GIỚI HẠN CỦA HÀM SỐ KHI $x \to \infty$

DẠNG 1: Tính giới hạn trực tiếp

Ví dụ: Tính giới các giới hạn sau:

a). $\lim\limits_{x\to+\infty}(2x^3-3x)$ b). $\lim\limits_{x\to\pm\infty}\sqrt{x^2-3x+4}$ c). $\lim\limits_{x\to-\infty}\left(\sqrt{2x^2+1}+x\right)$

LỜI GIẢI

a). $\lim\limits_{x\to+\infty}(2x^3-3x)=\lim\limits_{x\to+\infty}x^3\left(2-\frac{3}{x^2}\right)=\lim\limits_{x\to+\infty}2x^3=+\infty$

b). $\lim\limits_{x\to\pm\infty}\sqrt{x^2-3x+4}=\lim\limits_{x\to\pm\infty}|x|\sqrt{1-\frac{3}{x}+\frac{4}{x^2}}=\left[\begin{array}{l}\lim\limits_{x\to-\infty}\left[-x\sqrt{1-\frac{3}{x}+\frac{4}{x^2}}\right]\\ \lim\limits_{x\to+\infty}x\sqrt{1-\frac{3}{x}+\frac{4}{x^2}}\end{array}\right.=\left[\begin{array}{l}\lim\limits_{x\to-\infty}(-x)\\ \lim\limits_{x\to+\infty}x\end{array}\right.=\left[\begin{array}{l}+\infty\\ +\infty\end{array}\right.$

c). $\lim\limits_{x\to-\infty}\left(\sqrt{2x^2+1}+x\right)=\lim\limits_{x\to-\infty}\left(\sqrt{x^2\left(2+\frac{1}{x^2}\right)}+x\right)=\lim\limits_{x\to-\infty}\left(|x|\sqrt{2+\frac{1}{x^2}}+x\right)$

$=\lim\limits_{x\to-\infty}\left(-x\sqrt{2+\frac{1}{x^2}}+x\right)=\lim\limits_{x\to-\infty}x\left(-\sqrt{2}+1\right)=+\infty$.

DẠNG 2: $\frac{\infty}{\infty}$

PHƯƠNG PHÁP GIẢI TOÁN: Chia cả tử và mẫu cho x^k là lũy thừa cao nhất của tử và mẩu (hoặc đặt x^k làm nhân tử chung).

Ví dụ 1: Tìm giới hạn của các hàm số sau:

a). $\lim\limits_{x\to-\infty}\frac{3x\left(2x^2-1\right)}{(5x-1)\left(x^2+2x\right)}$ b). $\lim\limits_{x\to+\infty}\sqrt[3]{\frac{2x^5+x^3-1}{\left(2x^2-1\right)\left(x^3+x\right)}}$ c). $\lim\limits_{x\to+\infty}\frac{x\sqrt{x}+1}{x^2+x+1}$

d). $L=\lim\limits_{x\to-\infty}\frac{2|x|+3}{\sqrt{x^2+x+5}}$ e). $\lim\limits_{x\to-\infty}x.\sqrt{\frac{2x^3+x}{x^5-x^2+3}}$ f). $\lim\limits_{x\to+\infty}\frac{\sqrt{2x^4+x^2-1}}{1-2x}$.

LỜI GIẢI

a). $\lim\limits_{x\to-\infty}\frac{3x\left(2x^2-1\right)}{(5x-1)\left(x^2+2x\right)}=\lim\limits_{x\to-\infty}\frac{3x.x^2\left(2-\frac{1}{x^2}\right)}{x\left(5-\frac{1}{x}\right)x^2\left(1+\frac{2}{x}\right)}=\lim\limits_{x\to-\infty}\frac{3\left(2-\frac{1}{x^2}\right)}{\left(5-\frac{1}{x}\right)\left(1+\frac{2}{x}\right)}=\frac{3.2}{5.1}=\frac{6}{5}$.

b). $\lim\limits_{x\to+\infty}\sqrt[3]{\frac{2x^5+x^3-1}{\left(2x^2-1\right)\left(x^3+x\right)}}=\lim\limits_{x\to+\infty}\sqrt[3]{\frac{x^5\left(2+\frac{1}{x^2}-\frac{1}{x^5}\right)}{x^2\left(2-\frac{1}{x^2}\right)x^3\left(1+\frac{1}{x^2}\right)}}=\lim\limits_{x\to+\infty}\sqrt[3]{\frac{\left(2+\frac{1}{x^2}-\frac{1}{x^5}\right)}{\left(2-\frac{1}{x^2}\right)\left(1+\frac{1}{x^2}\right)}}=1$

c). $\lim\limits_{x\to+\infty}\frac{x\sqrt{x}+1}{x^2+x+1}=\lim\limits_{x\to+\infty}\frac{\frac{x\sqrt{x}+1}{x^2}}{\frac{x^2+x+1}{x^2}}=\lim\limits_{x\to+\infty}\frac{\frac{1}{\sqrt{x}}+\frac{1}{x^2}}{1+\frac{1}{x}+\frac{1}{x^2}}=\frac{0}{1}=0$.

d). $L=\lim_{x\to-\infty}\frac{2|x|+3}{\sqrt{x^2+x+5}}$

vì $x\to-\infty\Rightarrow x<0\Rightarrow|x|=-x$. Vậy $L=\lim_{x\to-\infty}\frac{-2x+3}{\sqrt{x^2+x+5}}$

$$=\lim_{x\to-\infty}\frac{-2x+3}{\sqrt{x^2\left(1+\frac{1}{x}+\frac{5}{x^2}\right)}}=\lim_{x\to-\infty}\frac{-2x+3}{|x|\sqrt{1+\frac{1}{x}+\frac{5}{x^2}}}=\lim_{x\to-\infty}\frac{x\left(-2+\frac{3}{x}\right)}{-x\sqrt{1+\frac{1}{x}+\frac{5}{x^2}}}=\lim_{x\to-\infty}\frac{-2+\frac{3}{x}}{-\sqrt{1+\frac{1}{x}+\frac{5}{x^2}}}=2$$

e). $\lim_{x\to-\infty}x.\sqrt{\frac{2x^3+x}{x^5-x^2+3}}=\lim_{x\to-\infty}x.\sqrt{\frac{x^3\left(2+\frac{1}{x^2}\right)}{x^5\left(1-\frac{1}{x^3}+\frac{3}{x^5}\right)}}=\lim_{x\to-\infty}x.\sqrt{\frac{\left(2+\frac{1}{x^2}\right)}{x^2\left(1-\frac{1}{x^3}+\frac{3}{x^5}\right)}}$

$$=\lim_{x\to-\infty}\frac{x}{|x|}\sqrt{\frac{\left(2+\frac{1}{x^2}\right)}{\left(1-\frac{1}{x^3}+\frac{3}{x^5}\right)}}=\lim_{x\to-\infty}-\sqrt{\frac{\left(2+\frac{1}{x^2}\right)}{\left(1-\frac{1}{x^3}+\frac{3}{x^5}\right)}}=-\sqrt{2}$$

f). $\lim_{x\to+\infty}\frac{\sqrt{2x^4+x^2-1}}{1-2x}=\lim_{x\to+\infty}\frac{\sqrt{x^4\left(2+\frac{1}{x^2}-\frac{1}{x^4}\right)}}{1-2x}$

$$=\lim_{x\to+\infty}\frac{x^2\sqrt{2+\frac{1}{x^2}-\frac{1}{x^4}}}{1-2x}=\lim_{x\to+\infty}\frac{x\sqrt{2+\frac{1}{x^2}-\frac{1}{x^4}}}{\frac{1}{x}-2}=\lim_{x\to+\infty}\left(-\frac{\sqrt{2}}{2}x\right)=-\infty$$