Câu 3: Tìm các giới hạn sau:

1). $\lim\limits_{x\to 0}\dfrac{\sin ax+\tan bx}{(a+b)x}$ $(a+b\neq 0)$

2). $\lim\limits_{x\to 0}\dfrac{\cos 3x-\cos 5x.\cos 7x}{x^2}$

3). $\lim\limits_{x\to 0}\dfrac{\cos ax-\cos bx.\cos cx}{x^2}$

4). $\lim\limits_{x\to 0}\dfrac{\sin(a+x)-\sin(a-x)}{\tan(a+x)-\tan(a-x)}$

5). $\lim\limits_{x\to 0}\dfrac{\sqrt{2x+1}-\sqrt[3]{x^2+1}}{\sin x}$

6). $\lim\limits_{x\to 0}\dfrac{\sin^2 2x-\sin x.\sin 4x}{x^4}$

7). $\lim\limits_{x\to 0}\dfrac{1-\cos 5x.\cos 7x}{\sin^2 11x}$

8). $\lim\limits_{x\to 0}\left(\dfrac{1}{\sin x}-\dfrac{1}{\tan x}\right)$

9). $\lim\limits_{x\to 0}\dfrac{\sin x-\sin 2x}{x\left(1-2\sin^2\dfrac{x}{2}\right)}$

10). $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x^2}-\cos x}{x^2}$

LỜI GIẢI

1). $\lim\limits_{x\to 0}\dfrac{\sin ax+\tan bx}{(a+b)x}=\lim\limits_{x\to 0}\dfrac{\sin ax+\dfrac{\sin bx}{\cos bx}}{(a+b)x}=\lim\limits_{x\to 0}\dfrac{\sin ax}{(a+b)x}+\lim\limits_{x\to 0}\dfrac{\sin bx}{(a+b)x.\cos bx}$

$=\lim\limits_{x\to 0}\dfrac{a}{a+b}.\dfrac{\sin ax}{ax}+\lim\limits_{x\to 0}\dfrac{b}{(a+b)\cos bx}.\dfrac{\sin bx}{bx}=\dfrac{a}{a+b}+\dfrac{b}{a+b}=1$

2). $\lim\limits_{x\to 0}\dfrac{\cos 3x-\cos 5x.\cos 7x}{x^2}=\lim\limits_{x\to 0}\dfrac{\cos 3x-1+(1-\cos 5x)\cos 7x+1-\cos 7x}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{\cos 3x-1}{x^2}\lim\limits_{x\to 0}\dfrac{(1-\cos 5x)\cos 7x}{x^2}+\lim\limits_{x\to 0}\dfrac{1-\cos 7x}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{-2\sin^2\dfrac{3x}{2}}{x^2}+\lim\limits_{x\to 0}\dfrac{2\sin^2\dfrac{5x}{2}\cos 7x}{x^2}+\lim\limits_{x\to 0}\dfrac{2\sin^2\dfrac{7x}{2}}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{-9}{2}.\left(\dfrac{\sin\dfrac{3x}{2}}{\dfrac{3x}{2}}\right)^2+\lim\limits_{x\to 0}\dfrac{25\cos 7x}{2}\left(\dfrac{\sin\dfrac{5x}{2}}{\dfrac{5x}{2}}\right)^2+\lim\limits_{x\to 0}\dfrac{49}{2}\left(\dfrac{\sin\dfrac{7x}{2}}{\dfrac{7x}{2}}\right)^2=-\dfrac{9}{2}+\dfrac{25}{2}+\dfrac{49}{2}=\dfrac{65}{2}$

3). $\lim\limits_{x\to 0}\dfrac{\cos ax-\cos bx.\cos cx}{x^2}=\lim\limits_{x\to 0}\dfrac{\cos ax-1-(\cos bx-1)\cos cx+1-\cos cx}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{-2\sin^2\dfrac{ax}{2}}{x^2}+\lim\limits_{x\to 0}\dfrac{2\sin^2\dfrac{bx}{2}\cos cx}{x^2}+\lim\limits_{x\to 0}\dfrac{2\sin^2\dfrac{cx}{2}}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{-a^2}{2}\left(\dfrac{\sin\dfrac{ax}{2}}{\dfrac{ax}{2}}\right)^2+\lim\limits_{x\to 0}\dfrac{b^2\cos cx}{2}.\left(\dfrac{\sin\dfrac{bx}{2}}{\dfrac{bx}{2}}\right)^2+\lim\limits_{x\to 0}\dfrac{c^2}{2}.\left(\dfrac{\sin\dfrac{cx}{2}}{\dfrac{cx}{2}}\right)^2=\dfrac{-a^2+b^2+c^2}{2}$

4). $\lim\limits_{x\to 0}\dfrac{\sin(a+x)-\sin(a-x)}{\tan(a+x)-\tan(a-x)}=\lim\limits_{x\to 0}\dfrac{2\cos a\sin x}{\dfrac{\sin 2x}{\cos(a+x)\cos(a-x)}}$

$$= \lim_{x\to 0} \frac{\cos a \cos(a+x)\cos(a-x)}{\cos x} = \cos^3 a$$

5). $\lim\limits_{x\to 0} \dfrac{\sqrt{2x+1} - \sqrt[3]{x^2+1}}{\sin x}$

$$\lim_{x\to 0} \frac{\sqrt{2x+1} - 1 + 1 - \sqrt[3]{x^2+1}}{\sin x} = \lim_{x\to 0} \frac{\sqrt{2x+1}-1}{\sin x} + \lim_{x\to 0} \frac{1-\sqrt[3]{x^2+1}}{\sin x}$$

$$= \lim_{x\to 0} \frac{2x}{\sin x\left(\sqrt{2x+1}+1\right)} + \lim_{x\to 0} \frac{-x^2}{\sin x\left[1+\sqrt[3]{x^2+1}+\left(\sqrt[3]{x^2+1}\right)^2\right]}$$

$$= \lim_{x\to 0} \frac{x}{\sin x} \cdot \frac{2}{\sqrt{2x+1}+1} + \lim_{x\to 0} \frac{x}{\sin x} \cdot \frac{-x}{1+\sqrt[3]{x^2+1}+\left(\sqrt[3]{x^2+1}\right)^2} = \frac{2}{1+1} + 0 = 1$$

6). $\lim\limits_{x\to 0} \dfrac{\sin^2 2x - \sin x.\sin 4x}{x^4} = \lim\limits_{x\to 0} \dfrac{\sin^2 2x - 2\sin x \sin 2x \cos 2x}{x^4}$

$$= \lim_{x\to 0} \frac{\sin 2x\left(2\sin x\cos x - 2\sin x\cos 2x\right)}{x^4}$$

$$= \lim_{x\to 0} \frac{2\sin 2x.\sin x\left(\cos x - \cos 2x\right)}{x^4} = \lim_{x\to 0} \frac{4\sin 2x.\sin x.\sin\dfrac{3x}{2}.\sin\dfrac{x}{2}}{x^4}$$

$$= \lim_{x\to 0} 6 \cdot \left(\frac{\sin 2x}{2x}\right) \cdot \left(\frac{\sin x}{x}\right) \cdot \left(\frac{\sin\dfrac{3x}{2}}{\dfrac{3x}{2}}\right) \cdot \left(\frac{\sin\dfrac{x}{2}}{\dfrac{x}{2}}\right) = 6$$

7). $\lim\limits_{x\to 0} \dfrac{1-\cos 5x.\cos 7x}{\sin^2 11x}$

$$= \lim_{x\to 0} \frac{\left(1-\cos 5x\right)\cos 7x + 1 - \cos 7x}{\sin^2 11x} = \lim_{x\to 0} \frac{2\sin^2\dfrac{5x}{2}\cos 7x}{\sin^2 11x} + \lim_{x\to 0} \frac{2\sin^2\dfrac{7x}{2}}{\sin^2 11x}$$

$$= \lim_{x\to 0} \frac{\left(\dfrac{\sin\dfrac{5x}{2}}{\dfrac{5x}{2}}\right)^2 \cos 7x}{\left(\dfrac{\sin 11x}{11x}\right)^2} \cdot \frac{25}{484} + \lim_{x\to 0} \frac{\left(\dfrac{\sin\dfrac{7x}{2}}{\dfrac{7x}{2}}\right)^2}{\left(\dfrac{\sin 11x}{11x}\right)^2} \cdot \frac{49}{484} = \frac{25}{484} + \frac{49}{484} = \frac{37}{242}$$

8). $\lim\limits_{x\to 0}\left(\dfrac{1}{\sin x} - \dfrac{1}{\tan x}\right) = \lim\limits_{x\to 0}\left(\dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x}\right) = \lim\limits_{x\to 0} \dfrac{1-\cos x}{\sin x} = \lim\limits_{x\to 0} \dfrac{2\sin^2\dfrac{x}{2}}{2\sin\dfrac{x}{2}\cos\dfrac{x}{2}}$

$$= \lim_{x\to 0} \tan\frac{x}{2} = 0 .$$

9). $\lim\limits_{x\to 0} \dfrac{\sin x - \sin 2x}{x\left(1-2\sin^2\dfrac{x}{2}\right)} = \lim\limits_{x\to 0} \dfrac{2\cos\dfrac{3x}{2}\sin\dfrac{-x}{2}}{x\cos x} = \lim\limits_{x\to 0} \dfrac{\sin\dfrac{x}{2}}{\dfrac{x}{2}} \cdot \dfrac{-\cos\dfrac{3x}{2}}{\cos x} = -1$

10). $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x^2}-\cos x}{x^2}=\lim\limits_{x\to 0}\dfrac{\sqrt{1+x^2}-1+1-\cos x}{x^2}=\lim\limits_{x\to 0}\dfrac{\sqrt{1+x^2}-1}{x^2}+\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{x^2}{x^2\left(\sqrt{1+x^2}+1\right)}+\lim\limits_{x\to 0}\dfrac{2\sin^2\dfrac{x}{2}}{x^2}=\lim\limits_{x\to 0}\dfrac{1}{\sqrt{1+x^2}+1}+\lim\limits_{x\to 0}\dfrac{1}{2}\cdot\left(\dfrac{\sin\dfrac{x}{2}}{\dfrac{x}{2}}\right)^2=\dfrac{1}{2}+\dfrac{1}{2}=1.$