Câu 11: Tìm các giới hạn sau:

a). $\lim\left(\sqrt[3]{8n^3+3n^2-2}+5-2n\right)$ b) $\lim\left(\sqrt[3]{8n^3+3n^2-2}+\sqrt[3]{5n^2-8n^3}\right)$ c) $\lim\left[n.\left(\sqrt[3]{n^3+n}-n\right)\right]$

d). $\lim\left(\sqrt[3]{8n^3+2n^2-1}+3-2n\right)$

LỜI GIẢI

a). $\lim\left(\sqrt[3]{8n^3+3n^2-2}+5-2n\right)=\lim\left(\sqrt[3]{8n^3+3n^2-2}-2n\right)+5$

$$=\lim\frac{8n^3+3n^2-2-8n^3}{\left(\sqrt[3]{8n^3+3n^2-2}\right)^2+\sqrt[3]{8n^3+3n^2-2}.2n+4n^2}+5$$

$$=\lim\frac{3n^2-2}{\left(\sqrt[3]{n^3\left(8+\frac{3}{n}-\frac{2}{n^3}\right)}\right)^2+\sqrt{n^3\left(8+\frac{3}{n}-\frac{2}{n^3}\right)}.2n+4n^2}+5$$

$$=\lim\frac{n^2\left(3-\frac{2}{n^2}\right)}{n^2\left[\left(\sqrt[3]{8+\frac{3}{n}-\frac{2}{n^3}}\right)^2+\sqrt[3]{8+\frac{3}{n}-\frac{2}{n^3}}.2+4\right]}+5$$

$$=\lim\frac{3-\frac{2}{n^2}}{\left(\sqrt[3]{8+\frac{3}{n}-\frac{2}{n^2}}\right)+\sqrt[3]{8+\frac{3}{n}-\frac{2}{n^3}}.2+4}+5=\frac{3}{4+4+4}+5=\frac{1}{4}+5=\frac{21}{4}.$$

b). $\lim\left(\sqrt[3]{8n^3+3n^2-2}+\sqrt[3]{5n^2-8n^3}\right)$

$$=\lim\frac{8n^3+3n^2-2+5n^2-8n^3}{\left(\sqrt[3]{8n^3+3n^2-2}\right)^2-\sqrt[3]{8n^3+3n^2-2}.\sqrt[3]{5n^2-8n^3}+\left(\sqrt[3]{5n^2-8n^3}\right)^2}$$

$$=\lim\frac{8n^2-2}{\left(\sqrt[3]{n^3\left(8+\frac{3}{n}-\frac{2}{n^3}\right)}\right)^2-\sqrt[3]{n^3\left(8+\frac{3}{n}-\frac{2}{n^3}\right)}\sqrt[5]{n^3\left(\frac{5}{n}-8\right)}+\left(\sqrt[3]{n^3\left(\frac{5}{n}-8\right)}\right)^2}$$

$$=\lim\frac{n^2\left(8-\frac{2}{n^2}\right)}{n^2\left[\left(\sqrt[3]{8+\frac{3}{n}-\frac{2}{n^2}}\right)^2-\sqrt[3]{8+\frac{3}{n}-\frac{2}{n^3}}.\sqrt[3]{\frac{5}{n}-8}+\left(\sqrt[3]{\frac{5}{n}-8}\right)^3\right]}$$

$$=\lim\frac{8-\frac{2}{n^2}}{\left(\sqrt[3]{8+\frac{3}{n}-\frac{2}{n^2}}\right)^2-\sqrt[3]{8+\frac{3}{n}-\frac{2}{n^3}}.\sqrt[3]{\frac{5}{n}-8}+\left(\sqrt[3]{\frac{5}{n}-8}\right)^3}=\frac{8}{4+4+4}=\frac{2}{3}.$$

c) $\lim\left[n.\left(\sqrt[3]{n^3+n}-n\right)\right]=\lim\dfrac{n\left(n^3+n-n^3\right)}{\left(\sqrt[3]{n^3+n}\right)^2+\sqrt[3]{n^3+n}.n+n^2}$

$$= \lim \frac{n^2}{\left(\sqrt[3]{n^3\left(1+\frac{2}{n^2}\right)}\right)^2 + \sqrt[3]{n^3\left(1+\frac{1}{n^2}\right)}.n + n^2}$$

$$= \lim \frac{n^2}{n^2\left[\left(\sqrt[3]{1+\frac{2}{n^2}}\right)^2 + \sqrt[3]{1+\frac{1}{n^2}} + 1\right]} = \lim \frac{1}{\left(\sqrt[3]{1+\frac{2}{n^2}}\right)^2 + \sqrt[3]{1+\frac{1}{n^2}} + 1} = \frac{1}{3}.$$

d). Hoàn toàn tương tự câu a).

Câu 12: Tìm các giới hạn sau:

a). $\lim \frac{1}{\sqrt{n+2}-\sqrt{n+1}}$ b). $\lim \frac{1}{\sqrt{3n^2+2n}-\sqrt{3n^2+1}}$

c). $\lim\left(n+\sqrt[3]{1-n^3}\right)$ d) $\lim\left(\sqrt[3]{8n^3+3n^2+4}-2n+1\right)$

LỜI GIẢI

a). $\lim \frac{1}{\sqrt{n+2}-\sqrt{n+1}} = \lim \frac{\sqrt{n+2}+\sqrt{n+1}}{n+2-(n+1)}$

$$= \lim\left(\sqrt{n\left(1+\frac{2}{n}\right)} + \sqrt{n\left(1+\frac{1}{n}\right)}\right) = \lim \sqrt{n}\left(\sqrt{1+\frac{1}{n}} + \sqrt{1+\frac{1}{n}}\right)$$

$$= \lim\left(2\sqrt{n}\right) = +\infty.$$

b). $\lim \frac{1}{\sqrt{3n^2+2n}-\sqrt{3n^2+1}} = \lim \frac{\sqrt{3n^2+2n}+\sqrt{3n^2+1}}{\left(3n^2+2\right)-\left(3n^2+1\right)}$

$$= \lim \frac{\sqrt{n^2\left(3+\frac{2}{n}\right)} + \sqrt{n^2\left(3+\frac{1}{n^2}\right)}}{2n-1} = \lim \frac{n\left(\sqrt{3+\frac{2}{n}} + \sqrt{3+\frac{1}{n^2}}\right)}{n\left(2-\frac{1}{n}\right)}$$

$$= \lim \frac{\sqrt{3+\frac{2}{n}} + \sqrt{3+\frac{1}{n^2}}}{2-\frac{2}{n}} = \frac{\sqrt{3}+\sqrt{3}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}.$$

c). $\lim\left(n+\sqrt[3]{1-n^3}\right) = \lim \frac{n^3+1-n^3}{n^2-n\sqrt[3]{1-n^3}+\left(\sqrt[3]{1-n^3}\right)^2}$

$$= \lim \frac{1}{n^2 - n\sqrt[3]{n^3\left(\frac{1}{n^3}-1\right)} + \left(\sqrt[3]{n^3\left(\frac{1}{n^3}-1\right)}\right)^2}$$

$$= \lim \frac{1}{n^2\left[1-\sqrt[3]{\frac{1}{n^3}-1}+\left(\sqrt[3]{\frac{1}{n^3}-1}\right)^2\right]} = \lim \frac{1}{3n^2} = 0.$$

d). $\lim\left(\sqrt[3]{8n^3+3n^2+4}-2n+1\right) = \lim\left(\sqrt[3]{8n^3+3n^2+4}-2n\right)+1$

$$=\lim \frac{8n^3+3n^2+4-8n^3}{\left(\sqrt[3]{8n^3+3n^2+4}\right)^2+\sqrt[3]{8n^3+3n^2+4}.2n+4n^2}+1$$

$$=\lim \frac{3n^2+4}{n^2\left(\sqrt[3]{8+\frac{3}{n}+\frac{4}{n^3}}\right)^2+2n^2.\sqrt[3]{8+\frac{3}{n}+\frac{4}{n^3}}+4n^2}+1$$

$$=\lim \frac{n^2\left(3+\frac{4}{n^2}\right)}{n^2\left(\sqrt[3]{8+\frac{3}{n}+\frac{4}{n^3}}\right)^2+2n^2.\sqrt[3]{8+\frac{3}{n}+\frac{4}{n^3}}+4n^2}+1$$

$$=\lim \frac{3+\frac{4}{n^2}}{\left(\sqrt[3]{8+\frac{3}{n}+\frac{4}{n^3}}\right)^2+2.\sqrt[3]{8+\frac{3}{n}+\frac{4}{n^3}}+4}+1=\frac{1}{4}+1=\frac{5}{4}.$$

Câu 13*: Tìm các giới hạn sau:

a) $\lim \frac{\sqrt{4n^2+1}-2n}{\sqrt{n^2+4n+1}-n}$ b) $\lim \frac{\sqrt{4n^2+1}-2n}{\sqrt[3]{n^3+4n+1}-n}$

c) $\lim \frac{n\left(\sqrt[3]{4-n^3}+n\right)}{\sqrt{4n^2+1}-2n}$ d) $\lim \frac{n^2+\sqrt[3]{1-n^6}}{\sqrt{n^4+1}-n^2}$

LỜI GIẢI

a) $\lim \frac{\sqrt{4n^2+1}-2n}{\sqrt{n^2+4n+1}-n}$

Ta có: $\sqrt{4n^2+1}-2n=\frac{4n^2+1-4n^2}{\sqrt{4n^2+1}+2n}=\frac{1}{\sqrt{4n^2+1}+2n}$

Ta có: $\frac{1}{\sqrt{n^2+4n+1}-n}=\frac{\sqrt{n^2+4n+1}+n}{n^2+4n+1-n^2}=\frac{\sqrt{n^2+4n+1}+n}{4n+1}$

Vậy $\lim \frac{\sqrt{n^2+4n+1}+n}{\left(\sqrt{4n^2+1}+2n\right)(2n+1)}=\lim \frac{\sqrt{n^2\left(1+\frac{4}{n}+\frac{1}{n^2}\right)}+n}{\left(\sqrt{n^2\left(4+\frac{1}{n^2}\right)}+2n\right)(2n+1)}$

$$=\lim \frac{n\left[\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1\right]}{n\left(\sqrt{4+\frac{1}{n^2}}+2\right)n\left(2+\frac{1}{n}\right)}=\lim \frac{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1}{n\left(\sqrt{4+\frac{1}{n^2}}+2\right)\left(2+\frac{1}{n}\right)}$$

$$=\lim \frac{2}{n(2+2)2}=\lim \frac{1}{4n}=0.$$

b). $\lim\frac{\sqrt{4n^2+1}-2n}{\sqrt[3]{n^3+4n+1}-n}=\lim\frac{\left(4n^2+1-4n^2\right)\left[\left(\sqrt[3]{n^3+4n+1}\right)^2+n\sqrt[3]{n^3+4n+1}+n^2\right]}{\left(\sqrt{4n^2+1}+2n\right)\left(n^3+4n+1-n^3\right)}$

$=\lim\frac{\left(\sqrt[3]{n^3\left(1+\frac{4}{n^2}+\frac{1}{n^3}\right)}\right)^2+n\sqrt{n^3\left(1+\frac{4}{n^2}+\frac{1}{n^3}\right)}+n^2}{\left(\sqrt{n^2\left(4+\frac{1}{n^2}\right)}+2n\right)(4n+1)}$

$=\lim\frac{n^2\left[\left(\sqrt[3]{1+\frac{4}{n^2}+\frac{1}{n^3}}\right)^2+\sqrt{1+\frac{4}{n^2}+\frac{1}{n^3}}+1\right]}{n\left(\sqrt{4+\frac{1}{n^2}}+2\right)n\left(4+\frac{1}{n}\right)}$

$=\lim\frac{\left(\sqrt[3]{1+\frac{4}{n^2}+\frac{1}{n^3}}\right)^2+\sqrt{1+\frac{4}{n^2}+\frac{1}{n^3}}+1}{\left(\sqrt{4+\frac{1}{n^2}}+2\right)\left(4+\frac{1}{n}\right)}=\lim\frac{1+1+1}{(2+2)4}=\frac{3}{16}.$

c). $\lim\frac{n\left(\sqrt[3]{4-n^3}+n\right)}{\sqrt{4n^2+1}-2n}=\lim\frac{n\left(4-n^3+n^3\right)}{\left(\sqrt[3]{4-n^3}\right)^2-n\sqrt[3]{4-n^3}+n^2}\cdot\frac{\sqrt{4n^2+1}+2n}{4n^2+1-4n^2}$

$=\lim\frac{4n\left[\sqrt{n^2\left(1+\frac{1}{n^2}\right)}+2n\right]}{\left(\sqrt[3]{n^3\left(\frac{4}{n^3}-1\right)}\right)^2-n\sqrt[3]{n^3\left(\frac{4}{n^3}-1\right)}+n^2}=\lim\frac{4n^2\left(\sqrt{4+\frac{1}{n^2}}+2\right)}{n^2\left[\left(\sqrt[3]{\frac{4}{n^3}-1}\right)^2-\sqrt[3]{\frac{4}{n^3}-1}+1\right]}$

$=\lim\frac{4\left(\sqrt{4+\frac{1}{n^2}}+2\right)}{\left(\sqrt[3]{\frac{4}{n^3}-1}\right)^2-\sqrt[3]{\frac{4}{n^3}-1}+1}=\frac{4(2+2)}{1+1+1}=\frac{16}{3}.$

d). $\lim\frac{n^2+\sqrt[3]{1-n^6}}{\sqrt{n^4+1}-n^2}=\lim\frac{n^6+1-n^6}{n^4-n^2.\sqrt[3]{1-n^6}}\cdot\frac{\sqrt{n^4+1}+n^2}{n^4+1-n^4}$

$=\lim\frac{\sqrt{n^4+1}+n^2}{n^4-n^2\sqrt[3]{1-n^6}+\left(\sqrt[3]{1-n^6}\right)^2}=\lim\frac{\sqrt{n^4\left(1+\frac{1}{n^4}\right)}+n^2}{n^4-n^2\sqrt[3]{1-n^6}+\left(\sqrt[3]{1-n^6}\right)^2}$

$=\lim\frac{n^2\left(\sqrt{1+\frac{1}{n^4}}+1\right)}{n^4\left[1-\sqrt[3]{\frac{1}{n^6}-1}+\left(\sqrt[3]{\frac{1}{n^6}-1}\right)^2\right]}=\lim\frac{2}{3n^2}=0.$