

$$3). \begin{cases} u_2 + u_4 + u_6 = -42 \\ u_3 + u_5 = 20 \end{cases} \Leftrightarrow \begin{cases} u_1 \cdot q + u_1 \cdot q^3 + u_1 \cdot q^5 = -42. \\ u_1 \cdot q^2 + u_1 \cdot q^4 = 20 \end{cases} \Leftrightarrow \begin{cases} u_1 \cdot q(1 + q^2 + q^4) = -42 & (1) \\ u_1 \cdot q(1 + q^2) = 20 & (2) \end{cases}$$

$$\text{Lấy: } \frac{(1)}{(2)} \Leftrightarrow \frac{1 + q^2 + q^4}{q(1 + q^2)} = -\frac{21}{10} \Leftrightarrow 10 + 10q^2 + 10q^4 = -21q - 21q^3$$

$$\Leftrightarrow 10q^4 + 21q^3 + 10q^2 + 21q + 10 = 0 \Leftrightarrow 10q^2 + 21q + 10 + \frac{21}{q} + \frac{10}{q^2} = 10$$

$$\Leftrightarrow 10\left(q^2 + \frac{1}{q^2}\right) + 21\left(q + \frac{1}{q}\right) + 10 = 0 \quad (*)$$

$$\text{Đặt: } t = q + \frac{1}{q} \Rightarrow t^2 = \left(q + \frac{1}{q}\right)^2 \Leftrightarrow q^2 + \frac{1}{q^2} = t^2 - 2. \text{ Điều kiện } |t| \geq 2$$

$$(*) \Leftrightarrow 10(t^2 - 2) + 21t + 10 = 0 \Leftrightarrow 10t^2 + 21t - 10 = 0 \Leftrightarrow t = -\frac{5}{2} \vee t = \frac{2}{5} \text{ (loại).}$$

$$\text{Với } t = -\frac{5}{2} \Leftrightarrow q + \frac{1}{q} = -\frac{5}{2} \Leftrightarrow 2q^2 + 5q + 2 = 0 \Leftrightarrow q = -\frac{1}{2} \vee q = -2$$

- Nếu $q = -\frac{1}{2} \Rightarrow u_1 = \frac{20}{q^2 + q^4} = \frac{20}{\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^4} = 64$

- Nếu $q = -2 \Rightarrow u_1 = \frac{20}{q^2 + q^4} = \frac{20}{2^2 + 2^4} = 1.$

$$4). u_1 + u_6 = 165; u_3 + u_4 = 60.$$

$$\Leftrightarrow \begin{cases} u_1 + u_1 q^5 = 165 \\ u_1 q^2 + u_1 q^3 = 60 \end{cases} \Leftrightarrow \begin{cases} u_1(1 + q^5) = 165 & (1) \\ u_1 q^2(1 + q) = 60 & (2) \end{cases}$$

$$\text{Lấy } \frac{(1)}{(2)} \Leftrightarrow \frac{1 + q^5}{q^2(1 + q)} = \frac{11}{4} \Leftrightarrow \frac{(1 + q)(1 - q + q^2 - q^3 + q^4)}{q^2(1 + q)} = \frac{11}{4}$$

$$\Leftrightarrow 4(1 - q + q^2 - q^3 + q^4) = 11q^2 \Leftrightarrow 4q^4 - 4q^3 - 7q^2 - 4q + 4 = 0$$

$$\Leftrightarrow \frac{4q^4}{q^2} - \frac{4q^3}{q^2} - \frac{7q^2}{q^2} - \frac{4q}{q^2} + \frac{4}{q^2} = 0 \Leftrightarrow 4\left(q^2 + \frac{1}{q^2}\right) - 4\left(q + \frac{1}{q}\right) - 7 = 0 \quad (*)$$

$$\text{Đặt: } t = q + \frac{1}{q} \Rightarrow t^2 = \left(q + \frac{1}{q}\right)^2 \Rightarrow q^2 + \frac{1}{q^2} = t^2 - 2. \text{ Điều kiện: } |t| \geq 2.$$

$$(*) \Leftrightarrow 4(t^2 - 2) - 4t - 7 = 0 \Leftrightarrow 4t^2 - 4t - 15 = 0 \Leftrightarrow t = \frac{5}{2} \vee t = -\frac{3}{2} \text{ (loại).}$$

$$\text{Với } t = \frac{5}{2} \Leftrightarrow q + \frac{1}{q} = \frac{5}{2} \Leftrightarrow 2q^2 - 5q + 2 = 0 \Leftrightarrow q = 2 \vee q = \frac{1}{2}$$

- với $q = 2 \Rightarrow u_1 = \frac{165}{1 + q^5} = \frac{165}{1^2 + 2^5} = 5$
- với $q = \frac{1}{2} \Rightarrow u_1 = \frac{165}{1 + q^2} = \frac{165}{1 + \left(\frac{1}{2}\right)^5} = 160.$

$$5). \begin{cases} u_1 + u_2 + u_3 + u_4 = 15 \\ u_1^2 + u_2^2 + u_3^2 + u_4^2 = 85. \end{cases} \Leftrightarrow \begin{cases} u_1 + u_1 q + u_1 q^2 + u_1 q^3 = 15 \\ u_1^2 + u_1^2 q^2 + u_1^2 q^4 + u_1 q^6 = 85. \end{cases}$$

$$\Leftrightarrow \begin{cases} u_1(1+q+q^2+q^3)=15 \\ u_1^2(1+q^2+q^4+q^6)=85. \end{cases} \Leftrightarrow \begin{cases} u_1^2(1+q+q^2+q^3)^2=15^2 & (1) \\ u_1^2(1+q^2+q^4+q^6)=85 & (2). \end{cases}$$

$$\text{Lấy } \frac{(1)}{(2)} \Leftrightarrow \frac{(1+q+q^2+q^3)^2}{1+q^2+q^4+q^6} = \frac{45}{17} \Leftrightarrow \frac{[(1+q)+q^2(1+q)]^2}{(1+q^2)+q^4(1+q^2)} = \frac{45}{17}$$

$$\Leftrightarrow \frac{[(1+q)(1+q^2)]^2}{(1+q^2)+(1+q^4)} = \frac{45}{17} \Leftrightarrow \frac{(1+q)^2(1+q^2)}{1+q^4} = \frac{45}{17} \Leftrightarrow \frac{(1+2q+q^2)(1+q^2)}{1+q^4} = \frac{45}{17}$$

$$\Leftrightarrow 17(1+q^2+2q+2q^3+q^2+q^4) = 45(1+q^4)$$

$$\Leftrightarrow 28q^4 - 34q^3 - 34q^2 - 34q + 28 = 0 \Leftrightarrow \frac{28q^4}{q^2} - \frac{34q^3}{q^2} - \frac{34q^2}{q^2} - \frac{34q}{q^2} + \frac{28}{q^2} = 0 \text{ (vì dễ dàng thấy } q \neq 0)$$

$$\Leftrightarrow 28q^2 - 34q - 34 - \frac{34}{q} + 28 = 0 \Leftrightarrow 14\left(q^2 + \frac{1}{q^2}\right) - 17\left(q + \frac{1}{q}\right) - 17 = 0 \quad (*)$$

$$\text{Đặt } t = q + \frac{1}{q} \Rightarrow t^2 = \left(q + \frac{1}{q}\right)^2 \Rightarrow q^2 + \frac{1}{q^2} = t^2 - 2. \text{ Điều kiện: } |t| \geq 2.$$

$$(*) \Leftrightarrow 14(t^2 - 2) - 17t - 17 = 0 \Leftrightarrow 14t^2 - 17t - 45 = 0 \Leftrightarrow t = \frac{5}{2} \vee t = -\frac{9}{7} \text{ (loại)}$$

$$\text{Với } t = \frac{5}{2} \Rightarrow q + \frac{1}{q} = \frac{5}{2} \Leftrightarrow 2q^2 - 5q + 2 = 0 \Leftrightarrow q = 2 \vee q = \frac{1}{2}$$

$$\bullet \text{ với } q = 2 \Rightarrow u_1 = 1. \quad \bullet \text{ với } q = \frac{1}{2} \Rightarrow u_1 = \frac{15}{1+q+q^2+q^3} = 8.$$

$$6). \begin{cases} u_1 + u_2 + u_3 = 13 \\ u_4 + u_5 + u_6 = 351 \end{cases} \Leftrightarrow \begin{cases} u_1(1+q+q^2) = 13 & (*) \\ u_1q^3(1+q+q^2) = 351 & (**) \end{cases}$$

$$\text{Lấy } \frac{(**)}{(*)} \Leftrightarrow q^3 = 27 \Rightarrow q = 3 \Rightarrow u_1 = \frac{13}{1+q+q^2} = \frac{13}{1+3+9} = 1.$$

$$7). \begin{cases} 8u_2 + 5\sqrt{5}u_5 = 0 & (1) \\ u_1^3 + u_3^3 = 189. \end{cases}$$

$$(1) \Leftrightarrow \begin{cases} 8u_1q - 5\sqrt{5}u_1q^4 = 0 \\ u_1^3 + (u_1q^2)^3 = 189. \end{cases} \Leftrightarrow \begin{cases} 8 = 5\sqrt{5}q^3 \Rightarrow q^3 = \frac{8}{5\sqrt{5}} = \left(\frac{2}{\sqrt{5}}\right)^3 \Rightarrow q = \frac{2}{\sqrt{5}} \\ u_1^3(1+q^6) = 189 \Rightarrow u_1^3 = \frac{189}{1+q^6} = 125 \Rightarrow u_1 = 5. \end{cases}$$

$$8). \begin{cases} u_1u_2u_3 = 1728 & (1) \\ u_1 + u_2 + u_3 = 63 \end{cases}$$

$$(1) \Leftrightarrow \begin{cases} u_1 \cdot u_1 \cdot q \cdot u_1 \cdot q^2 = 1728 \\ u_1 + u_1q + u_1q^2 = \sqrt{3} \end{cases} \Leftrightarrow \begin{cases} (u_1q)^3 = 12^3 \\ u_1(1+q+q^2) = 63 \end{cases} \Leftrightarrow \begin{cases} u_1q = 12 \\ u_1(1+q+q^2) = 63 \end{cases}$$

$$\Leftrightarrow \begin{cases} u_1 = \frac{12}{q} \\ \frac{12}{q}(1+q+q^2) = 63 \end{cases} \Leftrightarrow \begin{cases} u_1 = \frac{12}{q} \\ 12q^2 - 51q + 12 = 0 \end{cases} \Leftrightarrow \begin{cases} q = 4 \Rightarrow u_1 = 3 \\ q = \frac{1}{4} \Rightarrow u_1 = 48. \end{cases}$$

$$9). \begin{cases} u_1 + u_3 = 3 \\ u_1^2 + u_3^2 = 5 \end{cases} \Leftrightarrow \begin{cases} u_1(1+q^2) = 3 \\ u_2(1+q^4) = 5 \end{cases} \Leftrightarrow \begin{cases} u_1^2(1+q^2)^2 = 9 \quad (*) \\ u_1^2(1+q^4) = 5 \quad (**) \end{cases}$$

Lấy $\frac{(*)}{(**)} \Leftrightarrow \frac{(1+q^2)^2}{1+q^4} = \frac{9}{5}$. Đặt: $t = q^2, t \geq 0$.

$$\Leftrightarrow 5(1+t)^2 = 9(1+t^2) \Leftrightarrow 4t^2 - 10t + 4 = 0 \Leftrightarrow t = 2 \vee t = \frac{1}{2}$$

Với $t = 2 \Rightarrow q = \pm\sqrt{2}$

• $q = \sqrt{2} \Rightarrow u_1 = \frac{3}{1+q^2} = 1$ • $q = -\sqrt{2} \Rightarrow u_1 = \frac{3}{1+q^2} = 1$

Với $t = \frac{1}{2} \Rightarrow q = \pm\frac{\sqrt{2}}{2}$

• $q = \frac{\sqrt{2}}{2} \Rightarrow u_1 = \frac{3}{1+q^2} = 2$ • $q = -\frac{\sqrt{2}}{2} \Rightarrow u_1 = \frac{3}{1+q^2} = 2$.

$$10). \begin{cases} u_1 + u_2 + u_3 = 7 \\ u_1^2 + u_2^2 + u_3^2 = 21 \end{cases} \Leftrightarrow \begin{cases} u_1 + u_1q + u_1q^2 = 7 \\ u_1^2 + (u_1q)^2 + (u_1q^2)^2 = 21 \end{cases}$$

$$\Leftrightarrow \begin{cases} u_1(1+q+q^2) = 7 \\ u_1^2(1+q^2+q^4) = 21 \end{cases} \Leftrightarrow \begin{cases} u_1^2(1+q+q^2)^2 = 49 \quad (*) \\ u_1^2(1+q^2+q^4) = 21 \quad (**) \end{cases} \cdot \text{Lấy } \frac{(*)}{(**)} \text{ được:}$$

$$\frac{(1+q+q^2)^2}{1+q^2+q^4} = \frac{49}{21} \Leftrightarrow 21(1+q^2+q^4+2q+2q^2+2q^3) = 49(1+q^2+q^4)$$

$$\Leftrightarrow 21(1+2q+3q^2+2q^3+q^4) = 49(1+q^2+q^4) \Leftrightarrow 28q^4 - 42q^3 - 14q^2 - 42q + 28 = 0.$$

$$\Leftrightarrow \frac{28q^4}{q^2} - \frac{42q^3}{q^2} - \frac{14q^2}{q^2} - \frac{42q}{q^2} + \frac{28}{q^2} = 0 \Leftrightarrow 28q^2 - 42q - 14 - \frac{42}{q} + \frac{28}{q^2} = 0$$

$$\Leftrightarrow 28\left(q^2 + \frac{1}{q^2}\right) - 42\left(q + \frac{1}{q}\right) - 14 = 0 \quad (2)$$

Đặt: $t = q + \frac{1}{q} \Rightarrow t^2 = \left(q + \frac{1}{q}\right)^2 \Rightarrow q^2 + \frac{1}{q^2} = t^2 - 2$. Điều kiện: $|t| \geq 2$

$$(2) \Leftrightarrow 28(t^2 - 2) - 42t - 14 = 0 \Leftrightarrow 28t^2 - 42t - 70 = 0 \Leftrightarrow t = \frac{5}{2} \vee t = -1 \text{ (loại)}$$

Với $t = \frac{5}{2} \Leftrightarrow q + \frac{1}{q} = \frac{5}{2} \Leftrightarrow 2q^2 - 5q + 2 = 0 \Leftrightarrow q = 2 \vee q = \frac{1}{2}$

• $q = 2 \Rightarrow u_1 = \frac{7}{1+q+q^2} = 1$ • $q = \frac{1}{2} \Rightarrow u_1 = \frac{7}{1+q+q^2} = 4$