

Bài 3: Giải các phương trình sau:

$$\begin{aligned} \text{a) } \cos\left(4x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{4}\right) &= 0 & \text{b) } \cos^2\left(3x + \frac{\pi}{3}\right) &= \cos^2 x & \text{c) } \cos^2\left(2x - \frac{\pi}{4}\right) &= \frac{3}{4} \\ \text{d) } \cos^2\left(2x + \frac{\pi}{4}\right) &= \sin^2\left(x + \frac{\pi}{3}\right) & \text{e) } \cos\left(3x + \frac{\pi}{3}\right) + \sin\left(\frac{5\pi}{6} + 3x\right) &= 2 \end{aligned}$$

LỜI GIẢI

$$\begin{aligned} \text{a) } \cos\left(4x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{4}\right) &= 0 \\ \Leftrightarrow \cos\left(4x + \frac{\pi}{3}\right) &= -\sin\left(x - \frac{\pi}{4}\right) \Leftrightarrow \cos\left(4x + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{4} - x\right) \\ \Leftrightarrow \cos\left(4x + \frac{\pi}{3}\right) &= \cos\left(\frac{\pi}{2} - \left(\frac{\pi}{4} - x\right)\right) \Leftrightarrow \cos\left(4x + \frac{\pi}{3}\right) = \cos\left(x + \frac{\pi}{4}\right) \\ \Leftrightarrow \begin{cases} 4x + \frac{\pi}{3} = x + \frac{\pi}{4} + k2\pi \\ 4x + \frac{\pi}{3} = -x - \frac{\pi}{4} + k2\pi \end{cases} &\Leftrightarrow \begin{cases} x = -\frac{\pi}{36} + \frac{k2\pi}{3} \\ x = -\frac{7\pi}{60} + \frac{k2\pi}{5} \end{cases} \quad (k \in \mathbb{Z}). \end{aligned}$$

Kết luận nghiệm của phương trình: $x = -\frac{\pi}{36} + \frac{k2\pi}{3}, x = -\frac{7\pi}{60} + \frac{k2\pi}{3} \quad (k \in \mathbb{Z})$.

$$\begin{aligned} \text{b) } \cos^2\left(3x + \frac{\pi}{3}\right) &= \cos^2 x \\ \Leftrightarrow \frac{1 + \cos\left(6x + \frac{2\pi}{3}\right)}{2} &= \frac{1 + \cos 2x}{2} \Leftrightarrow 1 + \cos\left(6x + \frac{2\pi}{3}\right) = 1 + \cos 2x \\ \Leftrightarrow \cos\left(6x + \frac{2\pi}{3}\right) &= \cos 2x \Leftrightarrow \begin{cases} 6x + \frac{2\pi}{3} = 2x + k2\pi \\ 6x + \frac{2\pi}{3} = -2x + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = -\frac{\pi}{6} + \frac{k\pi}{2} \\ x = -\frac{\pi}{12} + \frac{k\pi}{4} \end{cases} \quad (k \in \mathbb{Z}). \end{aligned}$$

Vậy nghiệm của phương trình: $x = -\frac{\pi}{6} + \frac{k\pi}{2}, x = -\frac{\pi}{12} + \frac{k\pi}{4} \quad (k \in \mathbb{Z})$.

$$\begin{aligned} \text{c) } \cos^2\left(2x - \frac{\pi}{4}\right) &= \frac{3}{4} \Leftrightarrow \frac{1 + \cos\left(4x - \frac{\pi}{2}\right)}{2} = \frac{3}{4} \Leftrightarrow 1 + \cos\left(4x - \frac{\pi}{2}\right) = \frac{3}{2} \\ \Leftrightarrow \sin 4x &= \frac{1}{2} \Leftrightarrow \sin 4x = \sin \frac{\pi}{6} \quad (\text{vì } \sin \alpha = \cos(\alpha - \frac{\pi}{2})) \\ \Leftrightarrow \begin{cases} 4x = \frac{\pi}{6} + k2\pi \\ 4x = \pi - \frac{\pi}{6} + k2\pi \end{cases} &\Leftrightarrow \begin{cases} x = \frac{\pi}{24} + \frac{k\pi}{2} \\ x = \frac{5\pi}{24} + \frac{k\pi}{2} \end{cases} \quad (k \in \mathbb{Z}). \end{aligned}$$

Kết luận nghiệm của phương trình: $x = \frac{5\pi}{24} + \frac{k\pi}{2}, x = \frac{\pi}{24} + \frac{k\pi}{2} \quad (k \in \mathbb{Z})$.

$$\text{d) } \cos^2\left(2x + \frac{\pi}{4}\right) = \sin^2\left(x + \frac{\pi}{3}\right)$$

$$\Leftrightarrow \frac{1 + \cos\left(4x + \frac{\pi}{2}\right)}{2} = \frac{1 - \cos\left(2x + \frac{2\pi}{3}\right)}{2} \Leftrightarrow 1 + \cos\left(4x + \frac{\pi}{2}\right) = 1 - \cos\left(2x + \frac{2\pi}{3}\right)$$

$$\Leftrightarrow \cos\left(4x + \frac{\pi}{2}\right) = -\cos\left(2x + \frac{2\pi}{3}\right) \Leftrightarrow \cos\left(4x + \frac{\pi}{2}\right) = \cos\left(2x + \frac{2\pi}{3} + \pi\right)$$

$$\Leftrightarrow \begin{cases} 4x + \frac{\pi}{2} = 2x + \frac{5\pi}{3} + k2\pi \\ 4x + \frac{\pi}{2} = -2x - \frac{5\pi}{3} + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{7\pi}{12} + k\pi \\ x = -\frac{13\pi}{36} + \frac{k\pi}{3} \end{cases} \quad (k \in \mathbb{Z}).$$

Kết luận nghiệm của phương trình: $x = \frac{7\pi}{12} + k\pi, x = -\frac{13\pi}{36} + \frac{k\pi}{3} \quad (k \in \mathbb{Z})$.

e). $\cos\left(3x + \frac{\pi}{3}\right) + \sin\left(\frac{5\pi}{6} + 3x\right) = 2 \quad (*)$

Ta có: $\left(\frac{5\pi}{6} + 3x\right) - \left(3x + \frac{\pi}{3}\right) = \frac{\pi}{2} \Rightarrow \sin\left(\frac{5\pi}{6} + 3x\right) = \cos\left(3x + \frac{\pi}{3}\right)$

$$(*) \Leftrightarrow 2 \cos\left(3x + \frac{\pi}{3}\right) = 2 \Leftrightarrow \cos\left(3x + \frac{\pi}{3}\right) = 1 \Leftrightarrow 3x + \frac{\pi}{3} = k2\pi \Leftrightarrow x = -\frac{\pi}{9} + \frac{k2\pi}{3}, \quad (k \in \mathbb{Z}).$$

Kết luận nghiệm của phương trình: $x = -\frac{\pi}{9} + \frac{k2\pi}{3} \quad (k \in \mathbb{Z})$.