Bài 3: Tính đạo hàm của các hàm số sau:

a). $y=\left(x^7+x\right)^2$ b). $y=\left(2x^3-3x^2-6x+1\right)^2$ c). $y=\left(1-2x^2\right)^3$

d). $y=\left(x-x^2\right)^{32}$ e). $y=\left(x^2+x+1\right)^4$ f). $y=\left(x^2-x+1\right)^3.\left(x^2+x+1\right)^2$

g) $y=\left(\dfrac{2x+1}{x-1}\right)^3$ h). $y=\dfrac{1}{\left(x^2-x+1\right)^5}$ k). $y=\dfrac{\left(2-x^2\right)\left(3-x^3\right)}{1-x+x^2}$

l). $y=\left(1+2x\right)\left(2+3x^2\right)\left(3-4x^3\right)$

LỜI GIẢI

a). $y=\left(x^7+x\right)^2$. Sử dụng công thức $\left(u^\alpha\right)'=\alpha.u^{\alpha-1}.u'$ (với $u=x^7+x$)

$y'=2\left(x^7+x\right).\left(x^7+x\right)'=2\left(x^7+x\right)\left(7x^6+1\right)$

b). $y=\left(2x^3-3x^2-6x+1\right)^2$. Sử dụng công thức $\left(u^\alpha\right)'$ với $u=2x^3-3x^2-6x+1$

$y'=2\left(2x^3-3x^2+6x+1\right)\left(2x^3-3x^2+6x+1\right)'=2\left(2x^3-3x^2+6x+1\right)\left(6x^2-6x+6\right).$

c). $y=\left(1-2x^2\right)^3$. Sử dụng công thức $\left(u^\alpha\right)'$ với $u=1-2x^2$

$y'=3\left(1-2x^2\right)^2\left(1-2x^2\right)'=3\left(1-2x^2\right)^2\left(-4x\right)=-12x\left(1-2x^2\right)^2.$

d). $y=\left(x-x^2\right)^{32}$. Sử dụng công thức $\left(u^\alpha\right)'$ với $u=x-x^2$

$y'=32\left(x-x^2\right)^{31}.\left(x-x^2\right)'=32\left(x-x^2\right)^{31}.\left(1-2x\right)$

e). $y=\left(x^2+x+1\right)^4$. Sử dụng công thức $\left(u^\alpha\right)'$ với $u=x^2+x+1$

$y'=4\left(x^2+x+1\right)^3.\left(x^2+x+1\right)'=4\left(x^2+x+1\right)^3.\left(2x+1\right)$

f). $y=\left(x^2-x+1\right)^3.\left(x^2+x+1\right)^2$

Đầu tiên sử dụng quy tắc nhân.

$y'=\left[\left(x^2-x+1\right)^3\right]'\left(x^2+x+1\right)^2+\left[\left(x^2+x+1\right)^2\right]'\left(x^2-x+1\right)^3.$

Sau đó sử dụng công thức $\left(u^\alpha\right)'$

$y'=3\left(x^2-x+1\right)^2\left(x^2-x+1\right)'\left(x^2+x+1\right)+2\left(x^2+x+1\right)\left(x^2+x+1\right)'\left(x^2-x+1\right)^3$

$y'=3\left(x^2-x+1\right)^2\left(2x-1\right)\left(x^2+x+1\right)^2+2\left(x^2+x+1\right)\left(2x+1\right)\left(x^2-x+1\right)^3$

$y'=\left(x^2-x+1\right)^2\left(x^2+x+1\right)\left[3\left(2x-1\right)\left(x^2+x+1\right)+2\left(2x+1\right)\left(x^2-x+1\right)\right]$.

g) $y=\left(\dfrac{2x+1}{x-1}\right)^3$

Bước đầu tiên sử dụng $\left(u^\alpha\right)'$, với $u=\dfrac{2x+1}{x-1}$

$$y'=3.\left(\frac{2x+1}{x-1}\right)^2.\left(\frac{2x+1}{x-1}\right)'=3.\left(\frac{2x+1}{x-1}\right)^2.\frac{-1}{\left(x-1\right)^2}=-\frac{3\left(2x+1\right)^2}{\left(x-1\right)^4}.$$

h). $y=\dfrac{1}{\left(x^2-x+1\right)^5}$

Đầu tiên sử dụng công thức $\left(\dfrac{1}{u}\right)'$ với $u=\left(x^2-x+1\right)^5$

$$y'=-\frac{\left(\left(x^2-x+1\right)^5\right)'}{\left(\left(x^2-x+1\right)^5\right)^2}=\frac{-5\left(x^2-x+1\right)^4.\left(x^2-x+1\right)'}{\left(x^2-x+1\right)^{10}}=-\frac{5(2x-1)}{\left(x^2-x+1\right)^6}$$

k). $y=\dfrac{\left(2-x^2\right)\left(3-x^3\right)}{1-x+x^2}$

Đầu tiên sử dụng $\left(\dfrac{u}{v}\right)'$

$$y'=\frac{\left[\left(2-x^2\right)\left(3-x^3\right)\right]'.\left(1-x+x^2\right)-\left(1-x+x^2\right)'\left(2-x^2\right)\left(3-x^3\right)}{\left(1-x+x^2\right)^2}$$

Tính $\left[\left(2-x^2\right)\left(3-x^3\right)\right]'=\left(2-x^2\right)'\left(3-x^3\right)+\left(3-x^3\right)'\left(2-x^2\right)$

$=-2x\left(3-x^3\right)-3x^2\left(2-x^2\right)=5x^4-6x^2-6x.$

Vậy $y'=\dfrac{\left(5x^4-6x^2-6x\right)\left(1-x+x^2\right)-(-1+2x)\left(2-x^2\right)\left(3-x^3\right)}{\left(1-x+x^2\right)^2}$

l). $y=(1+2x)\left(2+3x^2\right)\left(3-4x^3\right)$

$y'=(1+2x)'\left(2+3x^2\right)\left(3-4x^3\right)+(1+2x)\left(2+3x^2\right)'\left(3-4x^3\right)+(1+2x)\left(2+3x^2\right)\left(3-4x^3\right)'$

$y'=2\left(2+3x^2\right)\left(3-4x^3\right)+(1+2x)(6x)\left(3-4x^3\right)+(1+2x)\left(2+3x^2\right)\left(-12x^2\right)$.

Bài 4: Tính đạo hàm của các hàm số sau:

a). $y=x^2+x\sqrt{x}+1$ b). $y=\sqrt{1+2x-x^2}$ c). $y=\sqrt{x^2+1}-\sqrt{1-x^2}$

d). $y=\sqrt{\dfrac{x^2+1}{x}}$ e). $y=\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)$ f) $y=\sqrt{x-1}+\dfrac{1}{\sqrt{x-1}}$

g). $y=\left(\sqrt{x}-\dfrac{1}{\sqrt{x}}\right)^5$ h). $y=\dfrac{1+x}{\sqrt{1-x}}$ i). $y=\sqrt{x+\sqrt{x+\sqrt{x}}}$

k). $y=\dfrac{4x+1}{\sqrt{x^2+2}}$ l). $y=\sqrt{\dfrac{x^3}{x-1}}$ m). $y=\sqrt{(x-2)^3}$ n) $y=\left(1+\sqrt{1-2x}\right)^3$

LỜI GIẢI

a). $y=x^2+x\sqrt{x}+1$

$$y'=\left(x^2\right)'+\left(x\sqrt{x}\right)'+1'=2x+x'.\sqrt{x}+\left(\sqrt{x}\right)'.x=2x+\sqrt{x}+\frac{1}{2\sqrt{x}}.x=2x+\frac{3\sqrt{x}}{2}.$$

b). $y=\sqrt{1+2x-x^2}$. Sử dụng công thức $\left(\sqrt{u}\right)'$ với $u=1+2x-x^2$

$$y' = \frac{\left(1+2x-x^2\right)'}{\sqrt{1+2x-x^2}} = \frac{1-x}{\sqrt{1+2x-x^2}}.$$

c). $y = \sqrt{x^2+1} - \sqrt{1-x^2}$

$$y' = \left(\sqrt{x^2+1}\right)' - \left(\sqrt{1-x^2}\right)' = \frac{\left(x^2+1\right)'}{2\sqrt{x^2+1}} - \frac{\left(1-x^2\right)'}{2\sqrt{1-x^2}} = \frac{x}{\sqrt{x^2+1}} + \frac{x}{\sqrt{1-x^2}}.$$

d). $y = \sqrt{\frac{x^2+1}{x}}$. Sử dụng công thức $\left(\sqrt{u}\right)'$ với $u = \frac{x^2+1}{x}$

$$y' = \frac{1}{2\sqrt{\frac{x^2+1}{x}}}.\left(\frac{x^2+1}{x}\right)' = \frac{1}{2\sqrt{\frac{x^2+1}{x}}}\left(1-\frac{1}{x^2}\right)$$

e). $y = \left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)$. Đầu tiên sử dụng công thức $\left(u^\alpha\right)'$ với $u = \frac{1-\sqrt{x}}{1+\sqrt{x}}$

$$y' = 2\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right).\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)'$$

Tính $\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)' = \frac{\left(1-\sqrt{x}\right)'\left(1+\sqrt{x}\right)-\left(1+\sqrt{x}\right)'\left(1-\sqrt{x}\right)}{\left(1+\sqrt{x}\right)^2}$

$$= \frac{\frac{-1}{2\sqrt{x}}\left(1+\sqrt{x}\right) - \frac{1}{2\sqrt{x}}\left(1-x\right)}{\left(1+\sqrt{x}\right)^2} = \frac{-1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}$$

Vậy $y' = 2\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right).\frac{-1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}$.

f). $y = \sqrt{x-1} + \frac{1}{\sqrt{x-1}}$

$$y' = \left(\sqrt{x-1}\right)' + \left(\frac{1}{\sqrt{x-1}}\right)' = \frac{1}{2\sqrt{x-1}} + \frac{-\left(\sqrt{x-1}\right)'}{\left(\sqrt{x-1}\right)^2} = \frac{1}{2\sqrt{x-1}} + \frac{-1}{2\sqrt{x-1}\left(x-1\right)}.$$

g). $y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^5$. Bước đầu tiên sử dụng $\left(u^\alpha\right)'$ với $u = \sqrt{x} - \frac{1}{\sqrt{x}}$

$$y' = 5\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^4.\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)' = 5\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^4.\left(\frac{1}{2\sqrt{x}} + \frac{\left(\sqrt{x}\right)'}{\left(\sqrt{x}\right)^2}\right)$$

$$= 5\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^4\left(\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x}.x}\right)$$

h). $y=\frac{1+x}{\sqrt{1-x}}$. Sử dụng $\left(\frac{u}{v}\right)'$ được: $y'=\frac{(1+x)'\sqrt{1-x}-\left(\sqrt{1-x}\right)'(1+x)}{\left(\sqrt{1-x}\right)^2}=\frac{\sqrt{1-x}-\frac{(1-x)'}{2\sqrt{1-x}}.(1+x)}{(1-x)}$

$=\frac{2(1-x)+(1+x)}{2\sqrt{1-x}.(1-x)}=\frac{3-x}{2\sqrt{1-x}(1-x)}$.

i) $y=\sqrt{x+\sqrt{x+\sqrt{x}}}$. Đầu tiên áp dụng $\sqrt{u}$ với $u=x+\sqrt{x+\sqrt{x}}$

$$y'=\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(x+\sqrt{x+\sqrt{x}}\right)'=\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}.\left(x+\sqrt{x}\right)'\right)$$

$$=\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}.\left[1+\frac{1}{2\sqrt{x+\sqrt{x}}}.\left(1+\frac{1}{2\sqrt{x}}\right)\right].$$

k). $y=\frac{4x+1}{\sqrt{x^2+2}}$ (áp dụng u chia v đạo hàm)

$$y'=\frac{(4x+1)'\sqrt{x^2+2}-\left(\sqrt{x^2+2}\right)'.(4x+1)}{\left(\sqrt{x^2+2}\right)^2}=\frac{4.\sqrt{x^2+2}-\frac{(x^2+2)'}{2\sqrt{x^2+2}}.(4x+1)}{(x^2+2)}$$

$$=\frac{4\sqrt{x^2+2}-\frac{x}{\sqrt{x^2+2}}(4x+1)}{x^2+2}=\frac{4(x^2+2)-x(4x+1)}{(x^2+2)\sqrt{x^2+2}}=\frac{-x+8}{(x^2+2)\sqrt{x^2+2}}$$

l). $y=\sqrt{\frac{x^3}{x-1}}$ (Áp dụng căn bậc hai của u đạo hàm).

$$y'=\frac{1}{2\sqrt{\frac{x^3}{x-1}}}.\left(\frac{x^3}{x-1}\right)'$$

Ta có: $\left(\frac{x^3}{x-1}\right)'=\frac{(x^3)'(x-1)-(x-1)'.x^3}{(x-1)^2}=\frac{3x^2(x-1)-x^3}{(x-1)^2}=\frac{2x^3-3x^2}{(x-1)^2}$

Vậy $y'=\frac{1}{2\sqrt{\frac{x^3}{x-1}}}.\frac{2x^3-3x^2}{(x-1)^2}$.

m). $y=\sqrt{(x-2)^3}$. Đầu tiên áp dụng $\left(\sqrt{u}\right)'$ với $u=(x-2)^3$

$$y'=\frac{1}{2\sqrt{(x-2)^3}}.\left((x-2)^3\right)'=\frac{1}{2\sqrt{(x-2)^3}}.3.(x-2)^2=\frac{3(x-2)}{2\sqrt{x-2}}.$$

n) $y=\left(1+\sqrt{1-2x}\right)^3$. Bước đầu tiên áp dụng $\left(u^\alpha\right)'$ với $u=1+\sqrt{1-2x}$

$$y'=3\left(1+\sqrt{1-2x}\right)^2.\left(1+\sqrt{1-2x}\right)'=3\left(1+\sqrt{1-2x}\right)^2.\frac{(1-2x)'}{2\sqrt{1-2x}}=\frac{-6\left(1+\sqrt{1-2x}\right)^2}{2\sqrt{1-2x}}.$$