

Bài 2: Giải các phương trình sau:

a) $\sin\left(3x + \frac{2\pi}{3}\right) = \cos\left(x - \frac{9\pi}{4}\right)$

b) $\sin\left(3x + \frac{2\pi}{3}\right) + \sin\left(x - \frac{7\pi}{5}\right) = 0$

c) $\sin\left(2x + \frac{\pi}{4}\right) + \cos x = 0$

d) $\sin^2\left(2x - \frac{\pi}{4}\right) = \frac{1}{2}$

e) $\sin^2\left(3x + \frac{2\pi}{3}\right) = \sin^2\left(\frac{7\pi}{5} - x\right)$

f) $\sin^2\left(5x + \frac{\pi}{3}\right) - \cos^2\left(3x + \frac{\pi}{4}\right) = 0$

g) $\sin\left(3x + \frac{\pi}{5}\right) + \sin\left(\frac{4\pi}{5} - 3x\right) = \sqrt{3}$

h) $\sin\left(\frac{4\pi}{9} + x\right) + \cos\left(\frac{\pi}{18} - x\right) = \sqrt{3}$

LỜI GIẢI

a). $\sin\left(3x + \frac{2\pi}{3}\right) = \cos\left(x - \frac{9\pi}{4}\right)$

$$\Leftrightarrow \sin\left(3x + \frac{2\pi}{3}\right) = \cos\left(x - 2\pi - \frac{\pi}{4}\right) \Leftrightarrow \sin\left(3x + \frac{2\pi}{3}\right) = \cos\left(x - \frac{\pi}{4}\right)$$

$$\Leftrightarrow \sin\left(3x + \frac{2\pi}{3}\right) = \sin\left[\frac{\pi}{2} - \left(x - \frac{\pi}{4}\right)\right] \Leftrightarrow \sin\left(3x + \frac{2\pi}{3}\right) = \sin\left(\frac{3\pi}{4} - x\right)$$

$$\Leftrightarrow \begin{cases} 3x + \frac{2\pi}{3} = \frac{3\pi}{4} - x + k2\pi \\ 3x + \frac{2\pi}{3} = \pi - \left(\frac{3\pi}{4} - x\right) + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{48} + \frac{k\pi}{2} \\ x = -\frac{5\pi}{24} + k\pi \end{cases} \quad (k \in \mathbb{Z}).$$

Kết luận: $x = \frac{\pi}{48} + \frac{k\pi}{2}, x = -\frac{5\pi}{24} + k\pi \quad (k \in \mathbb{Z})$.

b). $\sin\left(3x + \frac{2\pi}{3}\right) + \sin\left(x - \frac{7\pi}{5}\right) = 0 \Leftrightarrow \sin\left(3x + \frac{2\pi}{3}\right) + \sin\left(x - \pi - \frac{2\pi}{5}\right) = 0$

$$\Leftrightarrow \sin\left(3x + \frac{2\pi}{3}\right) - \sin\left(x - \frac{2\pi}{5}\right) = 0 \Leftrightarrow \sin\left(3x + \frac{2\pi}{3}\right) = \sin\left(x - \frac{2\pi}{5}\right)$$

$$\Leftrightarrow \begin{cases} 3x + \frac{2\pi}{3} = x - \frac{2\pi}{5} + k2\pi \\ 3x + \frac{2\pi}{3} = \pi - \left(x - \frac{2\pi}{5}\right) + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = -\frac{8\pi}{15} + k\pi \\ x = \frac{11\pi}{60} + \frac{k\pi}{2} \end{cases} \quad (k \in \mathbb{Z}).$$

Kết luận nghiệm của phương trình: $x = -\frac{8\pi}{15} + k\pi, x = \frac{11\pi}{60} + \frac{k\pi}{2}, (k \in \mathbb{Z})$.

c). $\sin\left(2x + \frac{\pi}{4}\right) + \cos x = 0 \Leftrightarrow \sin\left(2x + \frac{\pi}{4}\right) + \sin\left(\frac{\pi}{2} - x\right) = 0$

$$\Leftrightarrow \sin\left(2x + \frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{2} - x\right) \Leftrightarrow \sin\left(2x + \frac{\pi}{4}\right) = \sin\left(x - \frac{\pi}{2}\right)$$

$$\Leftrightarrow \begin{cases} 2x + \frac{\pi}{4} = x - \frac{\pi}{2} + k2\pi \\ 2x + \frac{\pi}{4} = \pi - \left(x - \frac{\pi}{2}\right) + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = -\frac{3\pi}{4} + k2\pi \\ x = \frac{5\pi}{12} + \frac{k2\pi}{3} \end{cases} \quad (k \in \mathbb{Z}).$$

$$\Leftrightarrow \begin{cases} 3x + \frac{\pi}{5} = \frac{\pi}{3} + k2\pi \\ 3x + \frac{\pi}{5} = \pi - \frac{\pi}{3} + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{2\pi}{45} + \frac{k2\pi}{3} \\ x = \frac{7\pi}{45} + \frac{k2\pi}{3} \end{cases}, (k \in \mathbb{Z}).$$

Kết luận nghiệm của phương trình: $x = \frac{2\pi}{45} + \frac{k2\pi}{3}, x = \frac{7\pi}{45} + \frac{k2\pi}{3} \quad (k \in \mathbb{Z})$.

h). $\sin\left(\frac{4\pi}{9} + x\right) + \cos\left(\frac{\pi}{18} - x\right) = \sqrt{3} (*)$

Các bạn để ý: $\left(\frac{4\pi}{9} + x\right) + \left(\frac{\pi}{18} - x\right) = \frac{\pi}{2} \Rightarrow \cos\left(\frac{\pi}{18} - x\right) = \sin\left(\frac{4\pi}{9} + x\right)$

$(*) \Leftrightarrow 2\sin\left(\frac{4\pi}{9} + x\right) = \sqrt{3} \Leftrightarrow \sin\left(x + \frac{4\pi}{9}\right) = \frac{\sqrt{3}}{2} \Leftrightarrow \sin\left(x + \frac{4\pi}{9}\right) = \sin\frac{\pi}{3}$

$$\Leftrightarrow \begin{cases} x + \frac{4\pi}{9} = \frac{\pi}{3} + k2\pi \\ x + \frac{4\pi}{9} = \pi - \frac{\pi}{3} + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = -\frac{\pi}{9} + k2\pi \\ x = \frac{2\pi}{9} + k2\pi \end{cases} \quad (k \in \mathbb{Z}).$$

Kết luận nghiệm của phương trình: $x = -\frac{\pi}{9} + k2\pi, x = \frac{2\pi}{9} + k2\pi \quad (k \in \mathbb{Z})$.