

Câu 2: Tìm các giới hạn sau:

$$\begin{array}{lll} \text{a). } \lim_{x \rightarrow -\infty} (\sqrt{2x^2 + 1} + x) & \text{b). } \lim_{x \rightarrow +\infty} \frac{3x^2 - x + 3}{x - 4} & \text{c). } \lim_{x \rightarrow +\infty} \frac{x^4 - x^3 + 11}{2x - 7} \\ \text{d). } \lim_{x \rightarrow +\infty} \frac{\sqrt{2x^4 + x^2 - 1}}{1 - 2x} & \text{e). } \lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 - x}}{1 - 2x} & \text{f). } \lim_{x \rightarrow -\infty} \frac{2x^4 + 7x^3 - 15}{x^4 + 1}. \end{array}$$

### LỜI GIẢI

$$\text{a). } \lim_{x \rightarrow -\infty} (\sqrt{2x^2 + 1} + x) = \lim_{x \rightarrow -\infty} \left( \sqrt{x^2 \left( 2 + \frac{1}{x^2} \right)} + x \right) = \lim_{x \rightarrow -\infty} (\sqrt{2x^2} + x)$$

$$= \lim_{x \rightarrow -\infty} (|x| \sqrt{2} + x) = \lim_{x \rightarrow -\infty} (-\sqrt{2}x + x) = \lim_{x \rightarrow -\infty} x(-\sqrt{2} + 1) = +\infty$$

$$\text{b). } \lim_{x \rightarrow +\infty} \frac{3x^2 - x + 3}{x - 4} = \lim_{x \rightarrow +\infty} \frac{x^2 \left( 3 - \frac{1}{x} + \frac{3}{x^2} \right)}{x \left( 1 - \frac{1}{x} \right)} = \lim_{x \rightarrow +\infty} 3x = +\infty$$

$$\text{c). } \lim_{x \rightarrow +\infty} \frac{x^4 - x^3 + 11}{2x - 7} = \lim_{x \rightarrow +\infty} \frac{x^4 \left( 1 - \frac{1}{x} + \frac{11}{x^3} \right)}{x \left( 2 - \frac{7}{x} \right)} = \lim_{x \rightarrow +\infty} \frac{1}{2} x^3 = +\infty$$

$$\text{d). } \lim_{x \rightarrow +\infty} \frac{\sqrt{2x^4 + x^2 - 1}}{1 - 2x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 \left( 2 + \frac{1}{x^2} - \frac{1}{x^4} \right)}}{x \left( \frac{1}{x} - 2 \right)} = \lim_{x \rightarrow +\infty} \frac{x \sqrt{2 + \frac{1}{x^2} - \frac{1}{x^4}}}{\frac{1}{x} - 2} = \lim_{x \rightarrow +\infty} \frac{\sqrt{2}x}{-2} = -\infty$$

$$\text{e). } \lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 - x}}{1 - 2x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 \left( 1 - \frac{1}{x^3} \right)}}{x \left( \frac{1}{x} - 2 \right)} = \lim_{x \rightarrow +\infty} \frac{x \sqrt{1 - \frac{1}{x^3}}}{\frac{1}{x} - 2} = +\infty$$

$$\text{f). } \lim_{x \rightarrow -\infty} \frac{2x^4 + 7x^3 - 15}{x^4 + 1} = \lim_{x \rightarrow -\infty} \frac{x^4 \left( 2 + \frac{7}{x} - \frac{15}{x^3} \right)}{x^4 \left( 1 + \frac{1}{x^4} \right)} = 2$$