6). $\sin\left(\frac{\pi}{2}+2x\right)+\sqrt{3}\sin\left(\pi-2x\right)=2 \Leftrightarrow \cos 2x+\sqrt{3}\sin 2x=2 \Leftrightarrow \frac{1}{2}\cos 2x+\frac{\sqrt{3}}{2}\sin 2x=1$

$\Leftrightarrow \cos 2x.\cos\frac{\pi}{3}+\sin 2x.\sin\frac{\pi}{3}=1 \Leftrightarrow \cos\left(2x-\frac{\pi}{3}\right)=1$

$\Leftrightarrow 2x-\frac{\pi}{3}=k2\pi \Leftrightarrow x=\frac{\pi}{6}+k\pi,\left(k\in\mathbb{Z}\right)$

7). $\Leftrightarrow \cos x+\sqrt{3}\sin x+2\cos\left(2x+\frac{\pi}{3}\right)=0$

$\Leftrightarrow \frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x=-\cos\left(2x+\frac{\pi}{3}\right) \Leftrightarrow \cos x\cos\frac{\pi}{3}+\sin x\sin\frac{\pi}{3}=\cos\left(2x+\frac{\pi}{3}+\pi\right)$

$\Leftrightarrow \cos\left(x-\frac{\pi}{3}\right)=\cos\left(2x+\frac{4\pi}{3}\right) \Leftrightarrow \left[\begin{array}{l} 2x+\frac{4\pi}{3}=x-\frac{\pi}{3}+k2\pi \\ 2x+\frac{4\pi}{3}=-\left(x-\frac{\pi}{3}\right)+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\frac{5\pi}{3}+k2\pi \\ x=-\pi+\frac{k2\pi}{3} \end{array}\right.$

8). $2\cos 2x=\left(1+\sqrt{3}\right)\left(\cos x-\sin x\right)$

$\Leftrightarrow 2\left(\cos^2 x-\sin^2 x\right)=\left(1+\sqrt{3}\right)\left(\cos x-\sin x\right)$

$\Leftrightarrow 2\left(\cos x-\sin x\right)\left(\cos x+\sin x\right)=\left(1+\sqrt{3}\right)\left(\cos x-\sin x\right)$

$\Leftrightarrow \left(\cos x-\sin x\right)\left[2\left(\cos x+\sin x\right)-\left(1+\sqrt{3}\right)\right]=0$

$\Leftrightarrow \left[\begin{array}{l} \cos x-\sin x=0 \\ 2\left(\cos x+\sin x\right)-\left(1+\sqrt{3}\right)=0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \sqrt{2}\cos\left(x+\frac{\pi}{4}\right)=0 \\ \sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=\frac{1+\sqrt{3}}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{ll} \cos\left(x+\frac{\pi}{4}\right)=0 & (1) \\ \cos\left(x-\frac{\pi}{4}\right)=\frac{1+\sqrt{3}}{2\sqrt{2}} & (2) \end{array}\right.$

Giải (1): $\Leftrightarrow x+\frac{\pi}{4}=\frac{\pi}{2}+k\pi \Leftrightarrow x=\frac{\pi}{4}+k\pi$.

Giải (2): $\cos\left(x-\frac{\pi}{4}\right)=\cos\frac{\pi}{12} \Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{4}=\frac{\pi}{12}+k2\pi \\ x-\frac{\pi}{4}=-\frac{\pi}{12}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{3}+k2\pi \\ x=\frac{\pi}{6}+k2\pi \end{array}\right.$

Vậy nghiệm của phương trình: $x=\frac{\pi}{4}+k\pi$, $x=\frac{\pi}{3}+k2\pi$, $x=\frac{\pi}{6}+k2\pi,\left(k\in\mathbb{Z}\right)$

9). $\left(\sqrt{3}-1\right)\sin x-\left(\sqrt{3}+1\right)\cos x=1-\sqrt{3}$.

Ta có $a=\sqrt{3}-1, b=\sqrt{3}+1, c=1-\sqrt{3} \Rightarrow \sqrt{a^2+b^2}=2\sqrt{2}$

$\Leftrightarrow \frac{\sqrt{3}-1}{2\sqrt{2}}\sin x-\frac{\sqrt{3}+1}{2\sqrt{2}}\cos x=\frac{1-\sqrt{3}}{2\sqrt{2}} \Leftrightarrow \sin x.\cos\frac{5\pi}{12}-\cos x.\sin\frac{5\pi}{12}=\frac{1-\sqrt{3}}{2\sqrt{2}}$

$\Leftrightarrow \sin\left(x-\frac{5\pi}{12}\right)=\sin\left(-\frac{\pi}{12}\right) \Leftrightarrow \left[\begin{array}{l} x-\frac{5\pi}{12}=-\frac{\pi}{12}+k2\pi \\ x-\frac{5\pi}{12}=\pi+\frac{\pi}{12}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{3}+k2\pi \\ x=\frac{3\pi}{2}+k2\pi \end{array}\right.$

Vậy nghiệm của phương trình: $x=\frac{\pi}{3}+k2\pi, x=\frac{3\pi}{2}+k2\pi,\left(k\in\mathbb{Z}\right)$

10). $3\sin 3x - \sqrt{3}\cos 9x = 1 + 4\sin^3 3x$

$\Leftrightarrow 3\sin 3x - 4\sin^3 3x - \sqrt{3}\cos 9x = 1 \quad \Leftrightarrow \sin 9x - \sqrt{3}\cos 9x = 1$

$\Leftrightarrow \frac{1}{2}\sin 9x - \frac{\sqrt{3}}{2}\cos 9x = \frac{1}{2} \quad \Leftrightarrow \sin 9x.\cos\frac{\pi}{3} - \cos 9x.\sin\frac{\pi}{3} = \frac{1}{2}$

$$\Leftrightarrow \sin\left(9x - \frac{\pi}{3}\right) = \sin\frac{\pi}{6} \Leftrightarrow \left[\begin{array}{l} 9x - \frac{\pi}{3} = \frac{\pi}{6} + k2\pi \\ 9x - \frac{\pi}{3} = \pi - \frac{\pi}{6} + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{18} + \frac{k2\pi}{9} \\ x = \frac{7\pi}{54} + \frac{k2\pi}{9} \end{array}\right.$$

Vậy nghiệm của phương trình: $x = \frac{\pi}{18} + \frac{k2\pi}{9}, x = \frac{7\pi}{54} + \frac{k2\pi}{9}, (k \in \mathbb{Z})$