Câu 6: Tìm các giới hạn sau:

a). $\lim\dfrac{n^2+4n-5}{3n^3+n^2+7}$ b). $\lim\dfrac{-2n^2+n+2}{3n^4+5}$ c). $\lim\dfrac{\sqrt{2n^2-n}}{1-3n^2}$ d). $\lim\left(\dfrac{\sin 3n}{4n}-1\right)$

LỜI GIẢI

a). $\lim\dfrac{n^2+4n-5}{3n^3+n^2+7}=\lim\dfrac{1+\dfrac{4}{n}-\dfrac{5}{n^2}}{3n+1+\dfrac{7}{n}}=\lim\dfrac{1}{3n+1}=0.$

b). $\lim\dfrac{-2n^2+n+2}{3n^4+5}=\lim\dfrac{-2+\dfrac{1}{n}+\dfrac{2}{n^2}}{3n^2+\dfrac{5}{n^2}}=\lim\dfrac{-2}{3n^2}=0.$

c). $\lim\dfrac{\sqrt{2n^2-n}}{1-3n^2}=\lim\dfrac{\dfrac{\sqrt{2n^2-n}}{n}}{\dfrac{1}{n}-3n}=\lim\dfrac{\sqrt{2-\dfrac{1}{n}}}{\dfrac{1}{n}-3n}=\lim\dfrac{\sqrt{2}}{-3n}=0.$

d). $\lim\left(\dfrac{\sin 3n}{4n}-1\right)=\lim\dfrac{\sin 3n}{4n}-1$

Ta có: $-1\le\sin 3n\le 1 \;\Leftrightarrow -\dfrac{1}{4n}\le\dfrac{\sin 3n}{4n}\le\dfrac{1}{4n}$

Mà: $\lim\left(-\dfrac{1}{4n}\right)=\lim\dfrac{1}{4n}=0\Rightarrow\lim\dfrac{\sin 3n}{4n}=0.$ Vậy $\lim\left(\dfrac{\sin 3n}{4n}-1\right)=-1.$

Câu 7: Tìm các giới hạn sau:

a). $\lim\dfrac{1}{\sqrt{3n+2}-\sqrt{2n+1}}$ b). $\lim\dfrac{5}{4^n+2^n}$ c). $\lim\dfrac{3^n+5.4^n}{7^n+2^n}$ d). $\lim\dfrac{(-5)^n+4^n}{(-7)^{n+1}+4^{n+1}}$

LỜI GIẢI

a). $\lim\dfrac{1}{\sqrt{3n+2}-\sqrt{2n+1}}=\lim\dfrac{1}{\sqrt{n\left(3+\dfrac{2}{n}\right)}-\sqrt{n\left(2+\dfrac{1}{n}\right)}}$

$=\lim\dfrac{1}{\sqrt{n}\left(\sqrt{3+\dfrac{2}{n}}-\sqrt{2+\dfrac{1}{n}}\right)}=\lim\dfrac{1}{\sqrt{n}\left(\sqrt{3}-\sqrt{2}\right)}=0.$

b). $\lim\dfrac{5}{4^n+2^n}=\lim\dfrac{5.\dfrac{1}{4^n}}{1+\left(\dfrac{1}{2}\right)^n}=0.$ Do $\lim\dfrac{1}{4^n}=\lim\left(\dfrac{1}{4}\right)^n=0$ và $\lim\left(\dfrac{1}{2}\right)^n=0$.

c). $\lim\frac{3^n+5.4^n}{7^n+2^n}=\lim\frac{4^n\left(\frac{3^n}{4^n}+5\right)}{7^n\left(1+\frac{2^n}{7^n}\right)}=\lim\left(\frac{4}{7}\right)^n\left(\frac{\left(\frac{3}{4}\right)^n+5}{1+\left(\frac{2}{7}\right)^n}\right)=0$. Do $\lim\left(\frac{3}{4}\right)^n=0$, $\lim\left(\frac{2}{7}\right)^n=0$ nên

$\lim\left(\frac{\left(\frac{3}{4}\right)^n+5}{1+\left(\frac{2}{7}\right)^n}\right)=5$ và $\lim\left(\frac{4}{7}\right)^n=0$. Nên $\lim u_n=0$.

d). $\lim\frac{(-5)^n+4^n}{(-7)^{n+1}+4^{n+1}}=\lim\frac{(-5)^n\left(1+\frac{4^n}{(-5)^n}\right)}{(-7)^n\left(-7+\frac{4.4^n}{(-7)^n}\right)}=\lim\left(\frac{5}{7}\right)^n\cdot\left(\frac{1+\left(-\frac{4}{5}\right)^n}{-7+4.\left(\frac{-4}{7}\right)^n}\right)$. Do $\lim\left(-\frac{4}{5}\right)^n=\lim\left(-\frac{4}{7}\right)^n=0$

nên $\lim\left(\frac{1+\left(-\frac{4}{5}\right)^n}{-7+4.\left(\frac{-4}{7}\right)^n}\right)=-\frac{1}{7}$ và $\lim\left(\frac{5}{7}\right)^n=0$.

Từ đó suy ra $\lim u_n=0$.

Câu 8: Tìm các giới hạn sau:

a). $\lim\left(\sqrt{n^2-n}-n\right)$ b). $\lim\left(\sqrt{n^2+n+1}-n\right)$

c). $\lim\left(\sqrt{4n^2+n}-\sqrt{4n^2+2}\right)$ d). $\lim\left[n\left(\sqrt{n^2+1}-\sqrt{n^2+2}\right)\right]$

LỜI GIẢI

a). $\lim\left(\sqrt{n^2-n}-n\right)=\lim\frac{n^2-n-n^2}{\sqrt{n^2-n}+n}$

$=\lim\frac{-n}{\sqrt{n^2\left(1-\frac{1}{n}\right)}+n}=\lim\frac{-n}{n\left(\sqrt{1-\frac{1}{n}}+1\right)}=\lim\frac{-1}{\sqrt{1-\frac{1}{n}}+1}=\frac{1}{2}$.

b). $\lim\left(\sqrt{n^2+n+1}-n\right)=\lim\frac{n^2+n+1-n^2}{\sqrt{n^2+n+1}+n}$

$=\lim\frac{n+1}{\sqrt{n^2\left(1+\frac{1}{n}+\frac{1}{n^2}\right)}+n}=\lim\frac{n\left(1+\frac{1}{n}\right)}{n\left(\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+1\right)}=\lim\frac{1+\frac{1}{n}}{\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+1}=\frac{1}{2}$.

c). $\lim\left(\sqrt{4n^2+n}-\sqrt{4n^2+2}\right)=\lim\frac{4n^2-\left(4n^2+2\right)}{\sqrt{4n^2+n}+\sqrt{4n^2+2}}$

$=\lim\frac{n-2}{\sqrt{n^2\left(4+\frac{1}{n}\right)}+\sqrt{n^2\left(4+\frac{2}{n^2}\right)}}=\lim\frac{n-2}{n\sqrt{4+\frac{1}{n}}+n\sqrt{4+\frac{2}{n^2}}}$

$$=\lim\frac{n\left(1-\frac{2}{n}\right)}{n\left(\sqrt{4+\frac{1}{n}}+\sqrt{4+\frac{2}{n^2}}\right)}=\lim\frac{1-\frac{2}{n}}{\sqrt{4+\frac{1}{n}}+\sqrt{4+\frac{2}{n^2}}}=\frac{1}{4}.$$

d). $\lim\left[n\left(\sqrt{n^2+1}-\sqrt{n^2+2}\right)\right]=\lim\frac{n\left[\left(n^2+1\right)-\left(n+2\right)\right]}{\sqrt{n^2+1}+\sqrt{n^2+2}}$

$$=\lim\frac{-n}{\sqrt{n^2\left(1+\frac{1}{n^2}\right)}+\sqrt{n^2\left(1+\frac{2}{n^2}\right)}}=\lim\frac{-n}{n\left(\sqrt{1+\frac{1}{n^2}}+\sqrt{1+\frac{2}{n^2}}\right)}$$

$$=\lim\frac{-1}{\sqrt{1+\frac{1}{n^2}}+\sqrt{1+\frac{2}{n^2}}}=\lim\frac{-1}{1+1}=-\frac{1}{2}.$$

Câu 9: Tìm các giới hạn sau:

a). $\lim\left(\sqrt{n^2+2n}-n+3\right)$ b). $\lim\left(\sqrt{4n^2+3n+1}-2n+1\right)$

c). $\lim\left(1+n^2-\sqrt{n^4+3n+1}\right)$ d). $\lim\left[n\left(\sqrt{n+1}-\sqrt{n}\right)\right]$.

LỜI GIẢI

a). $\lim\left(\sqrt{n^2+2n}-n+3\right)=\lim\left(\sqrt{n^2+2n}-n\right)+3$

$$=\lim\frac{n^2+2n-n^2}{\sqrt{n^2+2n}+n}+3=\lim\frac{2n}{\sqrt{n^2\left(1+\frac{2}{n}\right)}+n}+3$$

$$=\lim\frac{2n}{n\left(\sqrt{1+\frac{2}{n}}+1\right)}+3=\lim\frac{2}{\sqrt{1+\frac{2}{n}}+1}+3=\frac{2}{1+1}+3=4.$$

b). $\lim\left(\sqrt{4n^2+3n+1}-2n+1\right)=\lim\left(\sqrt{4n^2+3n+1}-2n\right)+1$

$$=\lim\frac{4n^2+3n+1-4n^2}{\sqrt{4n^2+3n+1}+2n}+1=\lim\frac{3n+1}{\sqrt{n^2\left(4+\frac{3}{n}+\frac{1}{n^2}\right)}+2n}+1=\frac{3}{2+2}+1=\frac{7}{4}.$$

c). $\lim\left(1+n^2-\sqrt{n^4+3n+1}\right)=1+\lim\left(n^2-\sqrt{n^4+3n+1}\right)$

$$=1+\lim\frac{n^4-\left(n^4+3n+1\right)}{n^2+\sqrt{n^4+3n+1}}=1+\lim\frac{-3n-1}{n^2+\sqrt{n^4\left(1+\frac{3}{n^3}+\frac{1}{n^4}\right)}}$$

$$=1+\lim\frac{n\left(-3-\frac{1}{n}\right)}{n^2\left(1+\sqrt{1+\frac{3}{n^2}+\frac{1}{n^4}}\right)}=1+\lim\frac{-3}{n}=1+0=1.$$

d). $\lim\left[n\left(\sqrt{n+1}-\sqrt{n}\right)\right]=\lim\frac{n\left(n+1-n\right)}{\sqrt{n+1}+\sqrt{n}}$

$$=\lim\frac{n}{\sqrt{n\left(1+\frac{1}{n}\right)}+\sqrt{n}}=\lim\frac{n}{\sqrt{n}\left(\sqrt{1+\frac{1}{n}}+1\right)}=\lim\frac{\sqrt{n}}{2}=+\infty.$$

Câu 10: Tìm các giới hạn sau:

a). $\lim\left(\sqrt[3]{n+2}-\sqrt[3]{n}\right)$ b). $\lim\left(\sqrt[3]{n-n^3}+n+2\right)$

c). $\lim\left(\sqrt[3]{2n-n^3}+n-1\right)$ d). $\lim\left(\sqrt[3]{n^3-2n^2}-n-1\right)$

LỜI GIẢI

a). $\lim\left(\sqrt[3]{n+2}-\sqrt[3]{n}\right)=\lim\dfrac{n+2-n}{\left(\sqrt[3]{n+2}\right)^2+\sqrt[3]{n+2}.\sqrt[3]{n}+\left(\sqrt[3]{n}\right)^2}$

$$=\lim\frac{2}{\left(\sqrt[3]{n\left(1+\frac{2}{n}\right)}\right)^2+\sqrt[3]{n\left(1+\frac{2}{n}\right)}.\sqrt[3]{n}+\left(\sqrt[3]{n}\right)^2}$$

$$=\lim\frac{2}{\left(\sqrt[3]{n}\right)^2\left[\left(\sqrt[3]{1+\frac{2}{n}}\right)^2+\sqrt[3]{1+\frac{2}{n}}+1\right]}=\lim\frac{2}{3\left(\sqrt[3]{n}\right)^2}=0.$$

b). $\lim\left(\sqrt[3]{n-n^3}+n+2\right)=\lim\left(\sqrt[3]{n-n^3}+n\right)+2$

$$=\lim\frac{n-n^3+n^3}{\left(\sqrt[3]{n-n^3}\right)^2-\sqrt[3]{n-n^3}.n+n^2}+2=\lim\frac{n}{\left(\sqrt[3]{n^3\left(\frac{1}{n^2}-1\right)}\right)^2-\sqrt[3]{n^3\left(\frac{1}{n^2}-1\right)}.n+n^2}+2$$

$$=\lim\frac{n}{n^2\left[\left(\sqrt[3]{\frac{2}{n^2}-1}\right)^2-\sqrt[3]{\frac{1}{n^2}-1}+1\right]}+2=\lim\frac{1}{3n}+2=0+2=2.$$

c). $\lim\left(\sqrt[3]{2n-n^3}+n-1\right)=\lim\left(\sqrt[3]{2n-n^3}+n\right)-1$

$$=\lim\frac{2n-n^3+n^3}{\left(\sqrt[3]{2n-n^3}\right)^2-\sqrt[3]{2n-n^3}.n+n^2}-1=\lim\frac{2n}{\left(\sqrt[3]{n^3\left(\frac{2}{n^2}-1\right)}\right)^3-\sqrt[3]{n^3\left(\frac{2}{n^2}-1\right)}.n+n^2}-1$$

$$=\lim\frac{2n}{n^2\left[\left(\sqrt[3]{\frac{2}{n^2}-1}\right)^2-\sqrt[3]{\frac{2}{n^2}-1}+1\right]}-1=\lim\frac{2}{3n}-1=0-1=-1.$$

d) $\lim\left(\sqrt[3]{n^3-2n^2}-n-1\right)=\lim\left(\sqrt[3]{n^3-2n^2}-n\right)-1$

$$=\lim\frac{n^3-2n^2-n^3}{\left(\sqrt[3]{n^3-2n}\right)^2+\sqrt[3]{n^3-2n}.n+n^2}-1=\lim\frac{-2n^2}{\left(\sqrt[3]{n^3\left(1-\frac{2}{n^2}\right)}\right)^2+\sqrt{n^3\left(1-\frac{2}{n^2}\right)}.n+n^2}-1$$

$$=\lim\frac{-2n^2}{n^2\left[\left(\sqrt[3]{1-\frac{2}{n^2}}\right)^2+\sqrt[3]{1-\frac{2}{n^2}}+1\right]}-1=\lim\frac{-2}{\left(\sqrt[3]{1-\frac{2}{n^2}}\right)^2+\sqrt[3]{1-\frac{2}{n^2}}+13}-1=-\frac{2}{3}+1=\frac{1}{3}.$$