac{\pi}{6} + \frac{k\pi}{2}; k \in \mathbb{Z}$

$$5). \cos 2x + (1 + 2 \cos x)(\sin x - \cos x) = 0$$

LỜI GIẢI

$$\Leftrightarrow (\cos^2 x - \sin^2 x) + (1 + 2 \cos x)(\sin x - \cos x) = 0$$

$$\Leftrightarrow (\cos x - \sin x)(\cos x + \sin x - 2 \cos x - 1) = 0 \Leftrightarrow (\cos x - \sin x)(\sin x - \cos x - 1) = 0$$

$$\text{Với } \cos x - \sin x = 0 \Leftrightarrow \cos\left(x + \frac{\pi}{4}\right) = 0 \Leftrightarrow x + \frac{\pi}{4} = \frac{\pi}{2} + k\pi \Leftrightarrow x = \frac{\pi}{4} + k\pi, (k \in \mathbb{Z})$$

$$\text{Với } \sin x - \cos x = 1 \Leftrightarrow \sin\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \Leftrightarrow x = \frac{\pi}{2} + k2\pi \text{ hoặc } x = \pi + k2\pi, (k \in \mathbb{Z})$$

Kết luận nghiệm của phương trình $x = \frac{\pi}{4} + k\pi, x = \frac{\pi}{2} + k2\pi, x = \pi + k2\pi, (k \in \mathbb{Z})$

$$6). \cos^3 x + \sin^3 x + 2 \sin^2 x = 1 \quad (1)$$

$$(1) \Leftrightarrow \cos^3 x + \sin^3 x = 1 - 2 \sin^2 x \Leftrightarrow (\sin x + \cos x)(1 - \sin x \cos x) = (\cos x - \sin x)(\cos x + \sin x)$$

$$\Leftrightarrow (\sin x + \cos x)[\sin x - \cos x - \sin x \cos x + 1] = 0$$

$$\Leftrightarrow (\sin x + \cos x)[(1 + \sin x) - \cos x(1 + \sin x)] = 0$$

$$\Leftrightarrow (\sin x + \cos x)(1 + \sin x)(1 - \cos x) = 0$$

$$\text{Với } \sin x + \cos x = 0 \Leftrightarrow \sin\left(x + \frac{\pi}{4}\right) = 0 \Leftrightarrow x = -\frac{\pi}{4} + k\pi, (k \in \mathbb{Z})$$

$$\text{Với } 1 + \sin x = 0 \Leftrightarrow \sin x = -1 \Leftrightarrow x = -\frac{\pi}{2} + k2\pi, (k \in \mathbb{Z})$$

$$\text{Với } 1 - \cos x = 0 \Leftrightarrow \cos x = 1 \Leftrightarrow x = k2\pi, (k \in \mathbb{Z})$$

Kết luận nghiệm của phương trình $x = -\frac{\pi}{4} + k\pi, x = -\frac{\pi}{2} + k2\pi, x = k2\pi, (k \in \mathbb{Z})$

$$7). 4\sin^3 x + 4\sin^2 x + 3\sin 2x + 6\cos x = 0 \quad (1)$$

$$\Leftrightarrow 4\sin^2 x(\sin x + 1) + 6\cos x(\sin x + 1) = 0 \Leftrightarrow (\sin x + 1)(4\sin^2 x + 6\cos x) = 0$$

$$\Leftrightarrow (\sin x + 1)[4(1 - \cos^2 x) + 6\cos x] = 0 \Leftrightarrow (\sin x + 1)(2\cos^2 x - 3\cos x - 2) = 0$$

$$\text{Với } \sin x + 1 = 0 \Leftrightarrow \sin x = -1 \Leftrightarrow x = -\frac{\pi}{2} + k\pi, (k \in \mathbb{Z})$$

$$\text{Với } 2\cos^2 x - 3\cos x - 2 = 0 \Leftrightarrow \cos x = -\frac{1}{2} \vee \cos x = 2 \text{ (loại)} \Leftrightarrow x = \pm \frac{2\pi}{3} + k2\pi, (k \in \mathbb{Z})$$

$$\text{Kết luận nghiệm của phương trình: } x = -\frac{\pi}{2} + k\pi, \quad x = \pm \frac{2\pi}{3} + k2\pi, (k \in \mathbb{Z})$$

$$8). (1 + \sin^2 x)\cos x + (1 + \cos^2 x)\sin x = 1 + \sin 2x$$

$$\Leftrightarrow \cos x + \sin^2 x \cos x + \sin x + \cos^2 x \sin x = (\sin x + \cos x)^2$$

$$\Leftrightarrow (\sin x + \cos x) + \sin x \cos x(\sin x + \cos x) - (\sin x + \cos x)^2 = 0$$

$$\Leftrightarrow (\sin x + \cos x)(1 + \sin x \cos x - \sin x - \cos x) = 0$$

$$\Leftrightarrow (\sin x + \cos x)(1 - \sin x)(1 - \cos x) = 0$$

$$\text{Với } \sin x + \cos x = 0 \Leftrightarrow \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) = 0 \Leftrightarrow x = -\frac{\pi}{4} + k\pi, (k \in \mathbb{Z})$$

$$\text{Với } 1 - \sin x = 0 \Leftrightarrow \sin x = 1 \Leftrightarrow x = \frac{\pi}{2} + k2\pi, (k \in \mathbb{Z})$$

$$\text{Với } 1 - \cos x = 0 \Leftrightarrow \cos x = 1 \Leftrightarrow x = k2\pi, (k \in \mathbb{Z})$$

$$\text{Kết luận nghiệm của phương trình } x = -\frac{\pi}{4} + k\pi, \quad x = \frac{\pi}{2} + k2\pi, \quad x = k2\pi, (k \in \mathbb{Z})$$