

Bước 2: Phân tích đa thức thành nhân tử, sau đó rút gọn hạng tử chung của cả tử và mẫu.

**Ví dụ 1: Tìm các giới hạn sau :**

a). $\lim_{x \rightarrow 1} \frac{\sqrt{x+3}-2}{x-1}$	b). $\lim_{x \rightarrow 7} \frac{2-\sqrt{x-3}}{x^2-49}$	c). $\lim_{x \rightarrow 3} \frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3}$
d). $\lim_{x \rightarrow 2} \frac{\sqrt{x+2}-2}{\sqrt{x+7}-3}$	e). $\lim_{x \rightarrow -1} \frac{\sqrt{x^2+x+2}-\sqrt{1-x}}{x^4+x}$	f). $\lim_{x \rightarrow 2} \frac{\sqrt{x+2}-\sqrt{2x}}{\sqrt{x-1}-\sqrt{3-x}}$
g). $\lim_{x \rightarrow 1} \frac{\sqrt{4x+5}-\sqrt{3x+6}}{\sqrt{x+3}-2}$	h). $\lim_{x \rightarrow 3} \frac{\sqrt{x+1}-\sqrt{3x-5}}{\sqrt{2x+3}-\sqrt{x+6}}$	

**LỜI GIẢI**

$$a). \lim_{x \rightarrow 1} \frac{\sqrt{x+3}-2}{x-1} = \lim_{x \rightarrow 1} \frac{x+3-2^2}{(x-1)(\sqrt{x+3}+2)} = \lim_{x \rightarrow 1} \frac{x-1}{(x-1)(\sqrt{x+3}+2)} = \lim_{x \rightarrow 1} \frac{1}{\sqrt{x+3}+2} = \frac{1}{4}.$$

$$b). \lim_{x \rightarrow 7} \frac{2-\sqrt{x-3}}{x^2-49} = \lim_{x \rightarrow 7} \frac{2^2-(x-3)}{(x^2-49)(2+\sqrt{x-3})} = \lim_{x \rightarrow 7} \frac{7-x}{(x-7)(x+7)(2+\sqrt{x-3})} = \lim_{x \rightarrow 7} \frac{-1}{(x+7)(2+\sqrt{x-3})} = -\frac{1}{56}$$

$$c). \lim_{x \rightarrow 3} \frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3} = \lim_{x \rightarrow 3} \frac{(x^2-2x+6)-(x^2+2x-6)}{(x^2-4x+3)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$

$$= \lim_{x \rightarrow 3} \frac{-4(x-3)}{(x-1)(x-3)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})} = \lim_{x \rightarrow 3} \frac{-4}{(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})} = -\frac{1}{3}.$$

$$d). \lim_{x \rightarrow 2} \frac{\sqrt{x+2}-2}{\sqrt{x+7}-3} = \lim_{x \rightarrow 2} \frac{(x+2-2^2)(\sqrt{x+7}+3)}{(x+7-3^2)(\sqrt{x+2}+2)} = \lim_{x \rightarrow 2} \frac{(x-2)(\sqrt{x+7}+3)}{(x-2)(\sqrt{x+2}+2)} = \lim_{x \rightarrow 2} \frac{\sqrt{x+7}+3}{\sqrt{x+2}+2} = \frac{3}{2}.$$

$$e). \lim_{x \rightarrow -1} \frac{\sqrt{x^2+x+2}-\sqrt{1-x}}{x^4+x} = \lim_{x \rightarrow -1} \frac{(x^2+x+2)-(1-x)}{(x^4+x)(\sqrt{x^2+x+2}+\sqrt{1-x})} = \lim_{x \rightarrow -1} \frac{x^2+2x+1}{x(x^3+1)(\sqrt{x^2+x+2}+\sqrt{1-x})}$$

$$= \lim_{x \rightarrow -1} \frac{(x+1)^2}{x(x+1)(x^2-x+1)(\sqrt{x^2+x+2}+\sqrt{1-x})}$$

$$= \lim_{x \rightarrow -1} \frac{x+1}{x(x^2-x+1)(\sqrt{x^2+x+2}+\sqrt{1-x})} = 0.$$

$$f). \lim_{x \rightarrow 2} \frac{\sqrt{x+2}-\sqrt{2x}}{\sqrt{x-1}-\sqrt{3-x}} = \lim_{x \rightarrow 2} \frac{(x+2-2x)(\sqrt{x-1}+\sqrt{3-x})}{(x-1-3+x)(\sqrt{x+2}+\sqrt{2x})}$$

$$= \lim_{x \rightarrow 2} \frac{-(x-2)(\sqrt{x-1}+\sqrt{3-x})}{2(x-2)(\sqrt{x+2}+\sqrt{2x})} = \lim_{x \rightarrow 2} \frac{-(\sqrt{x-1}+\sqrt{3-x})}{2(\sqrt{x+2}+\sqrt{2x})} = -\frac{1}{4}.$$

$$g). \lim_{x \rightarrow 1} \frac{\sqrt{4x+5}-\sqrt{3x+6}}{\sqrt{x+3}-2} = \lim_{x \rightarrow 1} \frac{(4x+5-3x-6)(\sqrt{x+3}+2)}{(x+3-4)(\sqrt{4x+5}+\sqrt{3x+6})}$$

$$= \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x+3}+2)}{(x-1)(\sqrt{4x+5}+\sqrt{3x+6})} = \lim_{x \rightarrow 1} \frac{\sqrt{x+3}+2}{\sqrt{4x+5}+\sqrt{3x+6}} = \frac{2}{3}$$

$$h). \lim_{x \rightarrow 3} \frac{\sqrt{x+1}-\sqrt{3x-5}}{\sqrt{2x+3}-\sqrt{x+6}} = \lim_{x \rightarrow 3} \frac{-2(x-3)(\sqrt{2x+3}+\sqrt{x+6})}{(x-3)(\sqrt{x+1}+\sqrt{3x-5})} = \lim_{x \rightarrow 3} \frac{-2(\sqrt{2x+3}+\sqrt{x+6})}{(\sqrt{x+1}+\sqrt{3x-5})} = -3$$

**Ví dụ 2: Tìm các giới hạn sau :**

a).  $\lim_{x \rightarrow 2} \frac{\sqrt[3]{4x-2}}{x-2}$       b).  $\lim_{x \rightarrow -1} \frac{\sqrt[3]{10+2x^3}+x-1}{x^2+3x+2}$       c).  $\lim_{x \rightarrow 3} \frac{x^3-27}{x+1-\sqrt[3]{4x^2+28}}$   
 d).  $\lim_{x \rightarrow 1} \frac{\sqrt[3]{x}-1}{\sqrt[3]{x-2}+1}$       e).  $\lim_{x \rightarrow 1} \frac{\sqrt[3]{2x-1}-\sqrt[3]{x}}{\sqrt{x}-1}$       f).  $\lim_{x \rightarrow 1} \frac{\sqrt[4]{4x-3}-1}{x-1}$ .

**LỜI GIẢI**

$$a). \text{Ta có } \sqrt[3]{4x-2} = \frac{(\sqrt[3]{4x-2}) \left[ (\sqrt[3]{4x-2})^2 + 2\sqrt[3]{4x-2} + 4 \right]}{\left[ (\sqrt[3]{4x-2})^2 + 2\sqrt[3]{4x-2} + 4 \right]} = \frac{(\sqrt[3]{4x-2})^3 - 2^3}{A} = \frac{4x-8}{A} = \frac{2(x-2)}{A}$$

$$\text{Do đó } \lim_{x \rightarrow 2} \frac{\sqrt[3]{4x-2}}{x-2} = \lim_{x \rightarrow 2} \frac{2(x-2)}{(x-2).A} = \lim_{x \rightarrow 2} \frac{2}{A} = \frac{2}{(\sqrt[3]{4.2})^2 + 2\sqrt[3]{4.2} + 4} = \frac{1}{6}$$

$$b). \text{Ta có } \sqrt[3]{10+2x^3} + (x-1)$$

$$= \frac{\left[ \sqrt[3]{10+2x^3} + (x-1) \right] \left[ (\sqrt[3]{10+2x^3})^2 - \sqrt[3]{10+2x^3} \cdot (x-1) + (x-1)^2 \right]}{\left[ (\sqrt[3]{10+2x^3})^2 - \sqrt[3]{10+2x^3} \cdot (x-1) + (x-1)^2 \right]}$$

$$= \frac{(\sqrt[3]{10+2x^3})^3 + (x-1)^3}{A} = \frac{3x^3 - 3x^2 + 3x + 9}{A} = \frac{3(x+1)(x^2 - 2x + 3)}{A}$$

$$\text{Và có } x^2 + 3x + 2 = (x+1)(x+2)$$

$$\text{Do đó } \lim_{x \rightarrow -1} \frac{\sqrt[3]{10+2x^3} + x - 1}{x^2 + 3x + 2} = \lim_{x \rightarrow -1} \frac{3(x+1)(x^2 - 2x + 3)}{(x+1)(x+2).A} = \lim_{x \rightarrow -1} \frac{3(x^2 - 2x + 3)}{(x+2).A} = \frac{3.6}{12} = \frac{3}{2}$$

$$c). \text{Ta có } x+1-\sqrt[3]{4x^2+28} = \frac{\left( (x+1) - \sqrt[3]{4x^2+28} \right) \left[ (x+1)^2 + (x+1)\sqrt[3]{4x^2+28} + (\sqrt[3]{4x^2+28})^2 \right]}{\left[ (x+1)^2 + (x+1)\sqrt[3]{4x^2+28} + (\sqrt[3]{4x^2+28})^2 \right]}$$

$$= \frac{(x+1)^3 - (\sqrt[3]{4x^2+28})^3}{A} = \frac{x^3 - x^2 + 3x - 27}{A} = \frac{(x-3)(x^2+2x+9)}{A}$$

Và  $x^3 - 27 = x^3 - 3^3 = (x-3)(x^2+3x+9)$ . Do đó  $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x+1 - \sqrt[3]{4x^2+28}} = \lim_{x \rightarrow 3} \frac{(x-3)(x^2+3x+9)}{(x-3)(x^2+2x+9)}$

$$= \lim_{x \rightarrow 3} \frac{(x^2+3x+9) \cdot A}{x^2+2x+9} = \frac{27 \cdot 48}{24} = 54.$$

d). Có  $\sqrt[3]{x}-1 = \frac{(\sqrt[3]{x}-1)[(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1]}{(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1} = \frac{(\sqrt[3]{x})^3 - 1}{A} = \frac{x-1}{A}$ , và

$$\sqrt[3]{x-2} + 1 = \frac{(\sqrt[3]{x-2} + 1)[(\sqrt[3]{x-2})^2 - \sqrt[3]{x-2} + 1]}{(\sqrt[3]{x-2})^2 - \sqrt[3]{x-2} + 1} = \frac{(\sqrt[3]{x-2})^3 + 1}{B} = \frac{x-1}{B}.$$

Từ đó  $\lim_{x \rightarrow 1} \frac{\sqrt[3]{x}-1}{\sqrt[3]{x-2}+1} = \lim_{x \rightarrow 1} \frac{\frac{x-1}{A}}{\frac{x-1}{B}} = \lim_{x \rightarrow 1} \frac{B}{A} = \frac{3}{3} = 1$ .

e). Có  $\sqrt[3]{2x-1} - \sqrt[3]{x} = \frac{(\sqrt[3]{2x-1} - \sqrt[3]{x})[(\sqrt[3]{2x-1})^2 + \sqrt[3]{2x-1}\sqrt[3]{x} + (\sqrt[3]{x})^2]}{(\sqrt[3]{2x-1})^2 + \sqrt[3]{2x-1}\sqrt[3]{x} + (\sqrt[3]{x})^2} = \frac{(\sqrt[3]{2x-1})^3 - (\sqrt[3]{x})^3}{A} = \frac{x-1}{A}$  và

$$\sqrt{x}-1 = \frac{x-1}{\sqrt{x}+1}.$$

Do đó  $\lim_{x \rightarrow 1} \frac{\sqrt[3]{2x-1} - \sqrt[3]{x}}{\sqrt{x}-1} = \lim_{x \rightarrow 1} \frac{\frac{x-1}{A}}{\frac{x-1}{\sqrt{x}+1}} = \lim_{x \rightarrow 1} \frac{\sqrt{x}+1}{A} = \frac{2}{3}$ .

f). Có  $\sqrt[4]{4x-3}-1 = \frac{(\sqrt[4]{4x-3}-1)(\sqrt[4]{4x-3}+1)}{\sqrt[4]{4x-3}+1} = \frac{\sqrt{4x-3}-1}{(\sqrt[4]{4x-3}+1)} = \frac{(\sqrt{4x-3}-1)(\sqrt{4x-3}+1)}{(\sqrt[4]{4x-3}+1)(\sqrt{4x-3}+1)} = \frac{4(x-1)}{A}$ .

Do đó  $\lim_{x \rightarrow 1} \frac{\sqrt[4]{4x-3}-1}{x-1} = \lim_{x \rightarrow 1} \frac{\frac{4(x-1)}{A}}{x-1} = \lim_{x \rightarrow 1} \frac{4}{A} = \frac{4}{4} = 1$ .