Câu 7: Tìm các giới hạn sau:

a). $\lim\limits_{x\to1}\dfrac{\sqrt{3x+1}-\sqrt{x+3}}{\sqrt{x+8}-3}$ b). $\lim\limits_{x\to9}\dfrac{3-\sqrt{x}}{\sqrt{x-5}-2}$ c). $\lim\limits_{x\to-1}\dfrac{\sqrt{3+2x}-\sqrt{x+2}}{3x+3}$

d). $\lim\limits_{x\to-1}\dfrac{\sqrt{4+x+x^2}-2}{x+1}$ e). $\lim\limits_{x\to-1}\dfrac{\sqrt{7-2x}+x-2}{x^2-1}$

LỜI GIẢI

a). $\lim\limits_{x\to1}\dfrac{\sqrt{3x+1}-\sqrt{x+3}}{\sqrt{x+8}-3}=\lim\limits_{x\to1}\dfrac{(3x+1-x-3)\left(\sqrt{x+8}+3\right)}{(x+8-9)\left(\sqrt{3x+1}+\sqrt{x+3}\right)}$

$=\lim\limits_{x\to1}\dfrac{2(x-1)\left(\sqrt{x+8}+3\right)}{(x-1)\left(\sqrt{3x+1}+\sqrt{x+3}\right)}=\lim\limits_{x\to1}\dfrac{2\left(\sqrt{x+8}+3\right)}{\sqrt{3x+1}+\sqrt{x+3}}=3$

b). $\lim\limits_{x\to9}\dfrac{3-\sqrt{x}}{\sqrt{x-5}-2}=\lim\limits_{x\to9}\dfrac{(9-x)\left(\sqrt{x-5}+2\right)}{(x-5-4)\left(3+\sqrt{x}\right)}=\lim\limits_{x\to9}\dfrac{-(x-9)\left(\sqrt{x-5}+2\right)}{(x-9)\left(3+\sqrt{x}\right)}=\lim\limits_{x\to9}\dfrac{-\left(\sqrt{x-5}+2\right)}{3+\sqrt{x}}=-\dfrac{2}{3}.$

c). $\lim\limits_{x\to-1}\dfrac{\sqrt{3+2x}-\sqrt{x+2}}{3x+3}=\lim\limits_{x\to-1}\dfrac{3+2x-(x+2)}{(3x+3)\left(\sqrt{3+2x}+\sqrt{x+2}\right)}$

$=\lim\limits_{x\to-1}\dfrac{x+1}{3(x+1)\left(\sqrt{3+2x}+\sqrt{x+2}\right)}=\lim\limits_{x\to-1}\dfrac{1}{3\left(\sqrt{3+2x}+\sqrt{x+2}\right)}=\dfrac{1}{6}.$

d). $\lim\limits_{x\to-1}\dfrac{\sqrt{4+x+x^2}-2}{x+1}=\lim\limits_{x\to-1}\dfrac{4+x+x^2-4}{(x+1)\left(\sqrt{4+x+x^2}+2\right)}=\lim\limits_{x\to-1}\dfrac{x(x+1)}{(x+1)\left(\sqrt{4+x+x^2}+2\right)}$

$=\lim\limits_{x\to-1}\dfrac{x}{\sqrt{4+x+x^2}+2}=-\dfrac{1}{4}.$

e). $\lim\limits_{x\to-1}\dfrac{\sqrt{7-2x}+x-2}{x^2-1}=\lim\limits_{x\to-1}\dfrac{\left(\sqrt{7-2x}\right)^2-(x-2)^2}{(x^2-1)\left[\sqrt{7-2x}-(x-2)\right]}=\lim\limits_{x\to-1}\dfrac{7-2x-(x^2-4x+4)}{(x^2-1)\left[\sqrt{7-2x}-(x-2)\right]}$

$=\lim\limits_{x\to-1}\dfrac{-x^2+2x+3}{(x^2-1)\left[\sqrt{7-2x}-(x-2)\right]}=\lim\limits_{x\to-1}\dfrac{-(x+1)(x-3)}{(x-1)(x+1)\left(\sqrt{7-2x}-x+2\right)}=\lim\limits_{x\to-1}\dfrac{-(x-3)}{(x-1)\left(\sqrt{7-2x}-x+2\right)}=-\dfrac{1}{3}$

Câu 8: Tìm các giới hạn sau:

a). $\lim\limits_{x\to-1}\dfrac{\sqrt[3]{5x-3}+2}{x+1}$ b). $\lim\limits_{x\to0}\dfrac{1-\sqrt[3]{1-x}}{x}$ c). $\lim\limits_{x\to3}\dfrac{\sqrt[3]{x^2-1}-2}{x-3}$

d). $\lim\limits_{x\to1}\dfrac{\sqrt[3]{x+7}-2}{\sqrt{x}-1}$ e). $\lim\limits_{x\to8}\dfrac{\sqrt[3]{x}-2}{\sqrt{2x+9}-5}$ f). $\lim\limits_{x\to-1}\dfrac{x^2-1}{2x+\sqrt{3x^2+1}}$

LỜI GIẢI

a). $\lim\limits_{x\to-1}\dfrac{\sqrt[3]{5x-3}+2}{x+1}=\lim\limits_{x\to-1}\dfrac{5x-3+8}{(x+1)\left[\left(\sqrt[3]{5x-3}\right)^2-2\sqrt[3]{5x-3}+4\right]}$

$=\lim\limits_{x\to-1}\dfrac{5(x+1)}{(x+1)\left[\left(\sqrt[3]{5x-3}-2.\sqrt[3]{5x-3}+4\right)\right]}=\lim\limits_{x\to-1}\dfrac{5}{\left[\left(\sqrt[3]{5x-3}\right)^2-2\sqrt[3]{5x-3}+4\right]}=\dfrac{5}{12}$

b). $\lim\limits_{x\to 0}\dfrac{1-\sqrt[3]{1-x}}{x}=\lim\limits_{x\to 0}\dfrac{1-(1-x)}{x\left[1+\sqrt[3]{1-x}+\left(\sqrt[3]{1-x}\right)^2\right]}=\lim\limits_{x\to 0}\dfrac{1}{1+\sqrt[3]{1-x}+\left(\sqrt[3]{1-x}\right)^2}=\dfrac{1}{3}$

c). $\lim\limits_{x\to 3}\dfrac{\sqrt[3]{x^2-1}-2}{x-3}=\lim\limits_{x\to 3}\dfrac{x^2-1-8}{(x-3)\left[\left(\sqrt[3]{x^2-1}\right)^2+2\sqrt[3]{x^2-1}+4\right]}$

$=\lim\limits_{x\to 3}\dfrac{(x-3)(x+3)}{\left(\sqrt[3]{x^2-1}\right)^2+2\sqrt[3]{x^2-1}+4}=\lim\limits_{x\to 3}\dfrac{x+3}{\left(\sqrt[3]{x^2-1}\right)^2+2\sqrt[3]{x^2-1}+4}=\dfrac{1}{2}$

d). $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x+7}-2}{\sqrt{x}-1}$

Ta có : $\sqrt[3]{x+7}-2=\dfrac{x+7-8}{\left(\sqrt[3]{x+7}\right)^2+2\sqrt[3]{x+7}+4}=\dfrac{x-1}{\left(\sqrt[3]{x+7}\right)^2+2\sqrt[3]{x+7}+4}$

Và $\dfrac{1}{\sqrt{x}-1}=\dfrac{\sqrt{x}+1}{x-1}$

Vậy $\lim\limits_{x\to 1}\dfrac{x-1}{\left(\sqrt[3]{x+7}\right)^2+2\sqrt[3]{x+7}+4}.\dfrac{\sqrt{x}+1}{x-1}=\lim\limits_{x\to 1}\dfrac{\sqrt{x}+1}{\left(\sqrt[3]{x+7}\right)^2+2\sqrt[3]{x+7}+4}=\dfrac{1}{6}$

e). $\lim\limits_{x\to 8}\dfrac{\sqrt[3]{x}-2}{\sqrt{2x+9}-5}=\lim\limits_{x\to 8}\left(\dfrac{x-8}{\left(\sqrt[3]{x}\right)^2+2\sqrt[3]{x}+4}.\dfrac{\sqrt{2x+9}+5}{2x+9-25}\right)$

$=\lim\limits_{x\to 8}\left(\dfrac{x-8}{\left(\sqrt[3]{x}\right)^2+2\sqrt[3]{x}+4}.\dfrac{\sqrt{2x+9}+5}{2(x-8)}\right)=\lim\limits_{x\to 8}\dfrac{\sqrt{2x+9}+5}{2\left[\left(\sqrt[3]{x}\right)^2+2\sqrt[3]{x}+4\right]}=\dfrac{10}{24}$

f). $\lim\limits_{x\to -1}\dfrac{x^2-1}{2x+\sqrt{3x^2+1}}$

$\lim\limits_{x\to -1}\dfrac{\left(x^2-1\right)\left(2x-\sqrt{3x^2+1}\right)}{4x^2-\left(3x^2+1\right)}=\lim\limits_{x\to -1}\dfrac{\left(x^2-1\right)\left(2x-\sqrt{3x^2+1}\right)}{x^2-1}=\lim\limits_{x\to -1}\left(2x-\sqrt{3x^2+1}\right)=-4$

Câu 9: Tìm các giới hạn sau:

a). $\lim\limits_{x\to 0}\dfrac{\sqrt{x+9}+\sqrt{x+16}-7}{x}$ b). $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x^3+7}-\sqrt{x^2+3}}{x-1}$ c). $\lim\limits_{x\to a}\dfrac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}}$, $(a>0)$ d). $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{8x+11}-\sqrt{x+7}}{x^2-3x+2}$ e). $\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}-\sqrt[3]{1+3x}}{x^2}$

LỜI GIẢI

a). $\lim\limits_{x\to 0}\dfrac{\sqrt{x+9}+\sqrt{x+16}-7}{x}=\lim\limits_{x\to 0}\dfrac{\sqrt{x+9}-3+\sqrt{x+16}-7}{x}$

$=\lim\limits_{x\to 0}\dfrac{\sqrt{x+9}-3}{x}+\lim\limits_{x\to 0}\dfrac{\sqrt{x+16}-4}{x}=\lim\limits_{x\to 0}\dfrac{x+9-9}{\left(\sqrt{x+9}+3\right)x}+\lim\limits_{x\to 0}\dfrac{x+16-16}{\left(\sqrt{x+16}+4\right)x}$

$$=\lim_{x\to 0}\frac{x}{\left(\sqrt{x+9}+3\right)x}+\lim_{x\to 0}\frac{x}{\left(\sqrt{x+16}+4\right)x}=\lim_{x\to 0}\frac{1}{\sqrt{x+9}+3}+\lim_{x\to 0}\frac{1}{\sqrt{x+16}+4}=\frac{7}{24}$$

b). $\lim_{x\to 1}\frac{\sqrt[3]{x^3+7}-\sqrt{x^2+3}}{x-1}=\lim_{x\to 1}\frac{\sqrt[3]{x^3+7}-2+2-\sqrt{x^2+3}}{x-1}$

$$=\lim_{x\to 1}\frac{\sqrt[3]{x^3+7}-2}{x-1}+\lim_{x\to 1}\frac{2-\sqrt{x^2+3}}{x-1}$$

$$=\lim_{x\to 1}\frac{x^3+7-8}{\left[\left(\sqrt[3]{x^3+7}\right)^2+2\sqrt[3]{x^3+7}+4\right](x-1)}+\lim_{x\to 1}\frac{2-x^2-3}{(2+\sqrt{x^2+3})(x-1)}$$

$$=\lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{\left[\left(\sqrt[3]{x^3+7}\right)^2+2\sqrt[3]{x^3+7}+4\right](x-1)}+\lim_{x\to 1}\frac{-(x-1)(x+1)}{(2+\sqrt{x^2+3})(x-1)}$$

$$=\lim_{x\to 1}\frac{x^2+x+4}{\left(\sqrt[3]{x^3+7}\right)^2+2\sqrt[3]{x^3+7}+4}+\lim_{x\to 1}\frac{x+1}{2+\sqrt{x^2+3}}=\frac{3}{4}$$

c). $\lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}}\quad (a>0)$

$$=\lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}}{\sqrt{x^2-a^2}}+\lim_{x\to a}\frac{\sqrt{x-a}}{\sqrt{x^2-a^2}}=\lim_{x\to a}\frac{x-a}{\sqrt{(x-a)(x+a)}\left(\sqrt{x}+\sqrt{a}\right)}+\lim_{x\to a}\frac{\sqrt{x-a}}{\sqrt{(x-a)(x+a)}}$$

$$=\lim_{x\to a}\frac{\left(\sqrt{x-a}\right)^2}{\sqrt{x-a}.\sqrt{x+a}.\left(\sqrt{x}+\sqrt{a}\right)}+\lim_{x\to a}\frac{1}{\sqrt{x+a}}=\lim_{x\to a}\frac{\sqrt{x-a}}{\sqrt{x+a}\left(\sqrt{x}+\sqrt{a}\right)}+\lim_{x\to a}\frac{1}{\sqrt{x+a}}=\frac{1}{\sqrt{2a}}$$

d). $\lim_{x\to 2}\frac{\sqrt[3]{8x+11}-\sqrt{x+7}}{x^2-3x+2}=\lim_{x\to 2}\frac{\sqrt[3]{8x+11}-3+3-\sqrt{x+7}}{x^2-3x+2}$

$$=\lim_{x\to 2}\frac{\sqrt[3]{8x+11}-3}{x^2-3x+2}+\lim_{x\to 2}\frac{3-\sqrt{x+7}}{x^2-3x+2}$$

$$=\lim_{x\to 2}\frac{8x+11-27}{\left[\left(\sqrt[3]{8x+11}\right)^2+3\sqrt[3]{8x+11}+9\right](x^2-3x+2)}+\lim_{x\to 2}\frac{9-x-7}{\left(3+\sqrt{x+7}\right)(x^2-3x+2)}$$

$$=\lim_{x\to 2}\frac{8(x-2)}{\left[\left(\sqrt[3]{8x+11}\right)^2+3\sqrt[3]{8x+11}+9\right](x-2)(x-1)}+\lim_{x\to 2}\frac{-(x-2)}{(3+\sqrt{x+7})(x-2)(x-1)}$$

$$=\lim_{x\to 2}\frac{8}{\left[\left(\sqrt[3]{8x+11}\right)^2+3\sqrt[3]{8x+11}+9\right](x-1)}+\lim_{x\to 2}\frac{-1}{\left(3+\sqrt{x+7}\right)(x-1)}=\frac{7}{54}$$

e). $L=\lim_{x\to 0}\frac{\sqrt{1+2x}-\sqrt[3]{1+3x}}{x^2}$

$$=\lim_{x\to 0}\left[\frac{1}{x}\left(\frac{\sqrt{1+2x}-1}{x}-\frac{\sqrt[3]{1+3x}-1}{x}\right)\right]=\lim_{x\to 0}\left[\frac{1}{x}\left(\frac{2}{\sqrt{1+2x}+1}-\frac{3}{\left(\sqrt[3]{1+3x}\right)^2+\sqrt[3]{1+3x}+1}\right)\right]$$

$$=\lim_{x\to 0}\left[\frac{1}{x}\cdot\frac{2\left(\sqrt[3]{1+3x}\right)^2+2\sqrt[3]{1+3x}+2-3\sqrt{1+2x}-3}{\underbrace{\left(\sqrt{1+2x}+1\right)\left(\left(\sqrt[3]{1+3x}\right)^2+\sqrt[3]{1+3x}+1\right)}_{A}}\right]$$

$$=\lim_{x\to 0}\left[\frac{1}{A}\left(2\frac{\left(\sqrt[3]{1+3x}\right)^2-1}{x}+2\frac{\sqrt[3]{1+3x}-1}{x}-3\frac{\sqrt{1+2x}-1}{x}\right)\right]$$

$$=\lim_{x\to 0}\left[\frac{1}{A}\left(2\left(\sqrt[3]{1+3x}+1\right)\frac{\sqrt[3]{1+3x}-1}{x}+2\frac{\sqrt[3]{1+3x}-1}{x}-3\frac{\sqrt{1+2x}-1}{x}\right)\right]$$

$$=\lim_{x\to 0}\left[\frac{1}{A}\left(\frac{6\left(\sqrt[3]{1+3x}+1\right)}{\left(\sqrt[3]{1+3x}\right)^2+\sqrt[3]{1+3x}+1}+\frac{6}{\left(\sqrt[3]{1+3x}\right)^2+\sqrt[3]{1+3x}+1}-\frac{6}{\sqrt{1+2x}+1}\right)\right]=\frac{1}{6}(4+2-3)=\frac{1}{2}.$$

CÁCH 2:

$$L=\lim_{x\to 0}\frac{\sqrt{1+2x}-(1+x)+(1+x)-\sqrt[3]{1+3x}}{x^2}=\lim_{x\to 0}\frac{\sqrt{1+2x}-(1+x)}{x^2}+\lim_{x\to 0}\frac{(1+x)-\sqrt[3]{1+3x}}{x^2}$$

$$=\lim_{x\to 0}\frac{-x^2}{x^2\left[\sqrt{1+2x}+(1+x)\right]}+\lim_{x\to 0}\frac{3x^3+3x^2}{x^2\left[(1+x)^2+(1+x)\sqrt[3]{1+3x}+\left(\sqrt[3]{1+3x}\right)^2\right]}$$

$$=\lim_{x\to 0}\frac{-1}{\sqrt{1+2x}+(1+x)}+\lim_{x\to 0}\frac{3(x+1)}{(1+x)^2+(1+x)\sqrt[3]{1+3x}+\left(\sqrt[3]{1+3x}\right)^2}=-\frac{1}{2}+1=\frac{1}{2}$$

Câu 10: Tính các giới hạn sau:

a). $\lim\limits_{x\to 1}\frac{x+x^2+...+x^n-n}{x-1}$ b). $\lim\limits_{x\to 1}\frac{x^n-1}{x^m-1}$ c). $\lim\limits_{x\to 1}\frac{x^n-nx+n-1}{(x-1)^2}$

LỜI GIẢI

a). $\lim\limits_{x\to 1}\frac{x+x^2+...+x^n-n}{x-1}=\lim\limits_{x\to 1}\frac{(x-1)+(x^2-1)+...+x^n-1}{x-1}$

$$=\lim_{x\to 1}\frac{(x-1)+(x-1)(x+1)+...+(x-1)(x^{n-1}+x^{n-x}+...+1)}{x-1}$$

$$=\lim_{x\to 1}\frac{(x-1)\left[1+(x+1)+...+(x^{n-1}+x^{n-x}+...+1)\right]}{x-1}=1+2+3+...+n=\frac{n(n+1)}{2}$$

b). $\lim\limits_{x\to 1}\frac{x^n-1}{x^m-1}=\lim\limits_{x\to 1}\frac{(x-1)(x^{n-1}+x^{n-2}+x^{n-3}+...+1)}{(x-1)(x^{m-1}+x^{m-2}+x^{m-3}+...+1)}=\frac{1+1+1+...+1}{1+1+1+...+1}=\frac{n}{m}$

c). $\lim\limits_{x\to 1}\frac{x^n-nx+n-1}{(x-1)^2}=\lim\limits_{x\to 1}\frac{(x^n-1)-n(x-1)}{(x-1)^2}$

$$=\lim_{x\to 1}\frac{(x-1)(x^{n-1}+x^{n-2}+x^{n-3}+...+1)-n(x-1)}{(x-1)^2}$$

$$=\lim_{x\to 1}\frac{(x-1)(x^{n-1}+x^{n-2}+x^{n-3}+...+1-n)}{(x-1)^2}=\lim_{x\to 1}\frac{x^{n-1}+x^{n-2}+x^{n-3}+...+1-n}{x-1}$$

$$=\lim_{x\to 1}\frac{(x^{n-1}-1)+(x^{n-2}-1)+(x^{n-3}-1)+...+(x-1)+(1-1)}{x-1}$$

$$=\lim_{x\to 1}\frac{(x-1)(x^{n-2}+x^{n-3}+...+1)+(x-1)(x^{n-3}+x^{n-4}+...+1)+...+(x-1)}{x-1}$$

$$=\lim_{x\to 1}\frac{(x-1)\left[(x^{n-2}+x^{n-3}+...+1)+(x^{n-3}+x^{n-4}+...+1)+...+1\right]}{x-1}$$

$$=(n-1)+(n-2)+...+1=\frac{(n-1).n}{2}.$$