

$$10). 3 \cos x - 4 \sin x + \frac{2}{3 \cos x - 4 \sin x - 6} = 3 \quad (1)$$

$$\text{Đặt } t = 3 \cos x - 4 \sin x - 6 \Rightarrow 3 \cos x - 4 \sin x = t + 6$$

$$(1) \Leftrightarrow t + 6 + \frac{2}{t} = 3 \Leftrightarrow t^2 + 3t + 2 = 0 \Leftrightarrow \begin{cases} t = -1 \\ t = -2 \end{cases}$$

$$\text{Với } t = -1 \Leftrightarrow 3 \cos x - 4 \sin x = 5 \Leftrightarrow \frac{3}{5} \cos x - \frac{4}{5} \sin x = 1. \text{ Đặt } \cos \alpha = \frac{3}{5} \Rightarrow \sin \alpha = \frac{4}{5}.$$

$$\Leftrightarrow \cos x \cdot \cos \alpha - \sin x \cdot \sin \alpha = 1$$

$$\Leftrightarrow \cos(x + \alpha) = 1 \Leftrightarrow x + \alpha = \frac{\pi}{2} + k2\pi \Leftrightarrow x = -\alpha + \frac{\pi}{2} + k2\pi.$$

$$\text{Với } t = -2 \Leftrightarrow 3 \cos x - 4 \sin x = 4 \Leftrightarrow \frac{3}{5} \cos x - \frac{4}{5} \sin x = \frac{4}{5}. \text{ Đặt } \cos \alpha = \frac{3}{5} \Rightarrow \sin \alpha = \frac{4}{5}.$$

$$\Leftrightarrow \cos x \cdot \cos \alpha - \sin x \cdot \sin \alpha = \sin \alpha \Leftrightarrow \cos(x + \alpha) = \sin \alpha$$

$$\Leftrightarrow \cos(x - \alpha) = \cos\left(\frac{\pi}{2} - \alpha\right) \Leftrightarrow \begin{cases} x + \alpha = \frac{\pi}{2} - \alpha + k2\pi \\ x + \alpha = -\left(\frac{\pi}{2} - \alpha\right) + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{2} - 2\alpha + k2\pi \\ x = -\frac{\pi}{2} + k2\pi \end{cases}, (k \in \mathbb{Z}).$$

$$\text{Nghiem phương trình: } x = -\alpha + \frac{\pi}{2} + k2\pi, x = \frac{\pi}{2} - 2\alpha + k2\pi, x = -\frac{\pi}{2} + k2\pi, (k \in \mathbb{Z})$$

$$11). 2 \sin\left(\frac{\pi}{4} + x\right) + \sin\left(x - \frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2}.$$

$$\Leftrightarrow 2 \cdot \frac{\sin x + \cos x}{\sqrt{2}} + \frac{\sin x - \cos x}{\sqrt{2}} = \frac{3\sqrt{2}}{2} \Leftrightarrow 3 \sin x + \cos x = 3$$

$$\Leftrightarrow \frac{3}{\sqrt{10}} \sin x + \frac{1}{\sqrt{10}} \cos x = \frac{3}{\sqrt{10}}. \text{ Đặt } \frac{3}{\sqrt{10}} = \cos \alpha \Rightarrow \frac{1}{\sqrt{10}} = \sin \alpha$$

$$\Leftrightarrow \sin x \cdot \cos \alpha + \cos x \cdot \sin \alpha = \cos \alpha \Leftrightarrow \sin(x + \alpha) = \sin\left(\frac{\pi}{2} - \alpha\right)$$

$$\Leftrightarrow \begin{cases} x + \alpha = \frac{\pi}{2} - \alpha + k2\pi \\ x + \alpha = \pi - \left(\frac{\pi}{2} - \alpha\right) + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{2} - 2\alpha + k2\pi \\ x = \frac{\pi}{2} + k2\pi \end{cases}, (k \in \mathbb{Z})$$

$$\text{Nghiem phương trình: } x = \frac{\pi}{2} - 2\alpha + k2\pi, x = \frac{\pi}{2} + k2\pi, (k \in \mathbb{Z})$$

Câu 2: Giải các phương trình sau:

1). $\frac{\sqrt{3}-\sqrt{3}\cos 2x}{2\sin x} = \cos x$.

2). $\tan \frac{\pi}{7} \cdot \sin x + 2\cos^2 \frac{x}{2} = 2$

3). $\sqrt{2}(\cos^4 x - \sin^4 x) = \sin x + \cos x$

4). $\sqrt{3}\cos 2x + \sin 2x + 2\sin\left(2x - \frac{\pi}{6}\right) = 2\sqrt{2}$

5). $\sqrt{3}\sin 7x - \cos 7x = 2\sin\left(5x - \frac{\pi}{6}\right)$

6). $\sin\left(\frac{\pi}{2} + 2x\right) + \sqrt{3}\sin(\pi - 2x) = 2$

7). $\cos x + \sqrt{3}\sin x + 2\cos\left(2x + \frac{\pi}{3}\right) = 0$

8). $2\cos 2x = (1 + \sqrt{3})(\cos x - \sin x)$

9). $(\sqrt{3}-1)\sin x - (\sqrt{3}+1)\cos x = 1 - \sqrt{3}$.

10). $3\sin 3x - \sqrt{3}\cos 9x = 1 + 4\sin^3 3x$

LỜI GIẢI

1). $\frac{\sqrt{3}-\sqrt{3}\cos 2x}{2\sin x} = \cos x$ (1). Điều kiện $\sin x \neq 0 \Leftrightarrow x \neq k\pi$

(1) $\Leftrightarrow \sqrt{3} - \sqrt{3}\cos 2x = 2\sin x \cos x \Leftrightarrow \sqrt{3}\cos 2x + \sin 2x = \sqrt{3}$

$\Leftrightarrow \frac{\sqrt{3}}{2}\cos 2x + \frac{1}{2}\sin 2x = \frac{\sqrt{3}}{2} \Leftrightarrow \cos 2x \cdot \cos \frac{\pi}{6} + \sin 2x \cdot \sin \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\Leftrightarrow \cos\left(2x - \frac{\pi}{6}\right) = \cos \frac{\pi}{6} \Leftrightarrow \begin{cases} 2x - \frac{\pi}{6} = \frac{\pi}{6} + k2\pi \\ 2x - \frac{\pi}{6} = -\frac{\pi}{6} + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{6} + k\pi \\ x = k\pi \end{cases}, (k \in \mathbb{Z})$

So với điều kiện thì nghiệm $x = k\pi$ loại.

Vậy nghiệm phương trình: $x = \frac{\pi}{6} + k\pi, (k \in \mathbb{Z})$

2). $\tan \frac{\pi}{7} \cdot \sin x + 2\cos^2 \frac{x}{2} = 2 \Leftrightarrow \tan \frac{\pi}{7} \cdot \sin x + 1 + \cos x = 2 \Leftrightarrow \frac{\sin \frac{\pi}{7}}{\cos \frac{\pi}{7}} \sin x + \cos x = 1$

$\Leftrightarrow \sin \frac{\pi}{7} \sin x + \cos \frac{\pi}{7} \cos x = \cos \frac{\pi}{7} \Leftrightarrow \cos\left(x - \frac{\pi}{7}\right) = \cos \frac{\pi}{7} \Leftrightarrow \begin{cases} x - \frac{\pi}{7} = \frac{\pi}{7} + k2\pi \\ x - \frac{\pi}{7} = -\frac{\pi}{7} + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{2\pi}{7} + k2\pi \\ x = k2\pi \end{cases}$

Nghiệm phương trình: $x = \frac{2\pi}{7} + k2\pi, x = k2\pi, (k \in \mathbb{Z})$

3). $\sqrt{2}(\cos^4 x - \sin^4 x) = \sin x + \cos x$

$\Leftrightarrow \sqrt{2}(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = \sin x + \cos x$

$\Leftrightarrow \sqrt{2}\cos 2x = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$

$$\Leftrightarrow \cos 2x = \cos\left(x - \frac{\pi}{4}\right) \Leftrightarrow \begin{cases} 2x = x - \frac{\pi}{4} + k2\pi \\ 2x = -x + \frac{\pi}{4} + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = -\frac{\pi}{4} + k2\pi \\ x = \frac{\pi}{12} + \frac{k2\pi}{3} \end{cases}, (k \in \mathbb{Z})$$

Nghiệm của phương trình: $x = -\frac{\pi}{4} + k2\pi, x = \frac{\pi}{12} + \frac{k2\pi}{3}, (k \in \mathbb{Z})$

$$4). \sqrt{3} \cos 2x + \sin 2x + 2 \sin\left(2x - \frac{\pi}{6}\right) = 2\sqrt{2}$$

$$\Leftrightarrow \frac{\sqrt{3}}{2} \cos 2x + \frac{1}{2} \sin 2x + \sin\left(2x - \frac{\pi}{6}\right) = \sqrt{2}$$

$$\Leftrightarrow \cos 2x \cdot \cos \frac{\pi}{6} + \sin 2x \cdot \sin \frac{\pi}{6} + \sin\left(2x - \frac{\pi}{6}\right) = \sqrt{2}$$

$$\Leftrightarrow \cos\left(2x - \frac{\pi}{6}\right) + \sin\left(2x - \frac{\pi}{6}\right) = \sqrt{2}$$

$$\Leftrightarrow \frac{1}{\sqrt{2}} \cos\left(2x - \frac{\pi}{6}\right) + \frac{1}{\sqrt{2}} \sin\left(2x - \frac{\pi}{6}\right) = 1$$

$$\Leftrightarrow \cos\left(2x - \frac{\pi}{6}\right) \cdot \cos \frac{\pi}{4} + \sin\left(2x - \frac{\pi}{6}\right) \sin \frac{\pi}{4} = 1$$

$$\Leftrightarrow \cos\left(2x - \frac{\pi}{6} - \frac{\pi}{4}\right) = 1 \Leftrightarrow 2x - \frac{\pi}{6} - \frac{\pi}{4} = k2\pi \Leftrightarrow x = \frac{5\pi}{24} + k\pi.$$

Nghiệm của phương trình: $x = \frac{5\pi}{24} + k\pi.$

$$5). \sqrt{3} \sin 7x - \cos 7x = 2 \sin\left(5x - \frac{\pi}{6}\right)$$

$$\Leftrightarrow \frac{\sqrt{3}}{2} \sin 7x - \frac{1}{2} \cos 7x = \sin\left(5x - \frac{\pi}{6}\right)$$

$$\Leftrightarrow \sin 7x \cos \frac{\pi}{6} - \cos 7x \sin \frac{\pi}{6} = \sin\left(5x - \frac{\pi}{6}\right) \Leftrightarrow \sin\left(7x - \frac{\pi}{6}\right) = \sin\left(5x - \frac{\pi}{6}\right)$$

$$\Leftrightarrow \begin{cases} 7x - \frac{\pi}{6} = 5x - \frac{\pi}{6} + k2\pi \\ 7x - \frac{\pi}{6} = \pi - \left(5x - \frac{\pi}{6}\right) + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = k\pi \\ x = \frac{\pi}{9} + \frac{k\pi}{6} \end{cases}$$