Câu 3: Tìm các giới hạn sau:

a). $\lim\limits_{x\to-\infty}\dfrac{(x-1)^2(5x+2)^2}{(3x+1)^4}$ b). $\lim\limits_{x\to-\infty}\dfrac{2|x|+3}{\sqrt{x^2+x+5}}$ c). $\lim\limits_{x\to-\infty}\dfrac{2|x|+3}{\sqrt{x^2+x+5}}$

d). $\lim\limits_{x\to+\infty}\sqrt{\dfrac{2x^5+x^3-1}{(2x^2-1)(x^3+x)}}$ e). $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+x}+2x}{2x+3}$ f). $\lim\limits_{x\to-\infty}\dfrac{|x|-\sqrt{x^2+x}}{x+10}$

LỜI GIẢI

a). $$\lim\limits_{x\to-\infty}\frac{(x-1)^2(5x+2)^2}{(3x+1)^4}=\lim\limits_{x\to-\infty}\frac{x^2\left(1-\frac{1}{x}\right)^2x^2\left(5+\frac{2}{x}\right)^2}{x^4\left(3+\frac{1}{x}\right)^4}=\lim\limits_{x\to-\infty}\frac{\left(1-\frac{1}{x}\right)^2\left(5+\frac{2}{x}\right)^2}{\left(3+\frac{1}{x}\right)^4}=\frac{25}{81}$$

b). $$\lim\limits_{x\to+\infty}\frac{\sqrt{x^6+2}}{3x^3-1}=\lim\limits_{x\to+\infty}\frac{\sqrt{x^6\left(1+\frac{2}{x^6}\right)}}{x^3\left(3-\frac{1}{x^3}\right)}=\lim\limits_{x\to+\infty}\frac{x^3\sqrt{1+\frac{2}{x^3}}}{x^3\left(3-\frac{1}{x^3}\right)}=\lim\limits_{x\to+\infty}\frac{\sqrt{\left(1+\frac{2}{x^3}\right)}}{3-\frac{1}{x^3}}=\frac{1}{3}$$

c). $$\lim\limits_{x\to-\infty}\frac{2|x|+3}{\sqrt{x^2+x+5}}=\lim\limits_{x\to-\infty}\frac{-2x+3}{\sqrt{x^2\left(1+\frac{1}{x}+\frac{5}{x^2}\right)}}=\lim\limits_{x\to-\infty}\frac{x\left(-2+\frac{3}{x}\right)}{x\sqrt{1+\frac{1}{x}+\frac{5}{x^2}}}=\lim\limits_{x\to-\infty}\frac{-2+\frac{3}{x}}{1+\frac{1}{x}+\frac{5}{x^2}}=-2$$

d). $$\lim\limits_{x\to+\infty}\sqrt{\frac{2x^5+x^3-1}{(2x^2-1)(x^3+x)}}=\lim\limits_{x\to+\infty}\frac{x^5\left(2+\frac{1}{x^2}-\frac{1}{x^5}\right)}{x^2\left(2-\frac{1}{x^2}\right)x^3\left(1+\frac{1}{x^2}\right)}=\lim\limits_{x\to+\infty}\frac{2+\frac{1}{x^2}-\frac{1}{x^3}}{\left(2-\frac{1}{x^2}\right)\left(1+\frac{1}{x^2}\right)}=1$$

e). $$\lim\limits_{x\to-\infty}\frac{\sqrt{x^2+x}+2x}{2x+3}=\lim\limits_{x\to-\infty}\frac{\sqrt{x^2\left(1+\frac{1}{x}\right)}+2x}{x\left(2+\frac{3}{x}\right)}=\lim\limits_{x\to-\infty}\frac{x\left(\sqrt{1+\frac{1}{x}}+2\right)}{x\left(2+\frac{3}{x}\right)}=\lim\limits_{x\to-\infty}\frac{\sqrt{1+\frac{1}{x}}+2}{2+\frac{3}{x}}=1$$

f). $$\lim\limits_{x\to-\infty}\frac{|x|-\sqrt{x^2+x}}{x+10}=\lim\limits_{x\to-\infty}\frac{-x-\sqrt{x^2\left(1+\frac{1}{x}\right)}}{x\left(1+\frac{10}{x}\right)}=\lim\limits_{x\to-\infty}\frac{-x\left(1+\sqrt{1+\frac{1}{x}}\right)}{x\left(1+\frac{10}{x}\right)}=\lim\limits_{x\to-\infty}\frac{1+\sqrt{1+\frac{1}{x}}}{1+\frac{10}{x}}=1$$

Câu 4: Tìm các giới hạn sau:

a). $\lim\limits_{x\to-\infty}x\sqrt{\dfrac{2x^3+x}{x^5-x^2+3}}$ b). $\lim\limits_{x\to+\infty}\left[(x+2)\sqrt{\dfrac{x-1}{x^3+x}}\right]$ c). $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+2x+3}+4x+1}{\sqrt{4x^2+1}-x+2}$

d). $\lim\limits_{x\to+\infty}\left(\sqrt{x^2-4x}-x\right)$ e). $\lim\limits_{x\to-\infty}\left(\sqrt[3]{8x^3+1}-2x+1\right)$

LỜI GIẢI

a). $\lim\limits_{x\to-\infty} x\sqrt{\dfrac{2x^3+x}{x^5-x^2+3}}=\lim\limits_{x\to-\infty} x\sqrt{\dfrac{x^3\left(2+\dfrac{1}{x^2}\right)}{x^5\left(1-\dfrac{1}{x^3}+\dfrac{3}{x^5}\right)}}=\lim\limits_{x\to-\infty} x\dfrac{\sqrt{2+\dfrac{1}{x^2}}}{\sqrt{x^2}\sqrt{1-\dfrac{1}{x^3}+\dfrac{3}{x^5}}}$

$=\lim\limits_{x\to-\infty}\dfrac{\sqrt{2+\dfrac{1}{x^2}}}{\sqrt{1-\dfrac{1}{x^3}+\dfrac{3}{x^5}}}=\sqrt{2}$

b). $\lim\limits_{x\to+\infty}\left[(x+2)\sqrt{\dfrac{x-1}{x^3+x}}\right]=\lim\limits_{x\to+\infty}\left[x\left(1+\dfrac{2}{x}\right)\sqrt{\dfrac{x\left(1-\dfrac{1}{x}\right)}{x^3\left(1+\dfrac{1}{x^2}\right)}}\right]=\lim\limits_{x\to+\infty}\left[x\left(1+\dfrac{2}{x}\right)\dfrac{\sqrt{1-\dfrac{1}{x}}}{x\sqrt{1+\dfrac{1}{x^2}}}\right]$

$=\lim\limits_{x\to+\infty}\left[\left(1+\dfrac{2}{x}\right)\dfrac{\sqrt{1-\dfrac{1}{x}}}{\sqrt{1+\dfrac{1}{x^2}}}\right]=1.$

c). $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+2x+3}+4x+1}{\sqrt{4x^2+1}-x+2}=\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2\left(1+\dfrac{2}{x}+\dfrac{3}{x^2}\right)}+4x+1}{\sqrt{x^2\left(4+\dfrac{1}{x^2}\right)}-x+2}=\lim\limits_{x\to-\infty}\dfrac{|x|\sqrt{\left(1+\dfrac{2}{x}+\dfrac{3}{x^2}\right)}+4x+1}{|x|\sqrt{4+\dfrac{1}{x^2}}-x+2}$

$=\lim\limits_{x\to-\infty}\dfrac{-x\sqrt{1+\dfrac{2}{x}+\dfrac{3}{x^2}}+4x+1}{-x\sqrt{4+\dfrac{1}{x^2}}-x+2}=\lim\limits_{x\to-\infty}\dfrac{-x+4x+1}{-4x-x+2}=\lim\limits_{x\to-\infty}\dfrac{-x\left(1-4-\dfrac{1}{x}\right)}{-x\left(4+1-\dfrac{2}{x}\right)}=\lim\limits_{x\to-\infty}\dfrac{-3-\dfrac{1}{x}}{5-\dfrac{2}{x}}=\dfrac{-3}{5}$

d). $\lim\limits_{x\to+\infty}\left(\sqrt{x^2-4x}-x\right)=\lim\limits_{x\to+\infty}\dfrac{x^2-4x-x}{\sqrt{x^2-4x}+x}=\lim\limits_{x\to+\infty}\dfrac{-4x}{\sqrt{x^2\left(1-\dfrac{1}{x}\right)}+x}=\lim\limits_{x\to+\infty}\dfrac{-4x}{x+x}=\lim\limits_{x\to+\infty}\dfrac{-4x}{2x}=-2$

e). $\lim\limits_{x\to-\infty}\left(\sqrt[3]{8x^3+1}-2x+1\right)=\lim\limits_{x\to-\infty}\dfrac{8x^3+1-8x^3}{\left(\sqrt[3]{8x^3+1}\right)^2+\sqrt[3]{8x^3+1}.2x+4x^2}+1$

$=\lim\limits_{x\to-\infty}\dfrac{1}{\left(\sqrt[3]{x^3\left(8+\dfrac{1}{x^3}\right)}\right)^2+\sqrt[3]{x^3\left(8+\dfrac{1}{x^3}\right)}.2x+4x^2}+1$

$=\lim\limits_{x\to-\infty}\dfrac{1}{4\left(\sqrt[3]{x^3}\right)^2+2\sqrt[3]{x^3}.2x+4x^2}+1=\lim\limits_{x\to-\infty}\dfrac{1}{4x^2+4x^2+4x^2}+1=\lim\limits_{x\to-\infty}\dfrac{1}{12x^2}+1=1$